Properties of the SU(N_c) Gluon Plasma

Saumen Datta[*] **and Sourendu Gupta**

Tata Institute of Fundamental Research
Homi Bhabha Road, Mumbai 400005, India.
E-mail: saumen@theory.tifr.res.in, sgupta@tifr.res.in

We investigate the deconfinement transition in SU(N_c) gauge theories, and properties of the deconfined phase. A detailed lattice study of SU(4) and SU(6) gauge theories are conducted, and finite volume and cutoff effects on thermodynamic observables are studied. The scaling of the deconfinement transition point with lattice spacing is used to calculate $\Lambda_{\overline{MS}}$. The continuum estimates of the thermodynamic quantities are used to study properties of the gluon plasma. In particular, the approach to conformal limit is studied. We do not find any evidence of a strongly coupled, conformal phase in these theories.

---

[*]Speaker.

1. Introduction

At very high temperatures, strongly interacting matter is known to exist in a deconfined, chirally symmetric state. The nature of the transition to this state is quite sensitive to the quark sector. For physical quarks, one has no phase transition, only a crossover. For very light quarks one expects a chiral symmetry restoring transition, whose order will depend on the mass and the number of quark flavors. On the other hand, for pure SU(3) gauge theory, which is the limit of QCD with infinitely heavy quarks, one has a first order, deconfining transition. See Ref. [1] for further discussions of the phase diagram.

While the nature of the transition is sensitive to the quark content, one can expect similarities in the properties of the high temperature phase of the pure gauge theory and of QCD. In particular, while at very high temperatures, the theories are expected to be weakly coupled due to asymptotic freedom, at moderately high temperatures $\sim 1 - 3T_c$, large deviations from the weak coupling theory are seen in both theories. An understanding of the interactions in the pure gauge theory plasma will surely help in our understanding of the QCD plasma.

A study of the SU(N_c) gauge theory in the limit of large N_c may be of help in this respect. Large N_c arguments have been used to understand various features of the SU(3) deconfinement transition [2]. Also, interesting phase structures have been suggested for SU(N_c) theory with quarks, when the baryon number chemical potential $\mu_B \sim N_c$ [3]. Analogies have also been drawn between the high temperature phase of QCD and the analytically tractable, $\mathcal{N} = 4$ supersymmetric SU(∞) theory [4]. Of course, connecting the SU($N_c \to \infty$) theory to SU(3) will require that the corrections in $1/N_c$ are small. An estimate of the size of the corrections can be obtained by studying SU(N_c) theories with different N_c on lattice.

In this paper, we report on a study of the SU(N_c) gauge theories at finite temperatures for $N_c = 4,6$. In Sec. 2 we study the deconfinement transition. Using a finite volume analysis at different lattice spacings, we establish the first order nature of the transition. We also study whether asymptotic scaling holds in the transition regime, and use it for setting the scale through $T_c/\Lambda_{\overline{MS}}$. More details about the material in this section can be found in Ref. [5]. In Sec. 3 we study the equation of state of the SU(N_c) gluon plasma. A comparison of the energy density and pressure for SU(3,4,6) gives us an idea of the size of the $1/N_c$ corrections. In the final section we discuss the physical implications of our results. Some of these results were earlier reported in ref. [1].

SU(N_c) gauge theories have been investigated previously on the lattice. In particular, the deconfinement transition has been studied in Ref. [6, 7], while the equation of state close to T_c has been studied in Ref. [8]. Our work focuses on the continuum limit, and also explores higher temperatures in the equation of state, which allows us to investigate the issue of approach to conformality. Equation of state at high temperatures have also been reported in this conference by Panero [9], who used a coarser lattice, but reported also results for SU(8).

