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# Meson decay constants from $N_{\mathrm{f}} = 2$ clover fermions

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We present recent results for meson decay constants calculated on configurations with two flavours of O(a)-improved Wilson fermions. Non-perturbative renormalisation is applied and quark mass dependencies as well as finite volume and discretisation effects are investigated. In this work we also present the first computation of the coupling of the light vector mesons to the tensor current using dynamical fermions.

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## 1. Introduction

The theoretical description of decay processes which include light mesons strongly relies on a non-perturbative and model independent determination of the relevant parameters. The experimentally most accurately known decay constant is the pseudo-scalar decay constant, $f_\mathrm{PS}$, which can be determined from the leptonic decay of charged pions or kaons. Attempts to compute this quantity from first principles on the lattice is an important validation of the lattice QCD methodology. It allows in particular to test control on the required extrapolations to the infinite volume, continuum and finally chiral limit. For phenomenology much more interesting is a lattice determination of the coupling of the light vector mesons to the vector current ($f_\mathrm{V}$) and the tensor current ($f_\mathrm{V}^\perp$). While the constant $f_\mathrm{V}$ can in principle be extracted from experimental studies of, e.g., leptonic decays like $\tau^- \to V^- \nu_\tau$, the coupling $f_\mathrm{V}^\perp$ can only be determined theoretically. It has been pointed out by the authors of [1] that the form factors describing the semi-leptonic B-decay $B \to \rho l \nu$ strongly depend on these couplings. A precise calculation of these couplings is hence an important step towards a more precise determination of one of the less well known CKM parameters, $|V_\mathrm{ub}|$.

## 2. Lattice Details

The configurations with $N_\mathrm{f}=2$ flavours of non-perturbatively improved Wilson fermions and Wilson glue which have been generated by the QCDSF and UKQCD collaborations in recent years now cover a larger range of quark masses and several lattice spacings. The simulations are performed for several values of the gauge coupling $\beta$ with the lattice spacing varying between $0.07-0.11$ fm. For each lattice spacing configurations are generated for 3-4 sea quark masses in the range of $590-1170$ MeV. For additional leverage on the quark mass dependence we performed simulations at a large number of valence quark masses which vary between 470 and 1200 MeV. Also the volumes are varied to explore finite size effects. Finally, a non-perturbative determination of the renormalisation constants is performed to further reduce systematic uncertainties.

Correlation functions for pseudo-scalar and vector mesons are calculated on configurations taken at a distance of 5-10 trajectories using 8-4 different locations of the fermion source. We use binning to obtain an effective distance of 20 trajectories. The size of the bins has little effect on the error, which indicates auto-correlations are small. To improve overlap with the ground state we employ Jacobi smearing.

In order to scale our results at different values of $\beta$ and $\kappa_\mathrm{sea}$ we use the Sommer-scale $r_0$, which can be determined with good precision on the lattice. When translating this into more common physical units, we face the problem that the static quark potential is difficult to determine experimentally. We therefore choose to use lattice results for $r_0 m_\mathrm{N}$ (where $m_\mathrm{N}$ is the nucleon mass) to obtain $r_0 = 0.467$ fm. We typically will use $r_0/a$ at the respective values of $\beta$ and $\kappa_\mathrm{sea}$. Only when stated explicitly we will use $r_0/a$ extrapolated to the chiral limit [2].

To fully eliminate $O(a)$ discretisation errors, it is not sufficient to improve the action, the operators also need to be improved. To determine the renormalised pseudo-scalar decay constant we therefore have to calculate

$$f_\mathrm{PS} = Z_\mathrm{A} \, (1 + b_\mathrm{A} \, am_\mathrm{q,sea}) \, f_\mathrm{PS}^{(0)} \left( 1 + c_\mathrm{A} \frac{f_\mathrm{PS}^{(1)}}{f_\mathrm{PS}^{(0)}} \right), \tag{2.1}$$

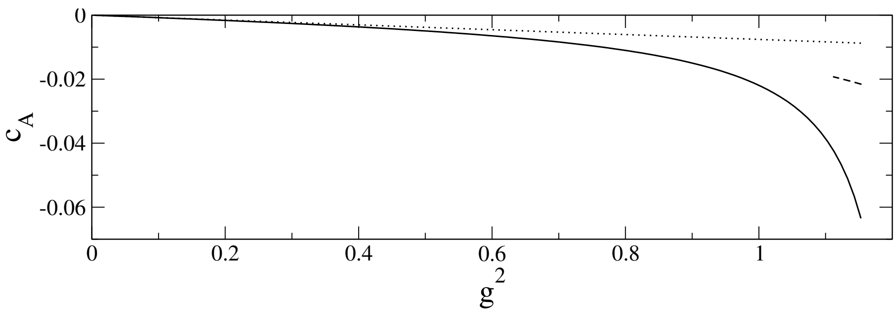


**Figure 1:** Comparison of $c_A$ determined from perturbation theory (dotted line, [6]), boosted perturbation theory (dashed line), and non-perturbative methods (solid line, [4]).

where $Z_A$ is the axial vector current renormalisation factor and $b_A$ a coefficient describing its dependency on the bare sea-quark mass $am_{q,sea} = \frac{1}{2}(1/\kappa_{sea} - 1/\kappa_{sea}^c)$. $c_A$ is the improvement coefficient which needs to be chosen such that $O(a)$ discretisation errors cancel.

