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Images of quark intrinsic motion in covariant parton model

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We discuss the relations between TMDs and PDFs in the framework of the covariant parton model. The quark OAM and its connection to TMDs are studied as well.

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1. Intrinsic 3D motion in covariant parton model

The transverse momentum dependent parton distribution functions (TMDs) [1, 2] open the new way to a more complete understanding of the quark-gluon structure of the nucleon. We studied this topic in our recent papers [3, 5, 4, 6]. We have shown, that requirements of symmetry (Lorentz invariance combined with rotationally symmetric parton motion in the nucleon rest frame) applied in the covariant parton model imply the relations between integrated unpolarized or polarized distribution functions and their unintegrated counterparts. Further part is devoted to the discussion on the quark orbital angular momentum and its relation to the pretzelosity distribution function.

2. Transversal motion

Formulation of the model in terms of the light–cone formalism is suggested in [3] and allows to compute the chiral-even leading-twist TMDs which are defined [2] by means of the light–front correlators $\phi(x,\mathbf{p}_T)_{ij}$ as:

$$\frac{1}{2}\,\mathrm{tr}\left[\gamma^+\,\phi(x,\mathbf{p}_T)\right] = f_1(x,\mathbf{p}_T) - \frac{\varepsilon^{jk}p_T^j S_T^k}{M} f_{1T}^{\perp}(x,\mathbf{p}_T). \tag{2.1}$$

$$\frac{1}{2}\mathrm{tr}\left[\gamma^+\gamma_5\phi(x,\mathbf{p}_T)\right] = S_L g_1(x,\mathbf{p}_T) + \frac{\mathbf{p}_T\mathbf{S}_T}{M} g_{1T}^{\perp}(x,\mathbf{p}_T). \tag{2.2}$$

In this section we assume mass of quark $m \to 0$. This assumption substantially simplifies calculation within the model and seems to be in a good agreement with experimental data – in all model relations and rules, where such comparison can be done. But in principle, more complicated calculation with $m > 0$ is possible [8].

The symmetry constraints applied in the model imply [4, 6] the relations between unintegrated distribution and its integrated counterparts:

$$f_1^q(x,\mathbf{p}_T) = -\frac{1}{\pi M^2}\left(\frac{f_1^q(\xi)}{\xi}\right)', \tag{2.3}$$

$$g_1^q(x,\mathbf{p}_T) = \frac{2x-\xi}{\pi M^2 \xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right), \tag{2.4}$$

$$g_{1T}^{\perp q}(x,\mathbf{p}_T) = \frac{2}{\pi M^2 \xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right), \tag{2.5}$$

where

$$\xi = x\left(1 + \left(\frac{\mathbf{p}_T}{Mx}\right)^2\right). \tag{2.6}$$

The time-reversal odd Sivers distribution function $f_{1T}^{\perp}$ requires explicit gluon degrees of freedom and is absent in our approach. Apparently, the last two functions are related:

$$\frac{g_1^q(x,\mathbf{p}_T)}{g_{1T}^{\perp q}(x,\mathbf{p}_T)} = \frac{x}{2}\left(1 - \left(\frac{\mathbf{p}_T}{Mx}\right)^2\right). \tag{2.7}$$

Notice that from this relation the "Wandzura-Wilczek-type approximation" [9] follows:

$$g_{1T}^{\perp(1)q}(x) = x\int_x^1 \frac{g_1^q(y)}{y}dy. \tag{2.8}$$

Now, using the input distributions $f_1^q(x)$ and $g_1^q(x)$ one can calculate corresponding TMDs.

