A new measurement of K_{e4}^+ decay and the s-wave $\pi\pi$-scattering length a_0^0

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Abstract: A sample of $4 \cdot 10^5$ events from the decay $K^+ \to \pi^+\pi^- e^+\nu_e$ (K_{e4}) has been collected in experiment E865 at the Brookhaven AGS. The analysis of these data yields new measurements of the K_{e4} branching ratio, the s-wave $\pi\pi$ scattering length a_0^0, and the form factors F, G, and H of the hadronic current.

The fourbody semileptonic decay K_{e4} ($K^+ \to \pi^+\pi^- e^+\nu_e$) has received a lot of attention, because the two pions are the only hadrons in the final state. The possibility of extracting the isospin zero, angular momentum zero scattering a_0^0 has been recognized more than thirty years ago [1]. The low branching ratio of 0.004 % made precise measurements of the decay parameters difficult, however. Early experiments [2] collected only a few hundred events. The presently generally accepted value of $a_0^0 = 0.26 \pm 0.05$ (in units of M_π^{-1}) [3] comes from a Geneva-Saclay experiment performed at CERN [4] prior to 1977.

On the theoretical side $\pi\pi$ scattering is the *golden reaction* for chiral perturbation theory (ChPT) [5]. This low-energy effective theory [6] for the strong interaction makes firm predictions for the scattering length. The pioneering tree level calculation by Weinberg [7] yielded $a_0^0 = 0.156$. The one-loop ($a_0^0 = 0.201$ [8]) and the recently completed two loop calculation ($a_0^0 = 0.217$ [9, 10]) show a satisfactory convergence. The most recent calculation [11] matches the known chiral perturbation theory representation of the $\pi\pi$ scattering amplitude to two loops with a phenomenological description that relies on the Roy equations [12]. The resulting prediction $a_0^0 = 0.220 \pm 0.005$ is in remarkable agreement with our new experimental result, which we will discuss in the following. If the experimental value $a_0^0 = 0.26$ had been confirmed with a smaller error, it could have been explained only by a significant reduction of the quark condensate $< 0|\bar{u}u|0 >$, as is possible in generalized chiral

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perturbation theory (GChPT) [13]. Remeasuring a_0^0 with higher precision hence allowed to reduce the bounds on this parameter [14].

The E865 detector [15] (see Fig. 1) was primarily designed to search for the lepton-flavor violating decay $K^+ \to \pi^+\mu^+e^-$ ($K_{\pi\mu e}$) below the 10^{-11} branching ratio level. It resided in a high-intensity, unseparated 6 GeV/c beam. The K^+ decay products were separated by charge by a first magnet and then momentum analysed by a second magnet sandwiched between two pairs of proportional chambers. The wire chambers with four planes each were kept at lower voltage in the region were the beam passes. This arrangement yielded a resolution of $\sigma_p/p^2 \approx 0.003$, where the momentum p of the decay products ($\pi^\pm, \mu^\pm$ or $e^\pm$) ranges between 0.6 and 3.5 GeV/c. Four gas filled Čerenkov counters were positioned inside and after the spectrometer magnet for $e^\pm$ identification. An electromagnetic calorimeter of Shashlyk type further helped to separate $e^\pm$ from other decay products. It was followed by an array of 12 planes of proportional tubes separated by iron plates used to discriminate pions against muons. The event trigger was based on the information from four scintillator

