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$U_A(1)$ symmetry at the phase transition - An update

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We study the $U_A(1)$ symmetry restoration above T_c using the Dirac operator spectrum and the spatial meson correlators. We show evidences that there is a gap in the Dirac operator spectrum and consequent degeneracy among meson correlators. These observations indicate an effective restoration of $U_A(1)$ symmetry at high temperature.

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1. Introduction

It is an interesting and long standing problem whether the $U_A(1)$ symmetry is restored or not above the phase transition temperature of QCD. At low temperature it is well known that the flavor non-singlet chiral symmetry is spontaneously broken according to the pattern:

$$SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_V \otimes U(1)_A \to U(1)_V \otimes SU(N_f)_V, \tag{1.1}$$

where $U(1)_V$ is the baryon number and V, A stand respectively for vector and axial sectors. Here the $U_A(1)$ symmetry is special because it is violated by a quantum effect through anomaly. In particular, it is broken by the presence of gauge configurations with topological charge Q different from zero, that generate an anomalous contribution to the divergence of flavor-singlet axial-vector current.

While the theoretical understanding of the $U_A(1)$ symmetry at zero temperature is well established, its finite temperature behavior has not been investigated in full detail yet. Some predictions come from the instanton gas models that are supposed to work only at very high temperatures, where the instanton density is exponentially suppressed and so the $U_A(1)$ symmetry effectively restores. An interesting question is then if this suppression occurs at the chiral phase transition. Only recently the problem was studied with ab-initio lattice QCD calculations. The problem is interesting because whether the $U_A(1)$ is restored or not could influence the order of the chiral phase transition [1].

We investigate the behavior of the $U_A(1)$ symmetry at high temperature by the numerical simulation of lattice QCD. For a systematic study of the $U_A(1)$ symmetry, chiral symmetry has to be realized to a good approximation. The current optimal answer to this problem is the overlap fermion [2], whose Dirac operator is

$$D_{\rm ov} = \frac{r_0}{a}[1 + \mathrm{sign}(H_W(-r_0/a)], \tag{1.2}$$

with $H_W(m)$ the massive hermitian Wilson operator. Overlap fermions realize exact chiral symmetry on the lattice.

Dynamical simulations with the overlap fermions are possible with current algorithms and machines and were performed by the JLQCD collaboration in the past years [3] at the cost of fixing topology throughout the HMC trajectories. A technique has been developed [4] to obtain the physical $\theta = 0$ results from these ensembles, which works nicely at zero temperature. It was shown that the effect of fixing topology is of order $O(1/V)$ on a lattice of volume V. Fixing topology is helpful also in checking results of chiral random matrix theory [5], that requires isolation of one topological sector. The theoretical predictions are nicely confirmed by the lattice data.

At finite temperature the application of the methods developed at zero temperature is not straightforward, as pointed out in [6]. We checked numerically in pure gauge theory [7] that even at finite temperature these systematic errors are under control.

In this work, we report our results on two-flavor QCD with overlap fermions.

2. Lattice setup

In Table 1 we list simulation parameters. The lattice size is $16^3 \times 8$ in lattice units. We use Iwasaki gauge action and two flavors of overlap fermions. The topology is fixed to $Q = 0$

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β	am	$a_{m=0}$ (fm)	T (MeV)	$T/T_c = 180$ MeV	N_{eigenval}	$N_{\text{correlators}}$
2.18	0.01	0.14379	171.5	0.95	118	100
2.18	0.05	0.14379	171.5	0.95	350	320
2.20	0.01	0.13909	177.3	0.985	187	187
2.20	0.025	0.13909	177.3	0.985	303	272
2.20	0.05	0.13909	177.3	0.985	279	279
2.25	0.01	0.12818	192.4	1.06	335	331
2.30	0.01	0.11834	208.4	1.15	512	479
2.30	0.025	0.11834	208.4	1.15	226	183
2.30	0.05	0.11834	208.4	1.15	281	281
2.40	0.01	0.10137	243.3	1.35	477	319
2.40	0.05	0.10137	243.3	1.35	210	210
2.45	0.05	0.09403	262.3	1.45	80	-

Table 1: Parameters for the $N_f = 2$ simulations. Lattice $16^3 \times 8$ and $Q = 0$. Reported also the number of configurations analyzed to produce the correlators.

