## Measurement of the CKM angle $\alpha$ at BABAR

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We present BABAR experiment studies to measure the CKM angle $\alpha$ of the Unitarity Triangle. The measurements are based on the $B$ meson decays into the two-body state $(\pi \pi)$, the quasi two-body state ( $\rho \rho$ ), and the three-body state $\left(\pi^{+} \pi^{-} \pi^{0}\right.$ ). The results are obtained from data samples of about 230 million $\Upsilon(4 S) \rightarrow B \bar{B}$ decays collected between 1999 and 2004 with the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC.
With $B^{0} \rightarrow \pi^{+} \pi^{-}$decays, we obtain for the $C P$-violating parameters: $S_{\pi \pi}=-0.30 \pm 0.17 \pm 0.03$ and $C_{\pi \pi}=-0.09 \pm 0.15 \pm 0.04$. A full isospin analysis based of the measurements of all the branching $B(\bar{B}) \rightarrow \pi \pi$, gives an upper bound on the angle difference $\left|\alpha_{\text {eff }}^{\pi \pi}-\alpha\right|$ of $35^{\circ}$ at $90 \%$ CL. With $B^{0} \rightarrow \rho^{+} \rho^{-}$decays, we measure the longitudinal polarization fraction $f_{L}=0.978 \pm$ $0.014_{-0.029}^{+0.021}$ and the $C P$-violating parameters $S_{\rho \rho}^{L}=-0.33 \pm 0.24_{-0.14}^{+0.08}$ and $C_{\rho \rho}^{L}=-0.03 \pm 0.18 \pm$ 0.09 . Using an isospin analysis of $B(\bar{B}) \rightarrow \rho \rho$ decays we constrain the angle $\alpha$. The solution compatible with the Standard Model is $\alpha=(100 \pm 13)^{\circ}$.
Finally, the time-dependent CP analysis of the $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay over the Dalitz plot gives a value of $\alpha=\left(113_{-17}^{+27} \pm 6\right)^{\circ}$.
Combining all constraints on $\alpha$ determined at $B A B A R$, we obtain a value of $\alpha=\left(103_{-9}^{+11}\right)^{\circ}$ in good agreement with the global CKM fit using other world measurements.

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## 1. Introduction

The study of $B$ meson decays into charmless hadronic final states $(b \rightarrow u)$ plays an important role in the understanding of $C P$ violation in the $B$ system. In the Standard Model, $C P$ violation arises from a single complex phase in the Cabibbo-Kobayashi-Maskawa quark-mixing matrix $V_{\mathrm{ij}}$ [1].

Measurements of the time-dependent $C P$-violating asymmetry in the $B^{0} \rightarrow \pi^{+} \pi^{-}$and $B^{0} \rightarrow$ $\rho^{+} \rho^{-}$decays provide information on the angle $\alpha \equiv \arg \left[-V_{\mathrm{td}} V_{\mathrm{tb}}^{*} / V_{\mathrm{ud}} V_{\mathrm{ub}}^{*}\right]$ of the Unitarity Triangle. However, in contrast to the theoretically clean determination of the angle $\beta$ in $B^{0}$ decays to charmonium final states, the extraction of $\alpha$ in $B^{0} \rightarrow h^{+} h^{-}$where $h^{+} h^{-}$is $\pi^{+} \pi^{-}$or $\rho^{+} \rho^{-}$, is complicated by the interference of tree and penguin amplitudes with different weak phases. The contribution of the latter may be evaluated assuming isospin symmetry and using measurements of the branching fractions of the isospin-related decays $B^{0}\left(\bar{B}^{0}\right) \rightarrow h^{0} h^{0}, B^{0}\left(\bar{B}^{0}\right) \rightarrow h^{+} h^{-}$and $B^{ \pm} \rightarrow h^{ \pm} h^{0}$ [2]. Another method [3], also based on isospin asymmetry, allows us to measure directly $\alpha$ by studying $B^{0}\left(\bar{B}^{0}\right) \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decays over the Dalitz plot.

This paper summarizes the analyses related to the determination of the $\alpha$ angle of BABAR experiment, for the three decay modes $\left(\pi \pi, \rho \rho\right.$ and $\left.\pi^{+} \pi^{-} \pi^{0}\right)$ of $B$ mesons. These analyses are performed with about 230 million $B \bar{B}$ pairs collected between 1999 and 2004 with the BABAR detector [4]. The three analyses share the same philosophy and use the same techniques. Full descriptions of these analyses are available in Ref. [5, 6, 7].