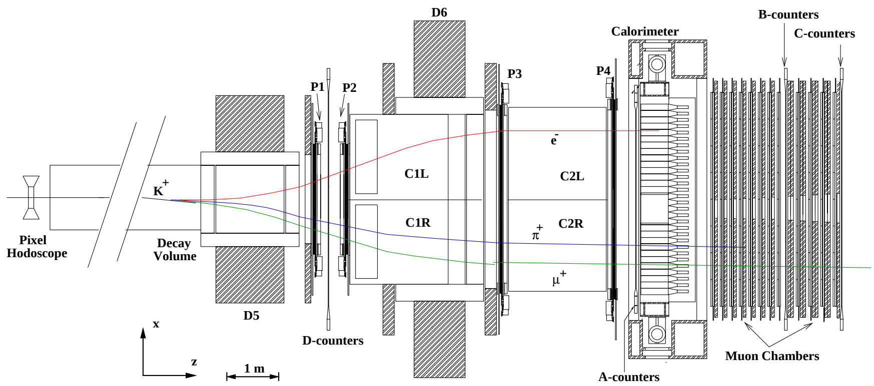

Figure 1: *Plan view of the E865 spectrometer, with a $K_{\pi\mu e}$ candidate event overlayed.*

hodoscopes (A-D), and the calorimeter. In the lowest trigger level three charged particles are required. Depending on the final state particle identification signals and topological information were added at higher levels. The flexibility of the trigger and particle identification system allowed to accumulate high statistics data samples either concurrently with $K_{\pi\mu e}$ high intensity running or in dedicated runs at lower intensity on K_{e4} and other decays like $\pi^+e^+e^-$ ($K_{\pi ee}$), $\pi^+\mu^+\mu^-$ ($K_{\pi\mu\mu}$) for physics analysis and on $\pi^+\pi^0$ ($e^+e^-\gamma$) (K_{dal}) and $\pi^+\pi^+\pi^-$ (K_τ) for normalisation.

K_{e4} events are selected with a vertex within the decay tank, a momentum reconstructed from the three daughter particles below the beam momentum, and an unambiguous identification of the e^+ and the π^- in both Čerenkov counters and the calorimeter. The momentum vector of the incoming K^+ is obtained from the hit in the pixel counter (see Fig. 1) and the vertex, and requiring the K^+ to come from the production target. The event sample contains a small amount of background events, mainly from K_τ decay (1.3 ± 0.3 %) and accidentals (2.6 ± 0.2 %). After event selection 406'103 events remain, of which we estimate 388270 ± 5025 to be K_{e4} events.

Quantitative understanding and analysis of the data relies on a Monte Carlo simulation of the detector response, which is supplemented with separately measured efficiencies of the various detector parts, and was extensively tested against data in the various decay channels under study. Figure 2 shows a few control plots demonstrating the good agreement between measured and simulated distributions of the kinematic variables.

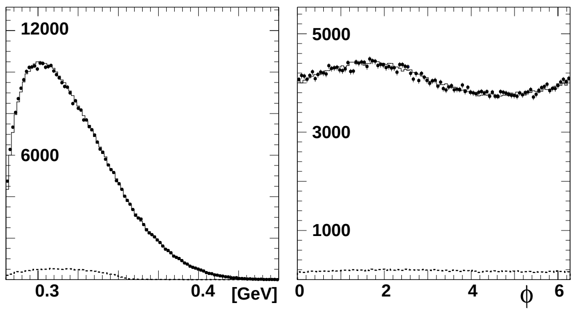

Figure 2: *Invariant dipion mass and angle distributions describing K_{e4} decay. The solid histogram represents the Monte Carlo simulation, the dashed histogram the experimental background, and the solid circles with errors bars the data.*

We also compare Monte Carlo with data distributions for the kinematically very distinct K_τ and K_{dal} decays. A chiral perturbation theory calculation on the one loop level [16] is used to model the physics of the decay at this point. Radiative corrections are included following Diamant-Berger [17]. The K_{e4} branching ratio was determined relative to K_τ decay, for which events were collected with a minimum bias trigger concurrently with K_{e4} events. The result (4109 $\pm$ 8 (stat.) $\pm$110 (syst.) $\cdot\, 10^{-8}$) is in agreement with the average of previous measurements ((3.91 $\pm$ 0.17)$\cdot 10^{-5}$ [2]). The largest of the 17 contributions to the systematic error (2.7%), which we have evaluated, stem from the background subtraction (1.2%), the Čerenkov efficiencies (1.5%) and the K_τ branching ratio (0.9%).