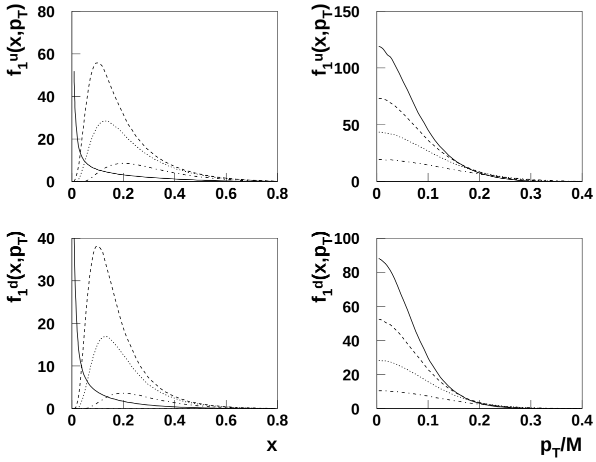

Figure 1: Transverse momentum dependent unpolarized distribution functions for u (*upper figures*) and $d-$quarks (*lower figures*). **Left part**: dependence on x for $p_T/M = 0.10, 0.13, 0.20$ is indicated by dash, dotted and dash-dot curves; solid curve correspods to the integrated distribution $f_1^q(x)$. **Right part**: dependence on p_T/M for $x = 0.15, 0.18, 0.22, 0.30$ is indicated by solid, dash, dotted and dash-dot curves.

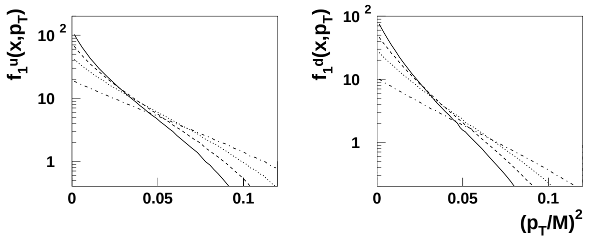

Figure 2: Transverse momentum dependent unpolarized distribution functions for u and $d-$quarks. Dependence on $(p_T/M)^2$ for $x = 0.15, 0.18, 0.22, 0.30$ is indicated by solid, dash, dotted and dash-dot curves.

2.1 Unpolarized distribution functions

For the unpolarized input we used the standard PDF parameterization [10] (LO at the scale $4GeV^2$). In Fig. 1 we have results obtained from relation (2.3) for u and $d-$quarks. The right part of this figure is shown again, but in different scale in Fig 2. One can observe the following:

i) For fixed x the p_T- distributions are very close to the Gauss Ansatz $f_1^q(x, p_T) \propto \exp\left(-p_T^2/\left\langle p_T^2\right\rangle\right)$. This is interesting result, since the Gaussian shape is supported by phenomenology [11].

ii) The width $\left\langle p_T^2\right\rangle$ depends on x. This result reflects to the fact, that in our approach, due to rotational symmetry, the parameters x and p_T are not independent.

iii) Figures suggest the typical values of transversal momenta, $\left\langle p_T^2\right\rangle \approx 0.01GeV^2$ or $\langle p_T\rangle \approx 0.1GeV$. These values correspond to the estimates based on the different analyses of the structure function $F_2(x, Q^2)$ [4]. On the other hand, much larger values $\langle p_T^2\rangle \sim 0.4GeV^2$ are inferred from SIDIS data referring to comparable scales [11], see also [12, 13]. Note also that in the statistical

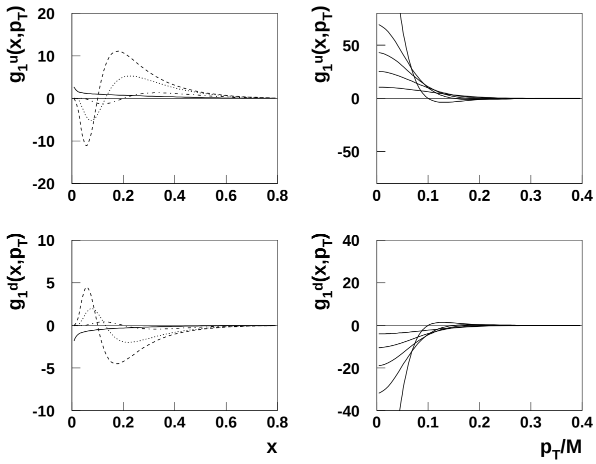

Figure 3: Transverse momentum dependent polarized distribution functions for u (*upper figures*) and $d-$quarks (*lower figures*). **Left part**: dependence on x for $p_T/M = 0.10, 0.13, 0.20$ is indicated by dash, dotted and dash-dot curves; solid curve correspods to the integrated distribution $g_1^q(x)$. **Right part**: dependence on p_T/M for $x = 0.10, 0.15, 0.18, 0.22, 0.30$ from top to down for $u-$quarks, and the same symmetrically for $d-$quarks.

model of TMDs [14] the parameter $\langle p_T \rangle$ may be interpreted as an effective [15] temperature of partonic "ensemble". In turn, it may be compared to the lattice calculations [16] of the QCD phase transition temperature $T \approx 175$ MeV.