2. T_c and scale setting

SU(3) gauge theory is known to have a weak first order transition, with the Polyakov loop,

$$L = \frac{1}{V} \sum_{\vec{x}} \frac{1}{N_c} \text{Tr} \prod_{i=1}^{N_t} U_t(\vec{x}, i) \tag{2.1}$$

acting as the order parameter. Here $V = N_s^3$ is the spatial volume and N_t is the temporal extent of the finite temperature lattice, and $U_\mu(\vec{x}, x_0)$ is the gauge link matrix connecting the sites $x = (\vec{x}, x_0)$ and $x + a\hat{\mu}$, where a is the lattice spacing.

We have carried out a detailed study of the deconfinement transition in SU(4) and SU(6) gauge theories, on lattices with spacings of $a^{-1} \leq 6T_c$ with the Wilson action. Finite volume analysis was carried out on lattices with $N_t = 6$ and 8 by using various aspect ratios, $\zeta = N_s/N_t$, in the range $\sim$ 2.5 - 4. In the transition regime, L shows a clear signal of a discontinuous transition, tunnelling between the low temperature phase around $L = 0$ and N_c non-zero values, related by rotations of $2\pi/N_c$, for the high temperature phase. Figure 1 shows the histogram for $|L|$ for SU(4) gauge theory on $N_t = 8$ lattices, showing clearly the tunnelling between the confined and deconfined phases, with the two-peak structure getting sharper as one goes to larger lattices. This is the expected behavior for a first order transition. For a more quantitative analysis, we look at the susceptibility of the order parameter,

$$\chi_L = N_s^3 \left\{ \langle |L|^2 \rangle - \langle |L| \rangle^2 \right\} \tag{2.2}$$

For a first order transition, $\chi_L \sim V$. In Fig. 1 we also show the result of a multihistogramming analysis of χ_L/V for the same set, confirming the first order nature of the transition. More details, and figures for SU(6), can be obtained in Ref. [5].
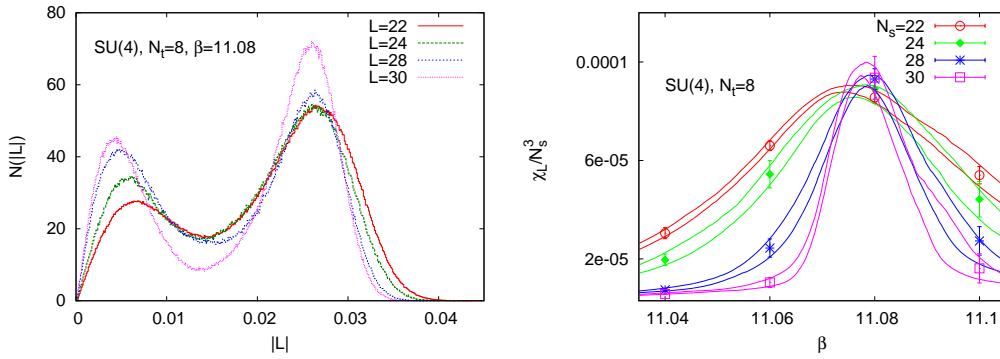

Figure 1: Finite size analysis of the deconfinement transition for SU(4) gauge theories, on $N_t = 8$ lattices. (Left) Histogram of $|L|$, for lattices with $N_s = 22$ - 30. (Right) Volume dependence of the $|L|$ susceptibility.

To confirm that one is looking at a physical transition, one needs to confirm that $\beta_c(N_t)$ has the proper scaling behavior, so that the transition temperature, $T_c = 1/N_t a(\beta_c)$, is independent of N_t. To study this scaling, we also add runs at $N_t = 10$ for both SU(4) and SU(6), and $N_t = 12$ for SU(4), at a single volume ($N_s = 24$). We use the two-loop renormalization group evolution (RGE), and use $\beta_c(N_t)$ for the different N_t to set a temperature scale. In Fig. 2 we show the temperature scale for $N_t = 6$, set by using the $\beta_c(N_t)$ for different N_t, for both SU(4) and SU(6), Here we have also included $\beta_c(N_t = 5)$, taken from Ref. [7]. As the figure reveals, while some deviations from scaling are seen at smaller N_t, in particular for $N_t = 5$, very good scaling is observed for $N_t \geq = 8$.

