NRQCD: A Critical Review

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ABSTRACT: In this talk I review some recent applications of NRQCD. I first discuss the unquenched NRQCD lattice extractions of the strong coupling constant, paying particular attention to the recent advances in reducing systematic errors. I then discuss the progress made in testing the NRQCD/factorization formalism for onium production. In particular, I gather the evidence, or lack thereof, for the universality of the production matrix elements. I also discuss the interesting question of polarized production, which is a crucial test of the formalism. I address the viability of this theory for the J/ψ system as well as what needs to be done before we can reach any definitive conclusions.

The study of heavy quarks is interesting from a phenomenological as well as formal standpoint. Indeed, the utility of the experiments designed to study the CKM sector of the standard model is bounded by our theoretical understanding of states containing heavy quarks. For theorists this presents a great challenge given that, in general, it is quite difficult to make predictions for strongly interacting particles from first principles. However, we do have one powerful tool at our disposal which saves us from despondency, namely effective field theories. These theories represent limits of QCD which make approximate symmetries manifest, thus greatly enhancing our predictive power. Furthermore, since the effective theory reproduces QCD in a well defined limit, it is possible to calculate corrections in a systematic fashion.

The effective theory I have been charged to review is non-relativistic QCD. As the moniker implies, this is a theory which approximates full QCD when applied to a bound state containing more than one heavy quark. The quarks necessarily have small velocities in the limit $m_Q \gg \Lambda_{QCD}$, due to asymptotic freedom. The theory is written down as a simultaneous expansion in $\alpha_S(2m_Q)$ and the relative velocity in the center of mass frame of the quarks, v. Indeed, if we wish, we may calculate corrections to arbitrary order in powers of v and α_s, at the price of introducing unknown, yet universal parameters. Thus, the relevant question is not "is NRQCD correct?", but rather, "how well do the v and α_s expansions converge for the system of interest?"

In this review I will discuss recent advances in the application of NRQCD. In particular, I will concentrate on applications for which we have new data. Thus, due to space limitations, I will not discuss the interesting topic of inclusive onia decays, nor will I discuss recent theoretical progress on threshold top quark production[1]. Instead, I will focus on the utilization of NRQCD to measure the strong coupling, and the predictions for onia production.

1. The Effective Field Theory

The effective Lagrangian is constructed [2] by ensuring that at the matching scale, m_Q, the effective field theory reproduces on shell Greens functions of QCD to some fixed order in $\alpha(2m_Q)$ and v. The operators in the effective field theory are then classified by their "effective v dimension" [3]. That is, to each operator we may assign a power of v which is determined by some velocity power counting rules. For our purposes, we will be interested in the fermion anti-fermion sector of the theory where the lowest order Lagrangian is given by

$$L_0 = \int dtd^3\mathbf{x}\Psi^\dagger(iD_0 + \frac{\vec{\mathbf{D}}^2}{2m})\Psi(t,\vec{\mathbf{x}}) + \int d^3\mathbf{y}\Psi^\dagger T^a\Psi(t,\vec{\mathbf{y}})\frac{4\pi\alpha_s C_V}{\mid \vec{\mathbf{x}} - \vec{\mathbf{y}} \mid^2}\chi^\dagger \bar{T}^a\chi(t,\vec{\mathbf{x}}) + L_g + L_q + \Psi \leftrightarrow \chi, \tag{1.1}$$

where ψ and χ are the fermion and anti-fermion two spinors respectively, L_g and L_q are the unscathed dimension four operators of the QCD Lagrangian for the gluons and light quarks respectively. C_V is unity at tree level but is corrected at higher order. The coefficients of the other operators in the heavy quark sector are fixed by reparameterization invariance [4]. Terms of higher order in v are easily calculated by expanding the full theory on-shell matrix elements in the v. In the one fermion sector this leads to the usual terms encountered in non-relativistic quantum mechanics, such as the magnetic moment and Darwin interactions. There is one technical detail which I should point out at this point. When matching beyond leading order in the strong coupling, the above Lagrangian must be modified if one wishes to preserve manifest power counting. In particular, if one wishes to regulates integrals using dimensional regularization, then the non-potential interactions must be dipole expanded[5, 6]. That is to say, we must make the replacement

$$A_\mu(t,\vec{\mathbf{x}}) \to A_\mu(t,0) + \vec{\mathbf{x}}\cdot\vec{\partial}A_\mu(t,0) + \ldots \tag{1.2}$$

This dipole expansion incorporates the fact that the typical gluon momentum is of order mv^2 whereas the size of the state is order mv. On a calculational level this leads to gluonic interactions which do not transfer three momentum, which in turn ensures manifest power counting. The situation is actually slightly more complicated that this, because there are also gluons which have typical momentum of order mv[7]. This leads to further modifications of the Lagrangian, which I will not have space to discuss in this review [8, 9].