2.2 Polarized distribution functions

With the use of standard input [17] on $g_1^q(x) = \Delta q(x)/2$ to the relation (2.4) we obtain the curves $g_1^q(x, p_T)$ displayed in Fig. 3. Let us remark, that the curves change the sign at the point $p_T = Mx$. This change is due to the term

$$2x - \xi = x\left(1 - \left(\frac{p_T}{Mx}\right)^2\right) = 2\tilde{p}_1/M \tag{2.9}$$

in relation (2.4). This term is proportional to the quark longitudinal momentum $\tilde{p}_1$ in the proton rest frame, which is defined by given x and p_T, see [4]. It means, that sign of the $g_1^q(x, p_T)$ is controlled by sign of the $\tilde{p}_1$. In fact, there is some similarity to the function $g_2^q(x)$, which also changes sign. The covariant parton model implies relation, which in the nucleon rest frame read [7]:

$$g_2(x) = -\frac{1}{2}\int \Delta G(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0}. \tag{2.10}$$

The $\delta-$function means, that large x is correlated with great and positive p_1 and on contrary the low x with great but negative p_1. The kinematic term inside the integral changes the sign between the extreme values of p_1, that is why the $g_2(x)$ changes the sign. Let us remark, that estimate of the $g_2(x)$ based on the relation (2.10) well agrees [8] with the experimental data.

3. Orbital motion

In the framework of covariant parton model we demonstrated that the 3D picture of parton momenta inside the nucleon is a necessary input for consistent accounting for quark OAM [7]. Let us repeat the main arguments. According to the rules of quantum mechanics the total angular momentum (in our case of a single quark) consists of the orbital and spin part $\mathbf{j} = \mathbf{l} + \mathbf{s}$ and in relativistic case the $\mathbf{l}$ and $\mathbf{s}$ are not conserved separately, but only the total angular momentum $\mathbf{j}$ is conserved. General solution of Dirac equation for $j = j_z = 1/2$ reads:

$$\Psi(\mathbf{p}) = \int a_k \psi_{kjlj_z}(\mathbf{p})\, dk; \quad \int a_k^\star a_k dk = 1, \tag{3.1}$$

where

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}} \begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix} \cos\theta \\ \sin\theta \exp(i\varphi) \end{pmatrix} \end{pmatrix}. \tag{3.2}$$

The average spin contribution to the total angular momentum is defined as

$$\langle s_z \rangle = \int \Psi^\dagger(\mathbf{p}) \Sigma_z \Psi(\mathbf{p})\, d^3p; \qquad \Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_z & \cdot \\ \cdot & \sigma_z \end{pmatrix}, \tag{3.3}$$

which implies

$$\langle s_z \rangle = \int a_p^\star a_p \frac{(p_0+m) + (p_0-m)\left(\cos^2\theta - \sin^2\theta\right)}{16\pi p^2 p_0} d^3p = \frac{1}{2}\int a_p^\star a_p \left(\frac{1}{3} + \frac{2m}{3p_0}\right) dp. \tag{3.4}$$

Since $\langle s_z \rangle + \langle l_z \rangle = j_z = 1/2$, this relation implies for the orbital momentum:

$$\langle l_z \rangle = \frac{1}{3}\int a_p^\star a_p \left(1 - \frac{m}{p_0}\right) dp. \tag{3.5}$$

In relativistic case, when $m \ll p_0$ in the nucleon rest frame, the role of OAM for generating nucleon spin is dominant,

$$\langle s_z \rangle \to 1/6, \qquad \langle l_z \rangle \to 1/3. \tag{3.6}$$

This result is related to the state $j = j_z = 1/2$, where the axis z represents direction of polarization. If the same state is polarized in any other direction, then $-1/2 < \langle j_z \rangle < 1/2$, but still it holds

$$\langle j_z \rangle = \langle s_z \rangle + \langle l_z \rangle, \qquad \langle l_z \rangle = 2\langle s_z \rangle. \tag{3.7}$$

