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Magnetic polarizability of hadrons in the background field method

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We present a calculation of hadron magnetic polarizability using the techniques of lattice QCD. This is carried out by introducing a uniform external magnetic field on the lattice and measuring the quadratic part of a hadron's mass shift. The calculation is performed on a 24^4 lattice with standard Wilson actions at beta=6.0 (spacing $a = 0.1$ fm) and pion mass down to about 500 MeV. Results are obtained for 30 particles covering the entire baryon octet (n, p, Σ^0, Σ^-, Σ^+, Ξ^-, Ξ^0, Λ) and decuplet (Δ^0, Δ^-, Δ^+, Δ^{++}, Σ^{*0}, Σ^{*-}, Σ^{*+}, Ξ^{*0}, Ξ^{*-}, Ω^-), plus selected mesons (π^0, π^+, π^-, K^0, K^+, K^-, ρ^0, ρ^+, ρ^-, K^{*0}, K^{*+}, K^{*-}). The results are compared with available values from experiments and other theoretical calculations.

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1. Introduction

Polarizabilities are fundamental properties of elementary particles. They are a measure of the rigidness of a bound system to external perturbations, and provide valuable insight into the internal structure of the system. In [1], the results of a lattice calculation of hadron electric polarizability for neutral hadrons, based on the methods of [2], has been presented. In this poster, we will focus on hadron magnetic polarizability from lattice QCD. Traditionally, the symbol α is used to represent the electric polarizability, and β is used for the magnetic polarizability.

A considerable number of experiments have been performed with the aim of measuring nucleon polarizabilities [3 – 12]. Most of these have specifically targeted the proton, some attempts have also been made at measuring the neutron polarizabilities. Despite some shortcomings, measurements for the proton polarizabilities in these experiments are in reasonable agreement with each other. The experimental situation regarding the polarizabilities of the neutron is still quite unsatisfactory. This is mainly because a neutron Compton scattering experiment cannot be directly performed.

The experimental situation on the nucleon can be approximately summarized as the following: the electric polarizability is roughly the same for the proton and neutron, with a value of around 10 in units of 10^{-4} fm^3; the magnetic polarizability is roughly the same for the proton and neutron, with a value of around 3 in the same units.

On the theoretical side, nucleon polarizabilities have been most studied in the framework of chiral perturbation theory (ChPT) [13 – 18]. Other approaches include quark models [19 – 21], chiral soliton models [22, 23]. For reviews on polarizabilities, see Refs. [24, 25].

2. Method

For small external magnetic fields, the mass shift

$$\Delta m(B) \equiv m(B) - m(0), \tag{2.1}$$

is given by

$$\Delta m(B) = -\vec{\mu}\cdot\vec{B} - \frac{1}{2}\beta\vec{B}^2. \tag{2.2}$$

By averaging $\Delta m(B)$ over the field , $\vec{B}$, and its inverse, $-\vec{B}$, we will form

$$\Delta m(B)_{even} = -\frac{1}{2}\beta\vec{B}^2. \tag{2.3}$$

After we get the even $\vec{B}$ mass shift from the lattice simulation, we do least-chi-square fits to the data points with a polynomial

$$\Delta m(B)_{even} = c_2 B^2 + c_4 B^4 + \cdots. \tag{2.4}$$

The magnetic polarizability is then the negative quadratic coefficient

$$\beta = -2c_2. \tag{2.5}$$

The quartic and higher terms in Eq.(2.4) are included in order to measure possible numerical contamination.

To place the external magnetic field on the lattice in the z-direction, we multiply each gauge field link variable in the y-direction with a x-dependent factor:

$$U_2(x) \to (1+i\eta\rho)U_2(x), \tag{2.6}$$

where the two dimensionless parameters are given by $\eta = qBa^2$ and $\rho = x_1/a$. The procedure we use is very similar but not exactly the same as the one presented in [2].