This allows us to use the data for $N_t \geq 8$ to get an estimate for the scale, $\Lambda_{\overline{MS}}$, of the theory [10]. We use the known expansions of plaquette to get the renormalized coupling, and use the two-loop running to convert it to $\Lambda_{\overline{MS}}$. Since we are performing the calculation at a finite order of the weak coupling expansion, we also investigate the scheme dependence, by calculating the renormalized
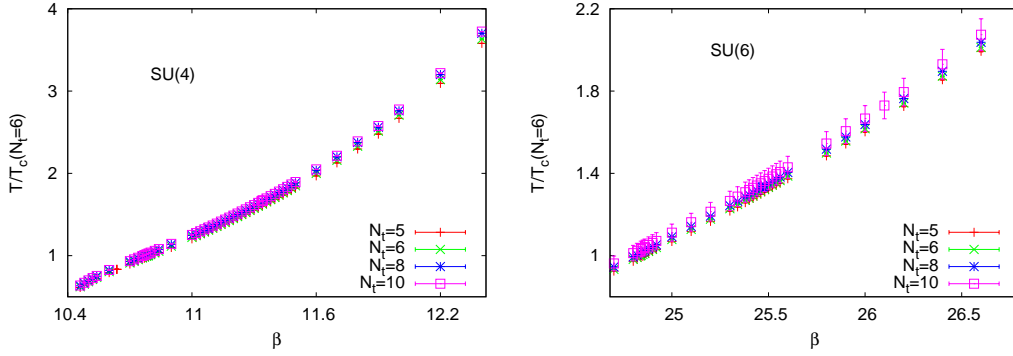
Figure 2: Temperature scale, $T/T_c(\beta)$, for $N_t = 6$, using $\beta_c(N_t)$ for different N_t, for (a) SU(4) and (b) SU(6).

N_c	E-scheme	V-scheme	$\overline{MS}$-scheme
3	1.19(3)	1.12(3)	1.20(2)
4	1.235(1)	1.153(1)	1.236(1)
6	1.222(1)	1.135(1)	1.217(1)

Table 1: Continuum estimate of $T_c/\Lambda_{\overline{MS}}$ for SU(N_c) gauge theories with $N_c = 3,4$ and 6. The renormalized couplings in different schemes are obtained from plaquette data, and converted to $a\Lambda$ using the two-loop RGE. These are converted to $T_c/\Lambda_{\overline{MS}}$ using the known relations between schemes. A constant fit to data for $N_t \geq 8$ is used for the continuum values.

coupling in some other scheme, getting the Λ parameter in this scheme, and then converting it to $\Lambda_{\overline{MS}}$. In table 1 we show the results of such estimates, where we have also used the E-scheme [11] and the V-scheme [12]. The data for $N_c = 3$ are taken from Ref. [13]. We find that there is considerable scheme dependence in the estimation of $T_c/\Lambda_{\overline{MS}}$. We also see that within a scheme, the dependence on the number of colors is quite weak.

3. Equation of State

For estimating the thermodynamic quantities, we use the integral method [13]. The pressure, p, and $\varepsilon - 3p$, where ε is the energy density, are calculated from the plaquette data, [13]

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = 6N_t^4 \int_{\beta_0}^{\beta} d\beta \, \Delta P(\beta, T), \qquad \frac{\varepsilon - 3p}{T^4} = 6N_t^4 \frac{\partial \beta}{\partial a} \Delta P(\beta, T) \qquad (3.1)$$

where $\Delta P(\beta, T)$ is the difference in the plaquette observables between the finte temperature lattice and the corresponding zero temperature (symmetric) lattice, calculated at the coupling β, $\Delta P(\beta, T) = P(\beta, T) - P(\beta, T = 0)$. T_0 is some reference temperature. We find that $p(T) \sim 0$ within our errors till temperatures $\sim 0.9 T_c$, and evaluate p/T^4 by taking $\beta_0(T < 0.8 T_c)$ as the lower limit of the integral. For $\frac{\partial \beta}{\partial a}$ we use the two-loop result, with the coupling defined through the V-scheme.