2. Lattice Extractions of α_s

As far a lattice calculations are concerned, the motivation for effective field theories is different than the one discussed in the introduction. In particular, by removing the heavy quark mass from the theory it is possible to work with lattice spacings a, which are larger than the Compton wavelength of the heavy quark $(ma) > 1$. This alleviates the problem of the quark literally falling through the cracks. Indeed, the use of coarser lattices allows for precision measurements which would otherwise be presently unmanageable.

Perhaps the most interesting use of lattice NRQCD is for the purpose of extracting the value of the strong coupling constant α_s. This low energy extraction is interesting, not only because it allows us to compare to results obtained from perturbative QCD, but also because any disparity between the low energy and high energy extractions, after renormalization group running, could imply the existence of new physics. Thus, a precise low energy extraction is of great scientific interest. Below we will discuss the results of the two collaborations, "NRQCD" [10] and "SESAM"[12], which used NRQCD to treat the heavy quarks. These are the first lattice extractions of α_s performed in the unquenched approximation. While the NRQCD collaborations results have been around for several years now, the important complimentary results of SESAM collaboration are more recent. The two collaborations use different techniques so that their agreement would provide strong corroboration of the results for this important measurement.

The strategy behind the extraction is to first define a short distance coupling constant $\alpha_s(a)$ via some lattice observable. Both collaborations utilized the so-called plaquette coupling [11], which is related to small Wilson loops via

$$-\log W_{1,1} = \frac{4\pi}{3}\alpha_p(3.41/a) + (1 - \alpha_p(1.1879 + (0.0249, 0.070)n_f)). \tag{2.1}$$

The first number in parenthesis is for Wilson fermion and the second is for staggered fermions, and

$$W_{m,n} = \frac{1}{3}\langle ReTr\left(P\exp\left[-ig_s\oint_{m,n} A\cdot dx\right]\right)\rangle. \tag{2.2}$$

$W_{m,n}$ is the Wilson around a rectangular path of size $(ma)\times(na)$. This observable has the further advantage that its leading non-perturbative correction,

$$\delta W_{m,n} = \frac{-\pi a^4 (mn)^2}{36} \langle \alpha_s G^2 \rangle . \tag{2.3}$$

is believed to be small due the anomalously small value of the gluon condensate $\langle \alpha_s G^2 \rangle \simeq 0.05\, GeV^4$. Note there are no higher order perturbative correction to the relation 2.1, as a matter of definition. This just shuffles higher order corrections into the relation between $\alpha_{\overline{MS}}$ and α_p. The scale $3.41/a$ is the BLM scale [13] determined by calculating the n_f dependent piece of the two loop perturbative expansion of the Wilson loop. The lattice spacing is then determined by measuring the the spin independent quarkonia splittings. These splittings are chosen since they are much less sensitive to tuning of the bare heavy quark mass. Once the value of $\alpha_p(3.41/a)$ has been extracted, it can be related to $\alpha_{\overline{MS}}$ via

$$\alpha_{\overline{MS}}(Q) = \alpha_P(e^{5/6}Q) \times \left[1 + \frac{2}{\pi}\alpha_P + C_2 \alpha_P^2\right] \tag{2.4}$$

The scale $e^{5/6}Q$ is chosen to absorb the BLM piece of the first order coefficient. The coefficient C_2 is only known in the pure gauge theory[14], in which it takes the value $C_2 = 0.96$. Using this relation its possible to determine a value of $\alpha^{(5)}_{\overline{MS}}(M_Z)$ by running with the three loop beta function and taking into account the relevant quark thresholds.

There are several sources of errors, and the SESAM and NRQCD collaborations incur them in different fashions, as I shall now discuss. As usual with lattice calculations there is the issue of cut-off artifacts. The NRQCD collaboration removed all $O(a^2)$ error from the heavy quark action by using an improved action. They did not improve the gluonic action to this order, but made a perturbative estimate of these corrections. They then checked that their spin splittings are independent of the lattice spacing (in the quenched approximation) by calculating with several different spacings in the range, $0.05 - 0.15$ fm, and found no variations. The use of staggered fermions also introduces errors on the order of $O(a^2)$. They estimated the net effect of lattice artifact errors to be at the level of 0.2%. The SESAM collaboration used dynamical Wilson fermions which incur lattice spacing errors at linear order in a. They worked at fixed lattice spacing and were therefore unable to perform a scaling analysis. They quote a larger error, due to discretization, than did the NRQCD collaboration, namely %5. This larger error can be attributed to the difference between the light fermion actions.

In addition, there are corrections due to truncating the Lagrangian at some fixed order in v. Both collaborations include $O(v^2)$ corrections to the effective action. $O(v^4)$ corrections should be negligibly small in the Υ system since the $O(v^2)$ corrections were found to shift the mass splitting by 10%. There is also an error incurred by truncation the perturbative expansion of the Wilson coefficients for these higher dimension operators. The coefficients of these operators are tree level "tadpole improved". An estimate for this error was made by the SESAM collaboration by varying the tadpole improvement prescription. They found that this error is of the same order as the relativistic errors, and estimated these combined errors to be at 1% level. Whereas the NRQCD collaboration estimated the relativistic errors to be at the 0.2% and the errors due to tadpole prescription to be 0.5%.