## 2. Analysis Overview

Signal decays are identified kinematically using two variables, the difference $\Delta E$ between the center-of-mass (CM) energy of the $B_{\mathrm{rec}}$ candidate and $\sqrt{s} / 2$, and $m_{\mathrm{ES}}=\sqrt{\left(s / 2+\mathbf{p}_{i} \cdot \mathbf{p}_{B}\right)^{2} / E_{i}^{2}-\mathbf{p}_{B}^{2}}$, the beam-energy substituted mass, where $\sqrt{s}$ is the total CM energy, and the $B_{\text {rec }}$ momentum $\mathbf{p}_{\mathbf{B}}$ and the four-momentum of the initial state $\left(E_{i}, \mathbf{p}_{\mathbf{i}}\right)$ are defined in the laboratory frame. The jet-like background from $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ (continuum) is suppressed by its topology. In the CM frame, we define discriminating variables based on the event shapes. These variables are combined in a single variable $x_{\text {sep }}$, either a Fisher discriminant [5] or a neural network output [6, 7].

The time difference $\Delta t$ is obtained from the known boost of the $e^{+} e^{-}$system and the measured distance between the $z$ positions of the $B_{\mathrm{rec}}$ and $B_{\mathrm{tag}}$ decay vertices. A detailed description of this algorithm is given in Ref. [8]. To determine the flavor of the $B_{\text {tag }}$ we use the tagging algorithm of Ref. [8]. This produces five mutually exclusive tagging categories.

We use unbinned extended maximum likelihood fits to extract yields and $C P$-violating parameters. The likelihood combines the various discriminating variables: $m_{\mathrm{ES}}, \Delta E, x_{\text {sep }}, \Delta t$, Dalitz plot variables for $\pi^{+} \pi^{-} \pi^{0}$ analysis, resonance mass and helicity angles for $\rho \rho$ analysis. Each component of the data sample (correctly and mis-reconstructed signal, continuum, charm and charmless $B$ backgrounds) has its own modeling in the likelihood.

## 3. Measurement of $\alpha$ with $B \rightarrow \pi \pi$ and $B \rightarrow \rho \rho$ decays

With $\Delta t \equiv t_{h h}-t_{\text {tag }}$ defined as the proper time interval between the decay of the reconstructed
$B_{h^{+} h^{-}}^{0}$ and that of the other meson $B_{\text {tag }}^{0}$, the time-dependent decay rates are given by

$$
\begin{equation*}
f_{Q_{\mathrm{tag}}}^{h^{+} h^{-}}=\frac{e^{-|\Delta t| / \tau}}{4 \tau}\left[1+Q_{\mathrm{tag}} S_{h h} \sin \left(\Delta m_{d} \Delta t\right)-Q_{\mathrm{tag}} C_{h h} \cos \left(\Delta m_{d} \Delta t\right)\right] \tag{3.1}
\end{equation*}
$$

where where $Q_{\mathrm{tag}}=1(-1)$ when the tagging meson $B_{\mathrm{tag}}^{0}$ is a $B^{0}\left(\bar{B}^{0}\right), \tau$ is the mean $B^{0}$ lifetime and $\Delta m_{d}$ is the mixing frequency due to the eigenstate mass difference. If the decay proceeds purely through the $b \rightarrow u$ tree amplitude, $C_{h h}=0$ and $S_{h h}=\sin (2 \alpha)$. In general, the $b \rightarrow d$ penguin amplitude may be not negligible, so that $C_{h h}$ which probes direct $C P$ violation, may be not equal to zero and $S_{h h}=\sqrt{1-C_{h h}^{2}} \sin 2 \alpha_{\text {eff }}^{h h}$, where $2 \alpha_{\text {eff }}^{h h}=\arg \left[(q / p)\left(\bar{A}_{h h} / A_{h h}\right)\right], \arg [q / p]$ is the $B^{0} \bar{B}^{0}$ mixing phase, and $A_{h h}\left(\bar{A}_{h h}\right)$ are the transition amplitudes of the processes $B^{0}\left(\bar{B}^{0}\right) \rightarrow h^{+} h^{-}$, respectively.
$B \rightarrow \rho \rho$ analyses are experimentally more challenging than the $B \rightarrow \pi \pi$ analyses as the final states consist of four pions, including two $\pi^{0}$ for the $\rho^{+} \rho^{-}$mode. The wide $\rho$ resonances result in more background. These vector-vector modes are also not CP eigenstates. But as they are almost longitudinally polarized ( $f_{L} \simeq 100 \%$ ), an analysis of the sole longitudinal (CP-even) component is adequate. On the other hand, as the $B \rightarrow \rho \rho$ branching ratio is about 6 times larger than for $B \rightarrow \pi \pi$, the sensitivity on $C P$-violating parameters $S_{h h}$ and $C_{h h}$ are quite similar. The maximum likelihood fits give $S_{\pi \pi}=-0.30 \pm 0.17 \pm 0.03, C_{\pi \pi}=-0.09 \pm 0.15 \pm 0.04$ for $\pi^{+} \pi^{-}$events [5] and $S_{\rho \rho}^{L}=-0.33 \pm 0.24_{-0.14}^{+0.08}, C_{\rho \rho}^{L}=-0.03 \pm 0.18 \pm 0.09$ with $f_{L}=0.978 \pm 0.014_{-0.029}^{+0.021}$ for $\rho^{+} \rho^{-}$ events [6].