The kinematic of K_{e4} decay can be fully described by five variables [18]: (1) $s_\pi = M^2_{\pi\pi}$, and (2) $s_e = M^2_{e\nu}$, the invariant mass squared of the dipion and the dilepton, resp.; (3) θ_π and (4) θ_e, the polar angles of π^+ and e^+ in the dipion and dilepton rest frames measured with respect to the flight direction of dipion and dilepton in the K^+ rest frame, resp.; (5) ϕ, the azimuthal angle between the dipion and dilepton planes. The matrix element in terms of the hadronic vector and axial vector current contributions V^μ and A^μ is then given by

$$M = \frac{G_F}{\sqrt{2}} V^*_{us} \overline{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_e)(V^\mu - A^\mu) \ , \tag{1}$$

$$A^\mu = FP^\mu + GQ^\mu + RL^\mu \ , \ \ V^\mu = H\epsilon^{\mu\nu\rho\sigma} L_\nu P_\rho Q_\sigma \ , \tag{2}$$

where $P = p_1 + p_2$, $Q = p_1 - p_2$, and $L = p_e + p_\nu$, and p_1, p_2, p_e, and p_ν are the four-momenta of the π^+, π^-, e^+, and ν_e in units of M_K, resp. The form factors F, G, R and H are dimensionless complex functions of s_π, s_e and θ_π. If terms $\propto M_e^2/s_e$ are neglected R does not contribute to the decay rate [19, 16]. A convenient partial wave expansion of the form factors in the variable θ_π gives the following [20]:

$$\begin{array}{l} F = \left(f_s + f'_s\, q^2 + f''_s\, q^4 + f_e\, s_e\right) e^{i\delta^0_0(s_\pi)} + \tilde{f}_p\, (q^2/s_\pi)^{1/2}\, (P\cdot L)\, \cos\theta_\pi\, e^{i\delta^1_1(s_\pi)} \ , \\ G = \left(g_p + g'_p\, q^2 + g_e\, s_e\right) e^{i\delta^1_1(s_\pi)} \ , H = \left(h_p + h'_p\, q^2\right) e^{i\delta^1_1(s_\pi)} \ , \end{array} \tag{3}$$

where $q = \sqrt{s_\pi/4M_\pi^2 - 1}$ is the pion momentum in the dipion rest frame, divided by M_π. This parameterization yields ten new form factors f_s, f'_s, f''_s, f_e, $\tilde{f}_p$, g_p, g'_p, g_e, h_p, and h'_p, which do not depend on any kinematical variables, plus the phases δ^0_0 and δ^1_1, which are still functions of s_π.

It is worth noting at this point, that the phase shifts enter into the decay rate only through interference terms, which are proportional to $\sin\phi$ or $\cos\phi$. The azimuthal ϕ asymmetry, which has an amplitude of only approximately 10% of the the average (Fig. 2), is principal source of the phase shift information.

The phase shifts at low energy can be related to the scattering lengths. A recent analysis [21] solved the Roy equations [12] numerically, with the free parameters expressed as functions of the scattering lengths using high energy $\pi\pi N$ data as additional constraint. The possible values of the scattering lengths are restricted to a band in the a_0^0 versus a_0^2 plane. The centroid of this band, the *universal curve* [22] relates a_0^0 and a_0^2:

$$a_0^2 = -0.0849 + 0.232\, a_0^0 - 0.0865\, (a_0^0)^2\, [\pm 0.0088]\ , \tag{4}$$

where the error given in the bracket indicates the width of the band permitted by analyticity [21]. The band can be narrowed, if chiral symmetry constraints [14] are imposed:

$$a_0^2 = -0.0444 + 0.236\, (a_0^0 - 0.22) - 0.61\, (a_0^0 - 0.22)^2 - 9.9(a_0^0 - 0.22)^3\, [\pm 0.0008]\ , \tag{5}$$