The use of dynamical light quarks was a crucial step in making these lattice predictions trustworthy. However, it is difficult, at this time, to perform calculations with physical masses for the light quarks. Therefore, the calculations are performed with unphysical light quark masses, and the results are extrapolated to the physical value. In [15], it was shown that the level splittings should grow linearly with the light quark mass. The SESAM collaboration calculated the splittings for several different light quark masses, between the radial as well as orbital excitations. The results for the $1S - 1P$ splitting are shown in figure 1 and seem to fit a linear relationship quite well. The result gives us confidence that the extrapolation to physical light quark masses is being performed correctly. Both collaborations estimate the error due to use of unphysical sea quark masses to be at the 1% level.

Both collaborations performed their unquenched calculations with $n_f = 2$, and extrapolated to

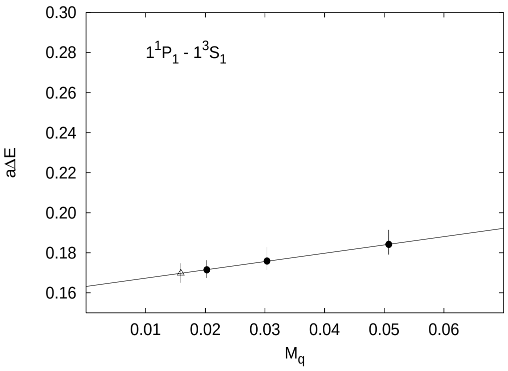

Figure 1: SESAM result for the ${}^1P_1 - {}^3S_1$ splitting as a function of the light quark mass.

$n_f = 3$. Indeed, the NRQCD collaboration found that fixing the lattice spacing using different Υ splittings led to results which varied by 3 σ when n_f is taken to be zero. This discrepancy essentially disappears when $n_f = 3$. So it seems reasonable to expect that these new unquenched calculations have the light quark effects well under control. We should point out that neither collaboration took into account $SU(3)$ violation , but these effects should be extremely small, given the size of the bound state.

A larger error arises due to the uncertainty in the, yet to be calculated, n_f dependent piece in the relation 2.4. The SESAM group varied the coefficient C_2 in 2.4, between 1 and -1 and found a 2%−3% variation in their result while the NRQCD collaboration quote a 1.9% error due to the uncertainty in C_2. The SESAM collaboration quotes the results

$$\alpha^{(5)}_{\overline{MS}}(m_Z) = \begin{cases} 0.1118\ (10)(12)(5) & \bar{\chi} - \Upsilon \text{ splitting} \\ 0.1124\ (13)(12)(15) & \Upsilon' - \Upsilon \text{ splitting} \end{cases} \tag{2.5}$$

where $\bar{\chi}$ is the spin average of the P wave states. The first error is statistical while the last two are from the NRQCD truncation and uncertainty in the sea quark mass dependence. The errors do not include any guess at the size of discretization error stemming from the light quarks, which the authors estimate to be at the 5% level. Nor does this error include the uncertainties due to our ignorance of the n_f dependent piece of C_2, which the authors estimate to be at the $2\% - 3\%$ level.

The NRQCD collaboration quotes

$$\alpha^{(5)}_{\overline{MS}}(m_Z) = \begin{cases} 0.1174\ (15)(19) & \bar{\chi} - \Upsilon \text{ splitting} \\ 0.1173\ (21)(18) & \Upsilon' - \Upsilon \text{ splitting} \end{cases} \tag{2.6}$$

The errors are due to lattice artifacts and perturbative truncation errors, respectively. Notice that the two collaborations differ by 3σ. The SESAM collaboration attributes this to light quark discretization errors.

We see that these calculations seem to have the errors well under control. My only true concern, is the issue of the convergence of the perturbative expansion at these scales. Given the precision of the measurement, a calculation of the complete C_2 coefficient would certainly increase the confidence level.

3. Onium Production

In [2] it was pointed out that by combining perturbative factorization with NRQCD it is possible to make "rigorous" [1] predictions for onium production. A general production process may be written as

$$d\sigma = \sum_n d\sigma_{i+j\to Q\bar{Q}[n]+X} \langle 0 \mid O_n^H \mid 0 \rangle. \tag{3.1}$$

Here $d\sigma_{i+j\to Q\bar{Q}[n]+X}$ is the short distance cross section for a reaction involving two partons i and j, in the initial state, and two heavy quarks, in a final state labeled by n, plus X. This part of the process is short distance dominated and completely calculable in perturbation theory, modulo the possible structure functions, in the initial state and may be considered as "matching coefficient". The long distance part of the process involved the hadronization of the heavy quarks in the state n into the hadron of choice H. Indeed, the matrix element which is written as

$$\langle 0 \mid O_n^H \mid 0 \rangle = \langle 0 \mid \psi^\dagger \Gamma^{n'} \chi \mid \sum_X H + X \rangle$$
$$\langle H + X \mid \chi^\dagger \Gamma^n \psi \mid 0 \rangle. \tag{3.2}$$