The results of Sec. 2 indicate that two-loop beta function should work fine when $a \leq 1/8 T_c$, while some cutoff effect may be seen in coarser lattices. Since we are interested in the thermodynamic and continuum limit results, we investigate the cutoff and finite volume effects by using two
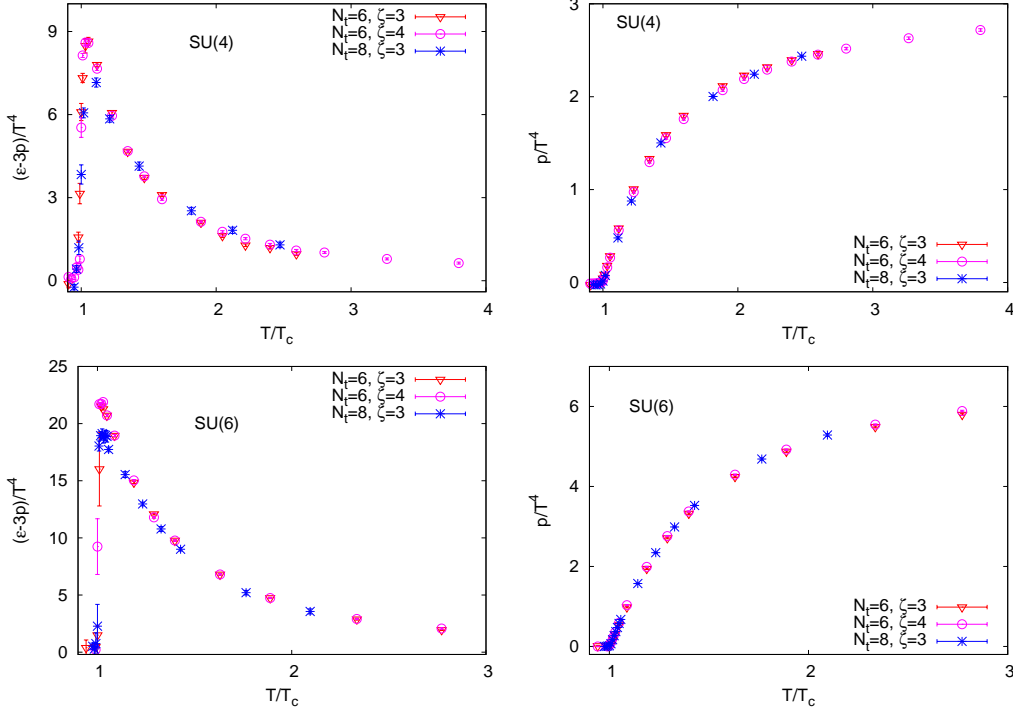
Figure 3: Volume and cutoff dependence on $(\varepsilon - 3p)/T^4$ (left) and p/T^4 (right) for SU(4) (top) and SU(6) (bottom) gauge theories. The results shown have been normalized by the known discretization error in free theory, so that the Stefan-Boltzmann limit for p/T^4 for both the N_t is $(N_c^2 - 1)\pi^2/45$ (see text).

aspect ratios and two cutoffs. Note that the Stefan-Boltzmann limit in the free theory has a known dependence on N_t: $\varepsilon_{SB} = 3p_{SB} = (N_c^2 - 1)\pi^2/15$. $R(N_t)$ where $R(N_t) = 1 + 8\pi^2/21N_t^2 + ...$ is the known discretization error in the integral method [14, 8]. When comparing data at different N_t in Fig. 3, we normalize them by this known discretization error.