3. Simulation Details

Most of our results are based on the standard Wilson quark action on a quenched 24^4 lattice with lattice spacing $a = 0.1$ fm. The lattice coupling $\beta = 6.0$. We have analyzed 150 configurations to extract hadron magnetic polarizabilities. Fermion propagators M^{-1} were constructed at six different quark masses, which corresponding to six κ values, κ= 0.1515, 0.1525, 0.1535, 0.1540, 0.1545, 0.1555. The critical kappa value is $\kappa_c = 0.157096$. The strange quark mass is set at $\kappa = 0.1535$. The corresponding pion masses are: 1000 MeV, 895 MeV, 782 MeV, 721 MeV, 657 MeV, 512 MeV. In the units of $10^{-3}e^{-1}a^{-2}$, the magnetic field takes the values -1.08, 2.16, -4.32, and 8.64. The lattice source is (x,y,z,t)= (12, 1, 1, 2).

We used six different values of the parameter in the units of 10^{-3} η = 0.0, +0.36, -0.72, +1.44, -2.88, +5.76. The η values in this sequence are related by a factor of -2. Thus we are able to study the response of a hadron composed of both up and down (strange) quarks, whose charges are related by the same factor, to four different nonzero magnetic fields. In the units of $10^{-3}e^{-1}a^{-2}$, the magnetic field takes the values -1.08, 2.16, -4.32, and 8.64. In physical units, the magnitude of the weakest magnetic field is 2.46 $\times 10^{13}$ tesla. This is a very strong magnetic field. On the other hand, in the sense of the mass shift, this is really not that strong. We can estimate the ratio of mass shift of proton to the mass of proton: $\frac{\delta m}{m} = \frac{\frac{1}{2}\beta_p B^2}{m} = 1.60 \times 10^{-5}$. Here we have used $2{\times}10^{-4}$ fm^3 as the value of β_p. From this rough estimation we can see that the mass shift is very small even in such a strong magnetic field.

4. Results

We investigated 30 particles sweeping through the baryon octet and decuplet and selected mesons. The results of our calculation are summarized in three tables 1, 2, and 3 Most of the polarizabilities have not been measured, except for the nucleon and pion [26, 27], so most of our results are predictions. Below is a summary of the main results of the calculation. Other theoretical calculations can be found in [28 – 33].

Our value for the proton magnetic polarizability agrees reasonably with the most recent world average value of about 3 in units of 10^{-4} fm^3, but suffers from relatively large errors. Our value for the neutron magnetic polarizability (about 15 to 20), which has smaller errors, is much greater than the world average value. This large difference between the proton and the neutron on the lattice is an interesting result that is worth further study.

In the decuplet sector, the most interesting result is the large and negative magnetic polarizability (about -60) for the Δ^{++}. Theoretical explanations, such as those from chiral effective theories, are called for. The Ω^- value of $-12.4(2)$ is a prediction free of uncertainty from chiral extrapolations, and can be directly compared with experiment. A measurement of the Ω^- magnetic polarizability is greatly desired.

Between the octet and the decuplet, we observe the following pattern. Positively-charged p and Σ^+ have relatively small and positive values, albeit with large errors, a situation opposite to that for the decuplet members Δ^+ and Σ^{*+} which have negative and slightly larger values along with smaller errors. It is important to increase the statistics to obtain a better signal for the p and Σ^+ on the lattice to confirm this difference. In addition, the charge-neutral particles have values on the order of 20, while the negatively-charged particles on the order of -20.

In the meson sector, we confirmed the expected result that a positively-charged particle has identical magnetic polarizability as its negatively-charged partner ($\pi^\pm$, $K^\pm$, $\rho^\pm$, $K^{*\pm}$). All charged mesons have negative magnetic polarizabilities, and all neutral mesons have positive ones. In terms of magnitude, the pseudoscalar mesons have about twice as large values as the vector mesons. Futhermore, the vector mesons have a weaker quark mass dependence than the pseudoscalar mesons.

Taken as a whole, our results demonstrate the efficacy of the methods used in computing the magnetic polarizabilities on the lattice. In addition to increasing statistics to the 300 to 500 configurations range, further studies should focus on assessing the systematic uncertainties.