The tensor Γ^n operates in color as well as spin space and also contains possible derivatives. This

[1] The calculation have a level of rigor equivalent to those in semi-inclusive hadro-production where one relies on perturbative factorization in the physical region.

tensor also determines the order, in v of the matrix element. The size of the matrix element is fixed by determining the perturbations necessary to give a non-vanishing result for the time ordered product(selection rules). The matrix elements are taken between states of the effective theory, which are given by the eigenstates of the dipole expanded version of 1.1. Thus, at leading order there is no overlap between operators with quantum numbers (usually labeled spectroscopically ${}^{2S+1}L_J(1,8)$) which differ from those of the state under consideration. Which is to say if H has the quantum numbers ${}^{2S+1}L_J$ then at leading order in v color singlet operators give

$$\langle 0 \mid O^H_{(1)}({}^{2S'+1}L'_{J'}) \mid 0\rangle \propto \delta_{SS'}\delta_{LL'}\delta_{JJ'}, \quad (3.3)$$

while all color octet operators give zero. Higher order contributions can be included by inserting higher multipole moment interactions into the time ordered product. For instance, the matrix element $\langle 0 \mid O^{J/\Psi}_{(8)}({}^3S_1) \mid 0\rangle$, would scale as v^4, since we need two $E1$ insertions at the the amplitude level, each costing a factor of v. The first insertion neutralizes the color, but it also changes L. So a second insertion is needed to bring us back to the S wave state. This is illustrated in figure 1.

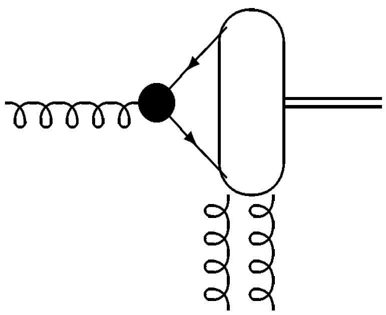

Figure 2: Gluon fragmentation into a ${}^3S_1$ state. The quark anti-quark pair are created at a short distance scale in a relative octet state. They then propagate into the J/ψ state by emitting two soft $E1$ gluons.

For the color singlet matrix elements, we may use factorization, which is typically correct up to v^4 corrections, [2] to write

$$\langle 0 \mid O^H_n \mid 0\rangle = \langle 0 \mid \psi^\dagger\Gamma^n\chi \mid H\rangle\langle H \mid \chi^\dagger\Gamma^n\psi \mid 0\rangle. \quad (3.4)$$

We may then interpret $\langle H \mid \chi^\dagger\Gamma^n\psi \mid 0\rangle$ as the "wave function" at the origin $\psi(0)$, or derivatives thereof, in a potential model (with some factorization scale dependence). This parameter is the same parameter which determines the annihilation rate and thus its value is easily extracted from the data.

Note that if we were to drop all color octet contributions, then we would end up with the old fashioned, color singlet model. In this model the production rate is calculated by weighting the partonic rate to produce two heavy quarks with zero relative velocity with the a potential wave function at the origin $\mid \psi(0) \mid^2$. Here I wish to emphasize that this method of calculation is indeed a *model*, in that there is no limit in which it reproduces the full QCD calculation. On the other hand, despite what might often be said, the NRQCD prediction is NOT a model. In the limit where the quark mass is much larger than the QCD scale the theory reproduces full QCD. Thus, those who call the NRQCD prediction, the "color octet" model, have been either gravely mislead or are using a definition of the term *model* with which I am not familiar.

The power of the NRQCD/factorization formalism for production lies in the universality of the matrix elements $\langle 0 \mid O^H_n \mid 0\rangle$. So that to make a prediction for a given process we must first extract the appropriate matrix elements from another process. This universality is a consequence of the factorization of the short distance production process from the relatively long distance hadronization process. At large p_T production is dominated by fragmentation processes [17]. In this case, the cross section is written as the probability to create a nearly on-shell parton with momentum q such that $q_0 \gg \sqrt{q^2}$, times the probability of the parton to fragment into the hadron of interest. This latter probability may be written in terms of an unphysical fragmentation function $D^H_i(z)$ which gives the probability

[2] This is true unless the L of the state created is different from the L of the operator under consideration, in which case the correction can be $O(v^2)$.

of forming a hadron H with momentum fraction z from an initial parton labeled by i. This factorization is analogous to case of factorization for semi-inclusive light hadron production[16]. In the case of onium production the fragmentation function itself factorizes into a product of the probability for an energetic parton with invariant mass on the order of $4m_Q^2$ to produce two heavy quarks in a state ${}^{2S+1}L_J^{(1,8)}$ times the probability of this heavy quark state to hadronize into the onium state of interest. Thus, as opposed to the case of light hadrons, the fragmentation function can be written in terms of a perturbative expansion in $\alpha_S(2m_Q)$ and an unknown constant(s) (our matrix elements from above). Writing the cross section in this way makes it simple to resum large logs of the form $\log p_T/m_Q$ (i.e. perform the Alterelli-Parisi running). In this case, the factorization holds up to corrections of order $4m_Q^2/p_T^2$.