As Fig. 3 indicates, there is a considerable cutoff effect in $\varepsilon - 3p$ in the temperature range $\leq 1.1T_c$ for the $N_t = 6$ lattice, for both SU(4) and SU(6). For $T \geq 1.3T_c$ for $N_t = 6$, the cutoff effect is small, which is also expected since $a = 1/7.8T_c$ and is near the scaling regime. For pressure, the cutoff effect is already seen to be small at T_c for $N_t = 6$, for both the theories. We also see that within the statistical uncertainties, volume dependence is small already for an aspect ratio $\zeta = 3$.

In Fig. 4 we show the results for SU(N_c) thermodynamics for $N_c = 3$, 4 and 6. For $N_c = 3$, we have taken the plaquette data from Ref. [13] and analyzed it similarly to $N_c = 4$ and 6. To emphasize both the deviation from the Stefan-Boltzmann limit, and to look at evidence of corrections to the leading N_c^2 behavior of the extensive quantities, we plot the energy density and pressure normalized by their Stefan-Boltzmann values, $\varepsilon/\varepsilon_{SB}$ and p/p_{SB}. Two comments are in order here. First, The thermodynamic quantities differ from their Stefan-Boltzmann values considerably even for temperatures near 4 T_c. Also, the approach to the Stefan-Boltzmann limit is slow. Second, the correction to the leading N_c^2 behavior is small, as there is no systematic change between $N_c = 3$ to 6. Note that a subleading correction is expected to be $\mathcal{O}(1/N_c^2)$, and therefore, four times larger in SU(3) than in SU(6).
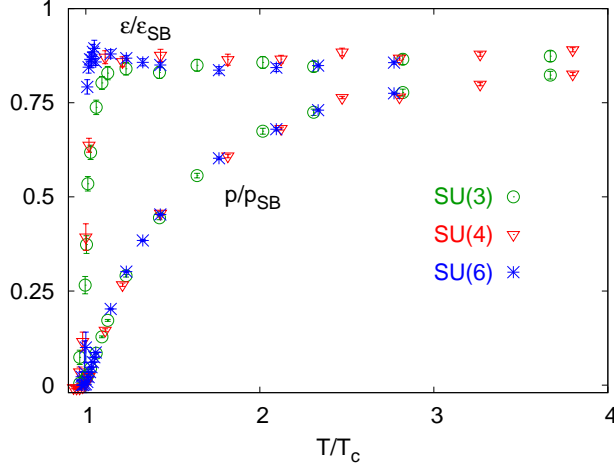
Figure 4: Results for energy density and pressure for SU(3-6) gauge theories, normalized to the corresponding Stefan-Boltzmann values. For SU(3), we have used the plaquette data of Ref. [13].

4. Summary and Discussion

In this report we have studied the deconfinement transition in SU(N_c) gauge theories, with emphasis on the continuum results. We have studied in detail the transition point, using a finite volume analysis to establish the first order nature of the transition for $N_c = 4$ and 6. We have also found that for $N_t \geq 8$, the two-loop RGE works quite well at the transition point. We have used this to set the scale of the theory, by transmuting the coupling at the deconfinement transition, β_c, to get $T_c/\Lambda_{\overline{MS}}$. We find this quantity to be very weakly dependent on N_c.

Next we have studied the thermodynamics of the deconfined plasma. Our results for scaling imply that for calculating the thermodynamic quantities near T_c, one needs lattices with $N_t \geq 8$. It was found that an aspect ratio of 3 was enough for the finite volume effects to stay under control.

Our estimates for the continuum thermodynamic quantities are shown in Fig. 4. The figure denotes that the correction to the leading N_c^2 contribution is rather small in both energy density and pressure, already for $N_c = 3$. It is also found that the energy density and pressure differ significantly from their Stefan-Boltzmann values, even deep in the deconfined phase.