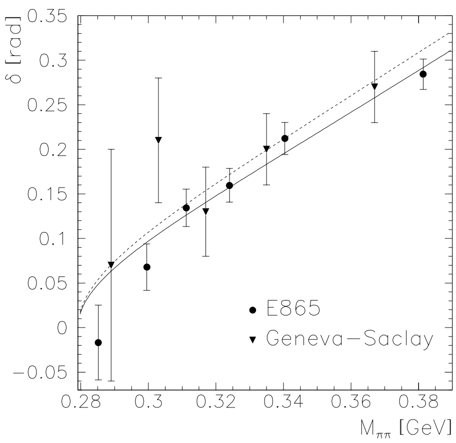

Figure 3: *The E865 phase shift results as function of $M_{\pi\pi}$ compared to the Geneva-Saclay data[4]. The solid curve is the result of the fit to our data.*

For the fits we divided our data into six bins in s_π, five in s_e, ten in $\cos\theta_\pi$, six in $\cos\theta_e$ and 16 in ϕ. In the χ^2 minimisation procedure, the number of measured events in each bin j is compared to the number of expected events given by:

$$r_j = Br(K_{e4}) \frac{N^K}{N^{MC}} \sum \frac{J_5(F,G,H)^{new}}{J_5(F,G,H)^{MC}}\ , \tag{6}$$

where the sum runs over all Monte Carlo events in bin j. N^K is the number of K^+ decays derived from the number of K_τ events. N^{MC} is the number of generated events. $J_5(F,G,H)^{MC}$ ($\equiv I$ [19]) is the five-dimensional phase space density generated at the momentum $q = q^{MC}$ with the form factors F, G, and H used to simulate the event. $J_5(F,G,H)^{new}$ is calculated at q^{MC} with F, G, H evaluated from the parameters of the fit. Thus, we apply the parameters on an event by event basis, and, at the same time, we divide out a possible bias caused by the matrix element, making the fit independent of the ChPT ansatz used to generate the MC. For the fit, we have assumed that F, G, and H do not depend on s_e and that F contributes to s-waves only, i.e. $f_e = g_e = \tilde{f}_p = 0$. Our first set of fits is done independently for each bin in s_π. The above assumptions then leave one parameter each to describe F, G, and H aside from the phase difference $\delta \equiv \delta_0^0 - \delta_1^1$. The results [23] for the phase shifts are given in Figure 3.

We have also made a single fit to the entire data sample. In this second fit we substituted δ in Eq. 3 by the expansion in terms of a_0^0 and a_0^2 [21]. With the relation between a_0^0 and a_0^2 given by Eq. 4 or Eq. 5 only f_s, f_s', f_s'', g_p, g_p', h_p, and a_0^0 then remain as free parameters. The results are listed in Table 1.

f_s: $575 \pm 2 \pm 8$	f'_s: $106 \pm 10 \pm 40$	f''_s: $-59 \pm 12 \pm 40$
g_p: $466 \pm 5 \pm 7$	g'_p: $67 \pm 10 \pm 4$	h_p: $-295 \pm 19 \pm 20$
a_0^0: $0.228 \pm 0.012 \pm 0.004^{+0.006}_{-0.012}$ (Eq. 4)		
a_0^0: $0.216 \pm 0.013 \pm 0.004 \pm 0.005$ (Eq. 5)		

Table 1: Form factors (in units of 10^{-2}) and scattering length a_0^0 in the parameterisation of Eq. 3 using either Eq. 4 or Eq. 5. The sequence of errors given is statistical, systematic and theoretical.

To summarize, experiment E865 has collected a K_{e4} event sample more than ten times larger than all previous experiments combined. From the model independent analysis of this data the momentum dependence of the form factors of the hadronic currents as well as $\pi\pi$ scattering phase shifts have been extracted. The form factors and phase shifts serve as an important input in the program to determine the couplings of the effective Hamiltonian of chiral QCD perturbation theory at low energies [24]. Using the relations between a_0^0 and a_0^2 given by the Roy equations [21] and chiral symmetry constraints [14], we have extracted the most precise value of the $\pi\pi$ scattering length a_0^0, which agrees well with predictions obtained in the framework of ChPT [11].

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