For smaller values of p_T fragmentation no longer dominates and one must consider the fusion of partons into the final state. This cross section is written as the probability to create a heavy quark anti-quark pair, in a given configuration, times the hadronic matrix element. In this regime higher twist effects are expected to scale as powers of $\Lambda/\sqrt{p_T^2+m_Q^2v^n}$ [3], though this has not been fully explored to date (n has not been fixed), since the quark pair can be strongly influenced by the jet in the forward direction as p_t gets smaller. Thus, our confidence in our calculation dwindles as p_T is reduced. We should thus keep in mind that when we test the formalism we are testing more than just the convergence of the α_s and v expansions. We are also testing factorization. Thus, at least for production, and to a lesser degree for decays, we should really say that we are testing the NRQCD/factorization formalism.

3.1 The Universality of the Matrix Elements

A first crucial test of the NRQCD/factorization formalism consists of checking the universality of the matrix elements. There are many places from which we may extract the relevant matrix elements. The first extractions were done using the Tevatron data. Let us, for the moment concentrate on the case of the J/ψ, where we presently have most of our data. As discussed above, at large p_T gluon fragmentation dominates and gives a leading order contribution proportional to $\langle 0 \mid O_8^{J/\psi}({}^3S_1) \mid 0\rangle$, as shown in figure 2, while at intermediate values of of p_T, the rate becomes sensitive to $\langle 0 \mid O_8^{J/\psi}({}^1S_0) \mid 0\rangle$ and $\langle 0 \mid O_8^{J/\psi}({}^3P_0) \mid 0\rangle$. Actually at lower values of p_T the rate is sensitive to a linear combination of these last two matrix elements. This combination is usually defined as

$$M_k^{J/\psi} = \langle O_8^{J/\psi}({}^1S_0)\rangle + \frac{k}{m_Q^2}\langle O_8^{J/\psi}({}^3P_0)\rangle, \quad (3.5)$$

k actually varies with p_T. The extraction of M_k is complicated by the fact that its value is very sensitive to the small x gluon distribution which makes it difficult to extract reliably. Varying parton distribution function can change the extracted values by a factor of four. Therefore, I will concentrate on extractions of $\langle O_8^{J/\psi}({}^3S_1)\rangle$.

Several groups have extracted this matrix element [18, 19, 20, 21, 22] for the J/ψ as well as the ψ'. I have collected the various extracted values from these references in tables 1 and 2. The extractions differed in several respects. References [20, 21, 22] included a Monte Carlo estimation for initial state radiation, which we expect to become more important for smaller values of p_T. Of these three references though, only [22] correctly accounted for the Alterelli-Parisi evolution, which is seen to be important for this particular matrix element. Noting the difference between [22] and [20, 21], we can see that the effect of resummation is only important, as we would expect, for small p_T. Only [19] made any estimates for the scale dependence of the results, which from the table can be seen to be substantial. The extractions also differ in the way in which they treat the interpolation between the fragmentation at high p_T and the direct production at low p_T. I think that it would reasonable to say that the dominant source of error will come from higher order corrections, as evidenced by the large scale dependence found by

[3] The first power correction would most likely scale as Λ^2.

J/ψ	CTEQ2L	CTEQ4L	CTEQ4M	GRVLO	GRVHO	MRS(R2)	MRSD0
[18]	-	-	-	-	-	-	0.66 ± 0.21
[19]	-	$1.06 \pm 0.14^{+1.05}_{-0.59}$	-	$1.12 \pm 0.14^{+0.99}_{-0.56}$	-	$1.40 \pm 0.22^{+1.35}_{-0.79}$	-
[20]	0.33 ± 0.05	-	-	-	0.34 ± 0.04	-	0.21 ± 0.05
[21]	-	-	0.27 ± 0.04	-	-	-	-
[22]	0.96 ± 0.15	-	-	-	0.92 ± 0.11	-	0.68 ± 0.16

Table 1: Extractions of $\langle O_8^{J/\psi}(^3S_1)\rangle$ in units of 10^{-2} GeV^3. The first error is statistical, while the second, when listed, is due scale dependence.