The large deviation from the Stefan-Boltzmann value, and the fact that some strongly coupled conformal field theories show similar deviations from Stefan-Boltzmann limit [4], have sometimes been used in the literature to speculate about a strongly coupled, conformal regime in pure SU(N_c) gauge theories. The thermodynamics results can be used to investigate the feasibility of such a phase. Following Ref. [15], in Fig. 5 we plot the energy density vs. pressure, normalized by the corresponding Stefan-Boltzmann values. By construction the point at (1,1) is the Stefan-Boltzmann limit, while the diagonal line denotes conformality. Also shown are the weak coupling lines for the theories with the different number of colors [16]. We find that the weak coupling line is reached before the conformal line, indicating the absence of a strongly coupled, conformal phase in the SU(N_c) gluon plasma.

We thank Mikko Laine for providing us with the weak coupling results of Fig. 5. The computations were carried out on the workstation farm of the department of theoretical physics, TIFR.
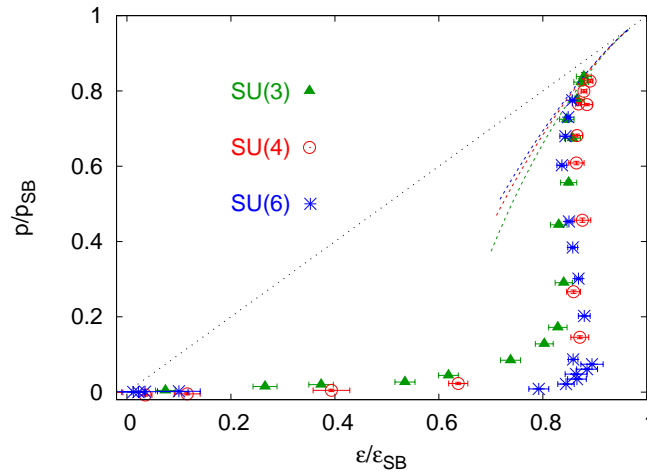
Figure 5: Approach to conformality in SU(N_c) gauge theories. Also shown are the weak coupling results from [16].

We thank Ajay Salve for technical support.

References

[1] S. Datta and S. Gupta, *Exploring the gluoN_c plasma*, in proceedings of *Quark Matter 2009* [arXiv:0906.3929]

[2] See, e.g., R. Pisarski, *Notes on the deconfining phase transition*, in: *Cargese 2001, QCD perspectives on hot and dense matter*, 353 [hep-ph/0203271] [hep-ph/9702340]

[3] R. Pisarski and L. Mclerran, *Nucl. Phys.* **A 796** (2007) 83.

[4] S. Gubser, I. Klebanov and A. Tseytlin, *Nucl. Phys.* **B 534** (1998) 202.

[5] S. Datta and S. Gupta, *Scaling and the continuum limit of gluoN_c plasmas*. arXiv:0909.5591.

[6] M. Wingate and S. Ohta, *Phys. Rev.* **D 63** (2001) 094502;
R.V. Gavai, *Nucl. Phys.* **B 633** (2002) 127.

[7] B. Lucini, M. Teper and U. Wenger, *Phys. Lett.* **B 545** (2002) 197; *JHEP* **0401** (2004) 061.
B. Lucini and M. Teper, *JHEP* **0502** (2005) 033.

[8] B. Bringoltz and M. Teper, *Phys. Lett.* **B 628** (2005) 113.

[9] M. Panero, *Thermodynamics of the QCD plasma and the large-N limit*. arXiv:0907.3719, and these proceedings, PoS(LAT2009)172.

[10] S. Gupta, *Phys. Rev.* **D 64** (2001) 034507.

[11] G. Martinelli, G. Parisi and R. Petronzio, *Phys. Lett.* **100B** (1981) 485.

[12] G. P. Lepage and P. B. Mackenzie, *Phys. Rev.* **D 48** (1993) 2250.

[13] G. Boyd, et al., *Nucl. Phys.* **B 469** (1996) 419.

[14] J. Engels, F. Karsch and T. Scheideler, *Nucl. Phys.* **B 564** (2000) 303.

[15] R. V. Gavai, S. Gupta and S. Mukherjee, *Pramana* **71** (2008) 487.

[16] M. Laine and Y. Schröder, *Phys.Rev.* **D 73** (2006) 085009.