throughout the HMC simulations by adding extra terms to the action [8]. The temporal size $N_t = 8$ was chosen in order to obtain configurations smooth enough to guarantee the localization properties of the overlap operator [9]. The aspect ratio N_s/N_t is not optimal for finite temperature simulation but we do not consider volume dependence in this exploratory work. Stored configurations were not enough to precisely estimate the transition point, so we assume a critical temperature of $T_c = 180$ MeV and quote the temperature relative to this value. An interpolating curve obtained at zero temperature is used for an estimate of the lattice spacing at several β's (in the chiral limit) and thus the temperature. We have only one measurement of the zero temperature pion mass at $\beta = 2.30$ and bare quark mass $am = 0.015$, where $m_\pi = 290$ MeV.

3. Dirac spectrum

We begin our discussion with the properties of the Dirac operator spectrum. The region we are interested in is the near-zero modes sector which represents the infrared behavior of QCD. At zero temperture we expect non-zero density of eigenvalues at zero as dictated by the Banks-Casher relation [10]. We expect that this density vanishes once we cross the chiral phase transition at T_c in the infinite volume limit.

The next question is what happens if we assume that the $U_A(1)$ symmetry is restored. It was recently shown [11] that the strongest constraint from the Ward-Takahashi identities in two-flavor QCD is:

$$\lim_{\lambda \to 0} \lim_{m \to 0} \frac{\rho(\lambda)}{\lambda^2} = 0. \tag{3.1}$$

As a consequence it can be shown that the difference of susceptibilities (volume integrals of the correlators) related by a parity transformation, such as:

$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \frac{4m^2\rho(\lambda)}{(m^2+\lambda^2)^2} \tag{3.2}$$

vanishes in the infinite volume limit. Then, the $U_A(1)$ symmetry appears to be restored if we measure such kind of operators. A gap in the spectrum $\rho(\lambda)$, a much stronger condition than (3.1), is also a sufficient condition for the $U(1)_A$ symmetry restoration in the chiral limit. Thus, the low-lying Dirac spectrum at different temperatures is a good observable to investigate the $U_A(1)$ symmetry restoration. We can look for the minimal λ^3 behavior or eventually for a stronger gap.

We calculate 50 lowest eigenpairs of the overlap Dirac operator ($D_{\rm ov}$ not $\gamma_5 D_{\rm ov}$). The result is plotted in figure 1 after a rescaling of the imaginary part to dimensionful quantities to compare the spectra at various temperatures.