Finally, as far as the cost of our calculation is concerned, it is equivalent to 11 standard mass-spectrum calculations using the same action (5 values of the parameter η to provide 4 non-zero magnetic fields, 5 to reverse the field, plus the zero-field to set the baseline). This factor can be reduced to 7 if only two non-zero values of magnetic field are desired. Reversing the field is well worth the cost: the magnetic moments can be extracted from the linear response in the mass shifts in the same data set [34].

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Table 1: The calculated magnetic polarizabilities for the octet baryons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
p	0.09 $\pm$ 0.29	0.14 $\pm$ 0.37	0.26 $\pm$ 0.48	0.40 $\pm$ 0.56	0.64 $\pm$ 0.67	2.36 $\pm$ 1.20	12-14
n	9.4 $\pm$ 0.2	10.4 $\pm$ 0.3	11.6 $\pm$ 0.4	12.3 $\pm$ 0.5	13.4 $\pm$ 0.6	17.0 $\pm$ 1.1	12-14
Σ^+	-0.15 $\pm$ 0.36	0.09 $\pm$ 0.42	0.24 $\pm$ 0.50	0.40 $\pm$ 0.56	0.61 $\pm$ 0.64	1.60 $\pm$ 1.00	12-14
Σ^0	8.0 $\pm$ 0.3	8.5 $\pm$ 0.3	9.1 $\pm$ 0.4	9.6 $\pm$ 0.5	10.1 $\pm$ 0.6	11.9 $\pm$ 0.9	12-14
Σ^-	-11.8 $\pm$ 0.3	-12.6 $\pm$ 0.3	-13.6 $\pm$ 0.4	-14.2 $\pm$ 0.4	-14.7 $\pm$ 0.4	-16.1 $\pm$ 0.5	12-14
Ξ^0	10.7 $\pm$ 0.3	11.3 $\pm$ 0.4	11.9 $\pm$ 0.4	12.3 $\pm$ 0.4	12.8 $\pm$ 0.5	13.9 $\pm$ 0.7	12-14
Ξ^-	-12.6 $\pm$ 0.3	-13.1 $\pm$ 0.3	-13.8 $\pm$ 0.4	-14.1 $\pm$ 0.4	-14.6 $\pm$ 0.4	-15.6 $\pm$ 0.5	12-14
Λ^8	9.1 $\pm$ 0.4	9.9 $\pm$ 0.5	10.8 $\pm$ 0.6	11.4 $\pm$ 0.7	12.1 $\pm$ 0.8	14.0 $\pm$ 1.2	12-14
Λ^C	9.3 $\pm$ 0.4	10.2 $\pm$ 0.5	11.2 $\pm$ 0.6	11.9 $\pm$ 0.7	12.6 $\pm$ 0.9	15.0 $\pm$ 1.4	12-14
Λ^S	3.6 $\pm$ 0.1	3.4 $\pm$ 0.2	3.3 $\pm$ 0.2	3.3 $\pm$ 0.3	3.2 $\pm$ 0.4	3.2 $\pm$ 1.0	5-7