ψ'	CTEQ2L	CTEQ4L	CTEQ4M	GRVLO	GRVHO	MRS(R2)	MRSD0
[18]	-	-	-	-	-	-	0.46 ± 0.10
[19]	-	$0.44 \pm 0.08^{+0.43}_{-0.24}$	-	$0.46 \pm 0.08^{+0.41}_{-0.23}$	-	$0.56 \pm 0.11^{+0.54}_{-0.32}$	-
[20]	0.14 ± 0.03	-	-	-	0.13 ± 0.02	-	0.11 ± 0.03

Table 2: Extractions of $\langle O_8^{\psi'}(^3S_1)\rangle$ in units of 10^{-2} GeV^3. The first error is statistical, while the second, when listed, is due scale dependence.

[19]. It would be interesting to see the extent to which this scale dependence is reduced by including higher order effects. However, given the present state of affairs, I don't think I would be overly conservative if I were to say that there is a factor of two uncertainty in the octet matrix element.

At LEP, we might hope that we can get a better handle on the octet matrix elements since the theoretical calculation is perhaps under better control. In this case we need not worry about factorization scale dependence in parton distribution functions or initial state gluon radiation. However, there is a complication do to final state soft gluon emission. Indeed, J/ψ production at LEP is a beautiful example of color coherence. Moreover, as opposed to hadronic collision, soft gluon effects are completely calculable in closed form in lepton initiated processes, as I will know discuss.

If one calculates the leading order differential cross section for octet production [23, 24] one finds schematically

$$\frac{d\Gamma}{dz}(Z \to J/\psi + X) \propto \alpha_s^2 \log(M_Z^2/M_{J/\psi}^2)/z + \quad , \tag{3.6}$$

where $z = 2E_{J/\psi}/M_Z$ and the terms left off are less singular in the $z \to 0$ limit. We see that the differential rate is dominated by an infrared enhancement coming from the small z region. This enhancement leads to large double logs in the total rate. Given that $\alpha \log(M_Z^2/M_{J/\psi}) \simeq 1$, a leading order prediction necessitates a resummation of these logs. Such a resummation can not be accomplished by standard ladder resummation techniques, due to the color coherence of the soft gluon radiation [25]. However, by imposing angular ordering it is possible to write down an integral equation for the resummed fragmentation function [26]. The resummed differential rate calculated in [27] is shown in figure 3. Notice that the total rate is dominated by the octet at small z. The peak at small z is an example of what is called the "hump-backed distribution" that arises in calculations of jet-multiplicities [25]. The prediction is not valid at smaller values of z due to the complete breakdown of the perturbative expansion, as well the the breakdown of a saddle point approximation used in [27]. Nonetheless, the region well below the peak contributes negligibly to the total rate. The data from LEP includes feed-down from excited states, so that the rate is not proportional to $\langle O_8^{J/\psi}\rangle$, but instead is proportional to a combination of matrix elements. This is the drawback of this study. The rate is in fact proportional to the combination

$$\langle \hat{O}_8^{\psi(m)}(^3S_1\ )\rangle \equiv \sum_m \langle O_8^{\psi(m)}(^3S_1)\rangle \times BR(\psi(m) \to J/\psi + X), \tag{3.7}$$

where $\psi(m)$ are excited states of the J/ψ. The authors of [27] found

$$\langle O_8^{J/\hat{\psi}}({}^3S_1)\rangle = (0.019 \pm 0.005_{stat} \pm 0.010_{theory})\ GeV^3. \tag{3.8}$$

The theoretical error quoted is conservative, and includes the uncertainty due to relativistic corrections, subleading logs and factorization scale dependence. If one compares this with an analogous extraction from the Tevatron data, which has errors on the order of 100%, one finds agreement. Before moving on I should point out that the peak is almost purely color octet. Thus the data seems to support the existence of this channel with a matrix element of order that found at the Tevatron.

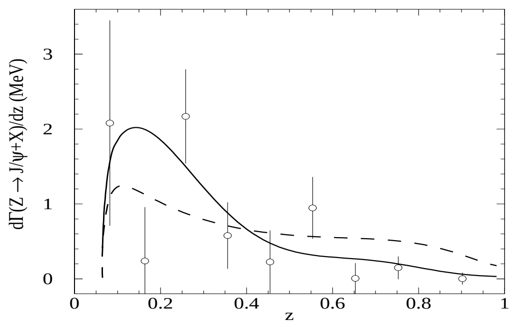

Figure 3: Results for $d\Gamma/dz$ at LEP taken from [27]. The dashed line shows the result without the resummation.