In the covariant parton model, we identify $\langle s_z \rangle$ and $\langle l_z \rangle$ with the quark spin and orbital momentum, so the sum over all quarks

$$J_z^{quarks} = \sum_q \langle j_z^q \rangle \tag{3.8}$$

gives the total quark contribution to the nucleon spin. Due to (3.7), only 1/3 of this sum is generated by quark spins.

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Now let us consider another representation of the quark spins and orbital momenta. The spin contribution of quarks inside the nucleon to its spin is defined as

$$\langle s^q \rangle = \int g_1^q(x)\,dx. \tag{3.9}$$

It has been suggested recently [18, 19], that the pretzelosity distribution $h_{1T}^{\perp(1)q}(x)$ is related to the quark orbital momentum as

$$\langle l^q \rangle = -\int h_{1T}^{\perp(1)q}(x)\,dx. \tag{3.10}$$

On the other hand, as we showed in [3] (Eq.22), expression for pretzelosity in the covariant model reads

$$h_{1T}^{\perp q}(x, p_T) = -M^2 \int \frac{\Delta G(p_0)}{p_0 + m}\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{dp_1}{p_0}, \tag{3.11}$$

from which we obtain the (1) – moment

$$h_{1T}^{\perp(1)q}(x) = \int \frac{p_T^2}{2M^2} h_{1T}^{\perp q}(x, p_T)\,d^2p_T = -\frac{1}{2}\int \Delta G(p_0)\frac{p_T^2}{p_0 + m}\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0} \tag{3.12}$$

and

$$\int h_{1T}^{\perp(1)q}(x)\,dx = -\frac{1}{2}\int \Delta G(p_0)\frac{p_T^2}{p_0 + m}\frac{d^3p}{p_0}. \tag{3.13}$$

After replacing $p_T^2 \to \frac{2}{3}|\mathbf{p}|^2$ and $|\mathbf{p}|^2 = p_0^2 - m^2$ one gets

$$-\int h_{1T}^{\perp(1)q}(x)\,dx = \frac{1}{3}\int \Delta G(p_0)\left(1 - \frac{m}{p_0}\right)d^3p. \tag{3.14}$$

For helicity the covariant model gives the relation

$$g_1(x) = \frac{1}{2}\int \Delta G(p_0)\left(m + p_1 + \frac{p_1^2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0}, \tag{3.15}$$

which implies

$$\int g_1^q(x)\,dx = \frac{1}{2}\int \Delta G_q(p_0)\left(\frac{1}{3} + \frac{2m}{3p_0}\right)d^3p. \tag{3.16}$$

We can arrange the two sets of results for average spin and orbital momentum calculated by means:

1. *wavefunctions and operators* (Eqs.(3.4),(3.5)):

$\langle s^q \rangle$	$\langle l^q \rangle$
$\frac{1}{2}\int a_p^* a_p \left(\frac{1}{3} + \frac{2m}{3p_0}\right)dp$	$\frac{1}{3}\int a_p^* a_p \left(1 - \frac{m}{p_0}\right)dp$

2. *structure functions and probabilistic distributions* (Eqs.(3.14),(3.16)):

$\int g_1^q(x)\,dx$	$-\int h_{1T}^{\perp(1)q}(x)\,dx$
$\frac{1}{2}\int \Delta G_q(p_0)\left(\frac{1}{3} + \frac{2m}{3p_0}\right)d^3p$	$\frac{1}{3}\int \Delta G_q(p_0)\left(1 - \frac{m}{p_0}\right)d^3p$

Obviously, if we identify probabilities

$$a_p^* a_p dp \Leftrightarrow \Delta G_q(p_0)\,d^3p; \quad \Delta G_q(p_0) = G_q^+(p_0) - G_q^-(p_0) \tag{3.17}$$

then the table implies, that relation (3.10) between orbital momentum and pretzelosity is valid also in the covariant model.

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