Assuming isospin symmetry [2], the measurement of the branching fractions of $B(\bar{B})$ meson decays to $h h$ allows us to estimate the shift between $\alpha_{\text {eff }}^{h h}$ and $\alpha$. From a maximum likelihood fit, we measure the following branching fractions [5]: $\mathscr{B}\left(B^{0} \rightarrow \pi^{+} \pi^{-}\right)=(4.7 \pm 0.6 \pm 0.2) \times 10^{-6}$, $\mathscr{B}\left(B^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}\right)=(5.8 \pm 0.6 \pm 0.4) \times 10^{-6}$ and $\mathscr{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=(1.17 \pm 0.32 \pm 0.10) \times 10^{-6}$ with $C_{00}=-0.12 \pm 0.56 \pm 0.06$. Similarly, we obtain $\mathscr{B}\left(B^{0} \rightarrow \rho^{+} \rho^{-}\right)=(30 \pm 4 \pm 5) \times 10^{-6}$ and an upper limit of $1.1 \times 10^{-6}$ at $90 \%$ Confidence Level (CL) on $\mathscr{B}\left(B^{0} \rightarrow \rho^{0} \rho^{0}\right)$. Finally, we use the averages of BELLE and BABAR measurements [9]: $B R\left(B^{+} \rightarrow \rho^{+} \rho^{0}\right)=[26.4 \pm 6.4] 10^{-6}$ and $f_{L}\left(\rho^{+} \rho^{0}\right)=0.96_{-0.07}^{+0.05}$.

With these branching fractions, for $\pi \pi$ decays, the isospin relations give an upper limit of $\left|\alpha_{\text {eff }}^{\pi \pi}-\alpha\right|<35^{\circ}$ at $90 \% \mathrm{CL}$. On the other hand, as $\rho \rho$ decays benefit from a very low upper limit on $\mathscr{B}\left(B^{0} \rightarrow \rho^{0} \rho^{0}\right)$ compared to $\mathscr{B}\left(B^{ \pm} \rightarrow \rho^{ \pm} \rho^{0}\right)$, the penguin pollution is much smaller. In consequence, we obtain a stronger constraint on $\alpha$ as shown on the left plot of Fig. 1. Selecting the solution closest to the CKM combined fit average, we find $\alpha=100^{\circ} \pm 13^{\circ}$

## 4. Measurement of $\alpha$ with $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decays.

The $B^{0} \rightarrow \rho^{+} \pi^{+}$decay has no final CP eigenstate like $\pi^{+} \pi^{-}$or $\rho^{+} \rho^{-}$. An isospin analysis would not constrain sufficiently the many amplitudes of the $B^{0,+}$ decays to $\rho^{+} \pi^{-}, \rho^{-} \pi^{+}, \rho^{0} \pi^{0}$, $\rho^{+} \pi^{0}, \rho^{0} \pi^{+}$and their charge conjugates. A better approach [3] is based on the time-dependent analysis of the $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decay over the Dalitz plot, using the isospin symmetry as an additional constraint. As this $B \rightarrow 3 \pi$ decay is dominated by $\rho \pi$ resonances, its amplitude is a function of well-known kinematic functions of the Dalitz variables and of the $B^{0} \rightarrow \rho \pi$ amplitudes, themselves functions of $\alpha$ and tree and penguin contributions. The time-dependent CP analysis of the
$B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ decays then provides enough constraints to extract $\alpha$ and the tree and penguin amplitudes. This measurement [7] gives a value of $\alpha=\left(113_{-17}^{+27} \pm 6\right)^{\circ}$ (see right plot of Fig. 1).



Figure 1: Confidence level on $\alpha$ for $B \rightarrow \rho \rho$ analysis (right plot) and for $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ analysis (left plot). The dashed lines correspond to the $68 \%$ (top) and $90 \%$ (bottom) CL intervals.

## 5. Summary of the BABAR results on $\alpha$

Combining all constraints on $\alpha$ obtained at $B A B A R$, gives a value of $\alpha=\left(103_{-9}^{+11}\right)^{\circ}$ in good agreement with the global CKM fit using other world measurements. The $\rho \rho$ mode gives the best single measurement, but has mirror solutions that are disfavored thanks to the Dalitz analysis results. The contribution to the constraint from the $\pi \pi$ modes is limited, mostly due to the large penguin pollution.

Accuracy will improve in the future with more data, and with updates of the $\rho^{+} \rho^{0}$ and $\rho^{+} \rho^{-}$ branching ratios using our full data sample. The measurement of the $\rho^{0} \rho^{0}$ branching ratio is the limiting factor. Once the $\rho^{0} \rho^{0}$ channel is observed, a time-dependent CP analysis could also provide additional constraints.

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