Table 2: The calculated magnetic polarizabilities for the decuplet baryons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
Δ^{++}	-39.9 $\pm$ 0.7	-43.9 $\pm$ 0.8	-48.7 $\pm$ 1.0	-51.5 $\pm$ 1.1	-54.7 $\pm$ 1.3	-63.1 $\pm$ 1.9	10-12
Δ^+	-2.5 $\pm$ 0.2	-3.0 $\pm$ 0.3	-3.5 $\pm$ 0.5	-3.9 $\pm$ 0.6	-4.3 $\pm$ 0.7	-5.1 $\pm$ 1.1	10-12
Δ^0	7.6 $\pm$ 0.2	8.2 $\pm$ 0.3	8.8 $\pm$ 0.4	9.2 $\pm$ 0.5	9.6 $\pm$ 0.6	10.9$\pm$ 1.0	10-12
Δ^-	-10.1 $\pm$ 0.2	-11.1 $\pm$ 0.2	-12.4 $\pm$ 0.2	-13.1 $\pm$ 0.3	-14.0 $\pm$ 0.3	-16.2 $\pm$ 0.5	10-12
Σ^{*+}	-2.9 $\pm$ 0.3	-3.2 $\pm$ 0.4	-3.6 $\pm$ 0.5	-3.9 $\pm$ 0.6	-4.2 $\pm$ 0.6	-5.1 $\pm$ 0.9	10-12
Σ^{*0}	7.9 $\pm$ 0.2	8.4 $\pm$ 0.3	8.9 $\pm$ 0.4	9.2 $\pm$ 0.5	9.5 $\pm$ 0.6	10.3 $\pm$ 0.8	10-12
Σ^{*-}	-10.9 $\pm$ 0.2	-11.7 $\pm$ 0.2	-12.6 $\pm$ 0.3	-13.1 $\pm$ 0.3	-13.7 $\pm$ 0.3	-15.0 $\pm$ 0.4	10-12
Ξ^{*0}	8.1 $\pm$ 0.3	8.5 $\pm$ 0.3	9.0 $\pm$ 0.4	9.2 $\pm$ 0.5	9.4 $\pm$ 0.5	9.8 $\pm$ 0.7	10-12
Ξ^{*-}	-11.9 $\pm$ 0.2	-12.4 $\pm$ 0.2	-12.8 $\pm$ 0.3	-13.1 $\pm$ 0.3	-13.4 $\pm$ 0.3	-14.0 $\pm$ 0.3	10-12
Ω^-			-12.4 $\pm$ 0.2				10-12

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Table 3: The calculated magnetic polarizabilities for the selected mesons as a function of the pion mass from the Wilson action. The pion mass is in GeV and the magnetic polarizability is in 10^{-4} fm^3. The time window from which each polarizability is extracted is given in the last column. The errors are statistical.

κ	0.1515	0.1525	0.1535	0.1540	0.1545	0.1555	fit range
m_π	1.000	0.895	0.782	0.721	0.657	0.512	
$\pi^\pm$	-16.0 $\pm$ 0.3	-17.8 $\pm$ 0.4	-19.8 $\pm$ 0.5	-21.1 $\pm$ 0.5	-22.5 $\pm$ 0.6	-26.4 $\pm$ 0.8	16-18
π^0	8.4 $\pm$ 0.2	9.1 $\pm$ 0.3	10.1 $\pm$ 0.4	10.8 $\pm$ 0.5	11.6 $\pm$ 0.5	14.1 $\pm$ 0.8	16-18
$K^\pm$	-18.4 $\pm$ 0.4	-19.4 $\pm$ 0.4	-20.5 $\pm$ 0.5	-21.1 $\pm$ 0.5	-21.7 $\pm$ 0.6	-23.1 $\pm$ 0.6	16-18
K^0	3.7 $\pm$ 0.1	3.8 $\pm$ 0.2	4.0 $\pm$ 0.2	4.1 $\pm$ 0.2	4.3 $\pm$ 0.2	4.7 $\pm$ 0.3	16-18
$\rho^\pm$	-11.8 $\pm$ 0.3	-12.5 $\pm$ 0.3	-13.1 $\pm$ 0.4	-13.3 $\pm$ 0.5	-13.3 $\pm$ 0.6	-12.5 $\pm$ 1.2	14-16
ρ^0	5.3 $\pm$ 0.2	5.6 $\pm$ 0.2	5.9 $\pm$ 0.3	6.0 $\pm$ 0.4	6.1 $\pm$ 0.4	6.5 $\pm$ 0.7	9-11
$K^{*\pm}$	-13.0 $\pm$ 0.3	-13.2 $\pm$ 0.4	-13.3 $\pm$ 0.5	-13.3 $\pm$ 0.5	-13.2 $\pm$ 0.6	-12.7 $\pm$ 0.8	14-16
K^{*0}	3.0 $\pm$ 0.2	3.2 $\pm$ 0.2	3.3 $\pm$ 0.3	3.5 $\pm$ 0.3	3.6 $\pm$ 0.3	4.1 $\pm$ 0.5	14-16

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