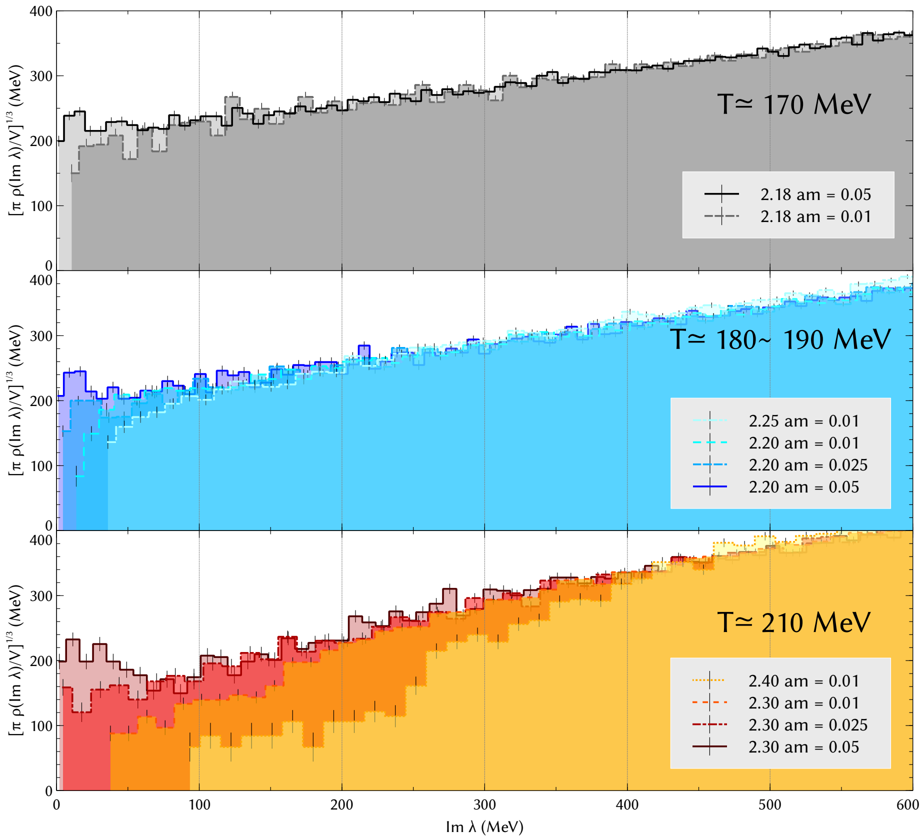

Figure 1: Spectral density of the massless overlap Dirac operator for $N_f = 2$. The several βs were isolated to emphasize the sea quark mass dependence of the density. Zero counting of eigenvalues is intended on the left when the filled area ends.

The outcome of our calculation is quite clear: in the chiral limit a gap opens in the spectrum at high temperatures implying a strong suppression of the $U(1)_A$ violating terms. The gap seems opening at the temperature around the chiral phase transition ~ 180 MeV, but is difficult to identify the exact point with current data due to possible volume effects. Certainly a gap is there starting from $\beta = 2.25$, i.e. $\simeq 192$ MeV. We do not observe any accumulation of near-zero modes in the chiral limit. A small peak is visible in the highest mass case, but could be most probably ascribed to the explicit, strong, violation of chiral symmetry by the mass term.

An evident limitation of this calculation is the volume dependence. A bigger volume could lead to a closure of the gap for example because there is more space to create the topological objects that are related to the near zero modes. This is an issue that needs to be investigated in the future.

4. Correlators

Further information can be obtained from degeneracies of four different meson channels. The scalar flavor-singlet (σ) and the pseudoscalar triplet (π) are related by the $SU(2)$ chiral transformation, like the pseudoscalar singlet (η) and the scalar triplet (δ), see figure 2. These two channels are expected to be degenerate at $T > T_c$ when chiral symmetry $SU(2)_L \otimes SU(2)_R$ is restored. If the $U_A(1)$ symmetry is effectively restored all correlators should be identical. This degeneracy implies that the disconnected parts in the singlet correlators should vanish. A gap in the spectrum is compatible with a strong suppression of the disconnected part but the opposite cannot be inferred.

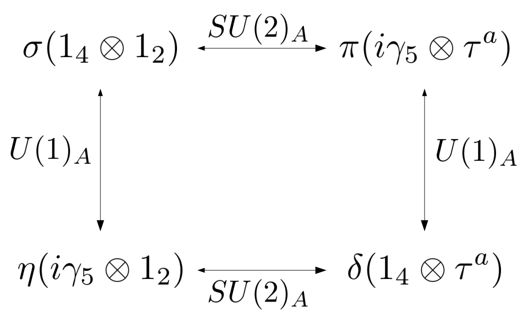

Figure 2: Diagram of symmetry relations between the lightest meson in two flavors QCD.

We measure the spatial correlators in the four channels: $\sigma, \pi, \delta, \eta$. The very short distance part of meson correlators is unreliable, since they are estimated using 50 eigenmodes only. Therefore, the screening mass could not be reliably extracted from the small spatial extension.

Nevertheless we can extract a qualitative estimate of the degeneracy of various channels. Looking at Figure 3 we again observe hints for the $U_A(1)$ symmetry restoration near the chiral limit. At the lowest mass all correlators are identical within errors and this becomes more evident as temperature increases. This means that the disconnected diagrams are zero. The conclusion in this case is that the diagram in Figure 2 is valid in every direction, i.e. all mesons are degenerate: the $U(1)_A$ symmetry is restored.

5. Conclusions

By measuring the eigenmodes of the overlap Dirac operator we have found that a gap is present once we approach the chiral limit. This is a convincing evidence of restoration of $U_A(1)$ symmetry above the chiral phase transition. The exact point where the symmetry is restored cannot be checked with current data, so we cannot settle the question whether the restoration is at or above T_c.