Before concluding our discussion of universality, I should address the so-called "HERA-anomaly". It has been pointed out that at HERA in the large $\hat{z}$[4] region, NRQCD predicts a sharp rise in the inelastic J/ψ direct photo-production cross-section [28, 29]. This behavior is due to the fact that the ${}^1S_0$ and ${}^3P_J$ configurations can be produced via a t-channel gluon at lowest order in α_s. A similar statement can be made of the ${}^3S_1$ configuration for resolved photo-production. No such peak near $\hat{z} \approx 1$ is seen in the data leading to the aforementioned "anomaly". However, as is now well appreciated, as $\hat{z}$ approaches 1, the theory for the spectrum becomes intractable for several reasons. Firstly, the endpoint region is sensitive to initial state intrinsic transverse momentum (equivalently initial state radiation[33]). Indeed, the authors of [34] found that by modeling the intrinsic transverse momentum with a Gaussian distribution with $\langle k_T\rangle \approx 0.7\ GeV$ they were able to fit the data. Secondly, as $\hat{z}$ approaches its endpoint ($\hat{z} > .75$) the spectrum becomes sensitive to the long distance hadronization process. This manifests itself as a breakdown in the NRQCD expansion [30, 31]. In [32] it was shown that the rate may be written in the following convoluted form

$$\frac{d\sigma}{d\hat{z}} = \int_{P^2_{T,min}} dP_T^2 \int_0^1 dx \int dy_+\, S(x,\hat{z},P_T^2)\, F_H[n](y_+)$$
$$\delta\left(s(1-\hat{z})x - \frac{M^2(1-\hat{z}) + P_T^2}{\hat{z}} - \frac{M^2(1-\hat{z})^2 + P_T^2}{\hat{z}(1-\hat{z})} y_+\right). \tag{3.9}$$

The lower cut on p_T is necessary to eliminate the diffractive contribution. The function $F_H[n](y_+)$ is a leading twist distribution function defined in [32] which physically accounts for the momentum carried by the non-perturbative gluons. For any non-zero P_T we see that the expansion parameter close to $\hat{z} = 1$ is $y_+/(1-\hat{z}) \sim v^2/(1-\hat{z})$. Thus, the NRQCD expansion breaks down for $1 - \hat{z} \sim O(v^2)$, because higher-order terms in v^2 grow more and more rapidly as $\hat{z} \to 1$. Consequently, the NRQCD factorization approach makes no prediction in the endpoint region and the discrepancy between leading order predictions and the data in this region does not allow us to draw any conclusion on the relevance of color-octet contributions to photo-production. If one averages the $\hat{z}$-distribution over a sufficiently large region containing the endpoint, the octet mechanisms contribute significantly to this average. However, the characteristic shape information is then lost and one has to deal with the more difficult and uncertain question of whether the absolute magnitude of the cross section requires the presence of octet contributions, and whether their magnitude is consistent with other production processes. One might hope to learn more about the octet matrix element from studying the smaller $\hat{z}$. However, there are large uncertainties in the

[4] $\hat{z} \equiv E_{J/\psi}/E_\gamma$ in the protons rest frame

color singlet contribution [35, 36] which make the prospect of learning much from this region rather bleak.

3.2 The question of polarization

NRQCD predicts that at large transverse momentum, when gluon fragmentation dominates, the onium should be dominantly transversely polarized [38]. This polarization arises as a consequence of the fact that the gluon is nearly on shell and thus is transversely polarized up to corrections of order $4m_c^2/p_T^2$. This transverse polarization is inherited by the onium, since the electric transitions preserve the spin of the heavy quarks. There are corrections coming from hard gluons which can flip the spin but these corrections turn out to be surprisingly small[39]. There are also spin symmetry breaking magnetic transitions which are suppressed by v^4 compared to the leading order transition. Thus, we have the rather robust prediction that ψ production at large p_T should be dominantly transversely polarized and this polarization should increase with p_T.

Presently there is data from CDF for both polarized J/ψ and ψ' production [41]. The data on the J/ψ shows no signs of polarization. However, this data includes feed-down from higher excited states, which we would expect would dilute the polarization [5]. The ψ' sample is perhaps more surprising. In this case we don't expect to have the dilution problem, and thus if our approximations are good, we should see a rise in the fraction of transversely polarized ψ''s as p_T increases. The preliminary data does not seem to show this trend. Indeed, it seems to indicate a trend towards *longitudinal* polarization with increasing p_T. Of course, the data still has large errors and thus it is perhaps as little too early to jump to any conclusions. However, the trend is rather disturbing.

4. Conclusions and Outlook

In this review, I have touched upon only two applications of NRQCD. There are several other applications which were discussed at this meeting [1]. As far as using NRQCD for extracting α_s, I think that the next important step will be to try to get a better handle on the perturbative relation between $\alpha_{\overline{MS}}$ and α_P, which will be a rather Herculean task. However, given that we believe that the series is asymptotic, there will always be doubts as to how well the series is behaving, and given the accuracy of these calculations, one always worries.