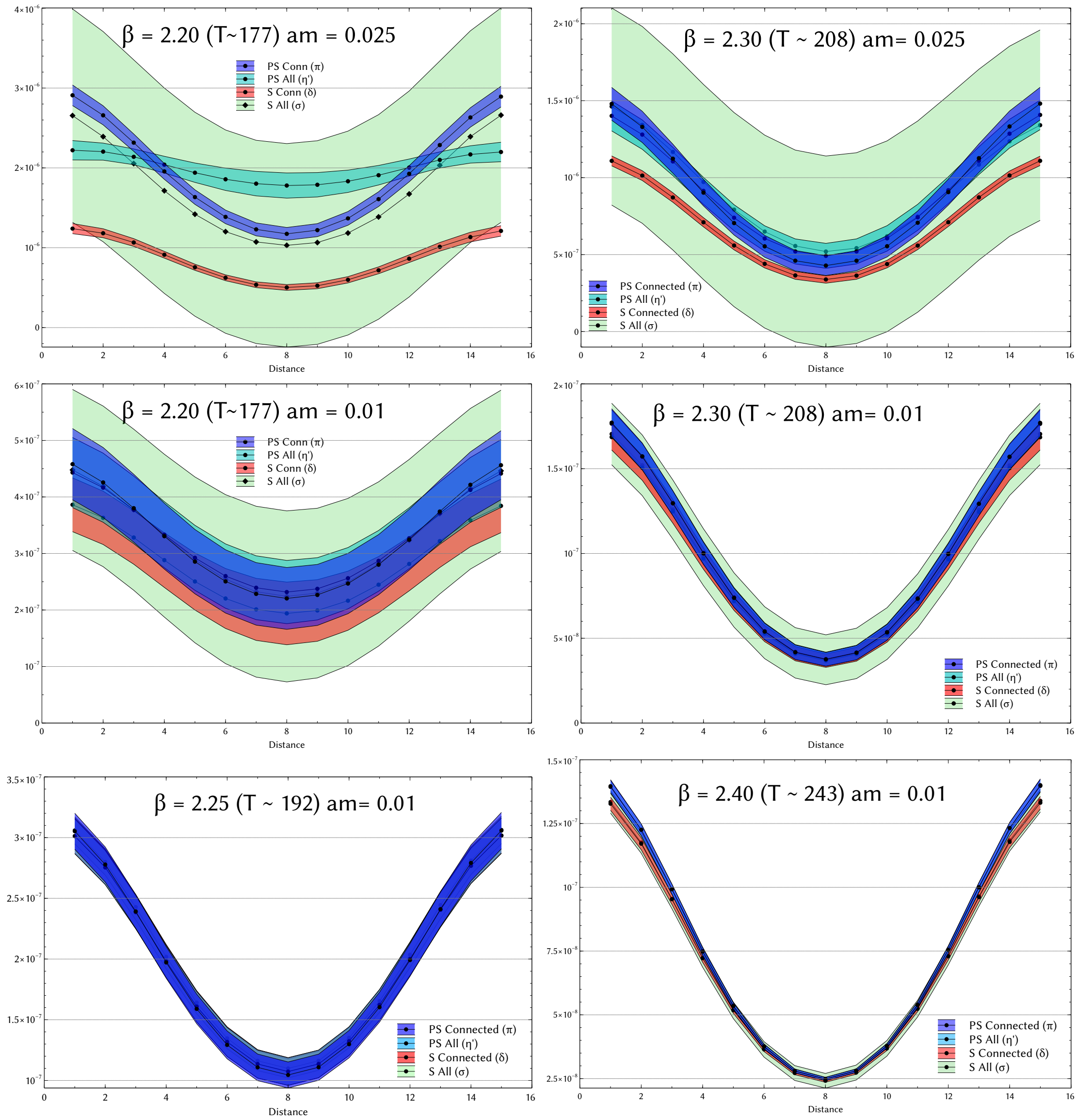

Figure 3: Meson correlators at temperatures 171 $\sim$ 208 MeV. Mass is decreasing towards the bottom of every column. PS and S stand for pseudoscalar and scalar respectively.

A second evidence comes from the related correlator degeneracy at the same temperatures. All the two point meson spatial correlators look the same in the chiral limit as soon as the critical temperature is crossed. In two flavor QCD this is a sign of $U_A(1)$ symmetry restoration [12].

Some systematics are to be addressed in future studies: volume dependence and deeper chiral limit and cross checking current results with an action that allows topology change while retaining a very good chiral symmetry.

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