As far as the NRQCD/factorization formalism, as applied to the charmed system, is concerned we are presently in a state of uncertainty[6]. Let us review the successes and failures of NRQCD in this context. To begin with let's consider predictions for cross-sections. The data at the Tevatron for ψ' seems to necessitate a color octet contribution [42]. The spectrum is well fit, once the octet contributions are included. This is not a terrible surprise since there are two octet contributions one which behaves as $1/p_T^4$ the other as $1/p_T^6$, nonetheless this is encouraging. The overall normalization of the prediction is uncertain at about a factor of two level. At LEP the theory is under better control, but the data is much more sparse. The theory predicts a large hump at small energies due to the octet contribution which seems to be supported by the data. A combined LEP analysis of the spectrum would certainly be welcomed. As far as the HERA data is concerned the data can be well fit without the contribution from the octet channel at next to leading order[36, 37]. The expected rise at large $\hat{z}$ from the octet, which is not seen, should not trouble us at this time, for reasons discussed above. Octet contributions to production in fixed target experiments[43, 44, 45], as well as in B decays [48, 49] have been analyzed. These studies also seem to yield matrix elements of order 10^{-3} GeV^3. However, we would expect higher twist contributions in these reactions to be especially important.

As far as the qualitative size of the extracted matrix element is concerned, NRQCD seems to have it right. If we use the relation 3.4 to extract the singlet production matrix element from a decay process, we find that the ratio of octet to

[5] A quantitative study of this dilution would be helpful.

[6] I have not discussed the Υ system due to space limitations and the fact that until recently the data was rather scant.

singlet matrix elements is of order 10^{-3}. Given that we expect $v^2 \simeq 0.3$ in the J/ψ system and that according to NRQCD scaling rules the ratio of these matrix elements should scale as v^4, we seem to have qualitative agreement. However, I would like to point out that the relation 3.4 itself has never been tested. Which is to say, we have yet to extract the size of the singlet production matrix element. Indeed, all extractions of octet matrix elements assume the relation 3.4 and then use the decay matrix element, either from a potential model, or from an NRQCD extraction. One could worry that the production singlet matrix element is actually smaller than expected. As far as I can tell, this can not be ruled out by the data, at least for the J/ψ [7]. One easy way to check, would be to extract the singlet production matrix element at CLEO, where the singlet dominates, away from the endpoint [7]. While I don't expect the singlet to be anomalously small, it is still important to extract the singlet production matrix element, since the perturbative series for decays seems to be very poorly behaved [51].

For more quantitative results, we should stick to the large p_T processes where we feel more confident that higher twist effects are small. This essentially leaves us with the Tevatron data and the LEP data. The Tevatron extraction is plagued by large factorization scale dependence, which could indicate a poorly behaved perturbative series. While the next to leading order correction to the total cross-section has been calculated [47], a next to leading order calculation of the differential cross-section $d\sigma/dp_T$ would allow us to study higher order corrections without worrying about higher twist contamination. The LEP extraction, on the other hand, is limited by statistics. Thus, at least for now, testing the universality of the matrix elements, at an accuracy level beyond 100% is difficult.

Finally, there is the issue of polarization. In addition to the studies at the Tevatron, polarization has also been investigated in fixed target experiments, where the theory is compatible with the experiment within errors, given the large uncertainty in the octet matrix elements. There have also been interesting theoretical studies of J/ψ polarization in B decays [52], LEP [50] and at HERA for photo-production [35] as well a lepto-production [53], which could in the future provide useful information. Presently, we should be concerned with the trend in the Tevatron data. If, once the statistics improves, the data continues to show no trend towards transverse polarization at larger p_T then we have a very intriguing puzzle on our hands. Where could the theory be going wrong? A naive guess would be the spin symmetry is badly violated. Which is to say that perhaps the magnetic transition operators are not as suppressed at we think. However, while possible, this seems unlikely given the fact that it seems to work so well in the D meson system. If spin symmetry were badly violated, then we would expect the splitting between the D and the D^* to be much larger than it is. Of course, the matrix element which determines this splitting is different than those which induce the spin flipping transitions in ψ' production. So we can't draw any hard conclusions from smallness of the splitting. The next naive guess would be that there are large perturbative corrections to the fragmentation which lead flip the spin. But, as discussed above, these corrections have been calculated [43], and are indeed small. Another possibility is that gluon fragmentation does not dominate at large p_T. For instance, it could be that the production singlet matrix elements $\langle O_1^{\psi'}(^3S_1)\rangle$ is 30 times larger than expected from relation 3.4. The problem with this proposal is that it would mean that the LEP data is too small by an order of magnitude at large energies. Thus, there is no obvious way to depolarize the ψ' at large p_T, no less get them to be longitudinally polarized. I should stress, that the prediction for polarization is based on very basic assumptions, which have been tested in other context many times. Those assumptions are: Factorization at large p_T, spin symmetry, and the standard parton model assumptions which are routinely tested in Drell-Yan processes. These assumptions are "derivable" from QCD, and have been tested repeatedly. So if the data persists we have a real puzzle on our hands. Much more light will be shed on this problem as more data

[7] Here I am not including the data from fixed targets, since the higher twist effects could be large. Also, at HERA for $\hat{z} \approx 0.5$, it might be difficult, but given the uncertainties, not impossible.

streams in. With more upsilons we may hope to gain information as well, since in that system we believe all of our expansions will be better behaved. However, we will have to go to larger values of p_T to see the expected transverse polarization[54]. Nonetheless we still should still see the trend.

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