The unimproved bare decay constant $f_{PS}^{(0)}$ and the improvement term are computed from the matrix elements $m_{PS} f_{PS}^{(0)} = \langle 0|A_4|PS\rangle$ and $m_{PS} a f_{PS}^{(1)} = \langle 0|a\partial_4 P|PS\rangle$. While the renormalisation constant $Z_A$ [3, 4] (we use [3]) and the the critical value of the hopping parameter $\kappa_{sea}^c$ [5] have been determined non-perturbatively, the coefficient $b_A$ is only known perturbatively [6]. As the expansion in the bare gauge coupling $g^2 = 6/\beta$ is known to have bad convergence properties, we use the boosted coupling $g^{\overline{MS}}(\mu)$ at the scale $\mu = 1/a$. This coupling is computed from the renormalisation group equations with $\mu/\Lambda^{\overline{MS}} = (r_0/a)/(r_0\Lambda^{\overline{MS}})$, taking $r_0/a$ in the chiral limit and $r_0\Lambda^{\overline{MS}} = 0.617(40)(21)$ [2]. To assess the differences between perturbation theory, boosted perturbation theory and non-perturbative methods, we compare the results for $c_A$. As can been seen from Fig. 1 the values from boosted perturbation theory are much closer to the non-perturbative results. However, there is still a significant discrepancy between the two results.

Similarly, we define the renormalised and improved vector decay constant $f_V$:

$$\frac{1}{f_V} = Z_V \ (1 + b_V \, am_{q,sea}) \ f_V^{(0)} \left(1 + c_V \frac{f_V^{(1)}}{f_V^{(0)}}\right), \tag{2.2}$$

where $e(\lambda)_i m_V^2 f_V^{(0)} = \langle 0|V_i|V,\lambda\rangle$ and $e(\lambda)_i m_V^2 a f_V^{(1)} = \langle 0|a\partial_4 T_{i4}|V,\lambda\rangle$ with $e(\lambda)_i$ being the polarization vector. The renormalisation constant $Z_V$ and the coefficient $b_V$ have been computed non-perturbatively [7], but we have to rely on a perturbative calculation of $c_V$ [6].

Finally, we define the renormalised and improved constant for the coupling of the vector meson to the tensor current, $f_V^\perp$:

$$f_V^\perp(\mu) = Z_T^{\mathscr{S}}(\mu) \ (1 + b_T \, am_{q,sea}) \ \left(f_V^{\perp(0)} + c_T \, f_V^{\perp(1)}\right), \tag{2.3}$$

where $e(\lambda)_i m_V f_V^{\perp(0)} = \langle 0|T_{\mu\nu}|V,\lambda\rangle$ and $e(\lambda)_i m_V a f_V^{\perp(1)} = \langle 0|a(\delta_{\mu\rho}\partial_\rho V_\nu - \delta_{\nu\rho}\partial_\rho V_\mu)|V,\lambda\rangle$. For the coefficients $b_T$ and $c_T$ only perturbative results are available [6]. The QCDSF collaboration has calculated the renormalisation constant (which depends on both, a scheme $\mathscr{S}$ and a scale $\mu$) non-perturbatively in the RI′-MOM scheme [7]. We use the 4-loop and 3-loop results for the $\beta$-function and anomalous dimension to translate our results into the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV [8].

## 3. Results

The results which are obtained for different values of the gauge coupling $\beta = 5.20$, 5.25, 5.29 and 5.40 can be well parametrised by the following ansatz:

$$r_0 f_x(a, r_0 m_{\mathrm{PS,sea}}, r_0 m_{\mathrm{PS,val}}) = r_0 F_x(a) \Big[ 1 + \alpha_0 \, (r_0 \, m_{\mathrm{PS,sea}})^2 + \alpha_1 \, (r_0 \, m_{\mathrm{PS,val}})^2 + \beta_0 \, (r_0 \, m_{\mathrm{PS,sea}})^4 + \beta_1 \, r_0^4 \, m_{\mathrm{PS,sea}}^2 \, m_{\mathrm{PS,val}}^2 + \beta_2 \, (r_0 \, m_{\mathrm{PS,val}})^4 \Big]. \quad (3.1)$$

Here $am_{\mathrm{PS,sea}}$ denotes the mass of the pseudo-scalar meson for $\kappa_{\mathrm{sea}} = \kappa_{\mathrm{val}}$, while for $am_{\mathrm{PS,val}}$ the valence and sea quark mass might be different. Given the small number of sea quark masses, we fixed the value of $\beta_0 = 0$. In case of $f_{\mathrm{V}}^{\perp}$ we find the valence quark mass dependency to be linear in $(am_{\mathrm{PS,val}})^2$ and therefore we also fix $\beta_2 = 0$. The results of our fits to this ansatz are shown in Fig. 2, 3 and 4. As can been seen from these figures, discretisation effects seem to be small, in particular compared to the statistical errors. For $f_{\mathrm{PS}}$ and $f_{\mathrm{V}}^{\perp}$ we therefore fit the results in the chiral limit to a constant. Only in case of $1/f_{\mathrm{V}}$ we find much better agreement when fitting to a polynomial linear in $(a/r_0)^2$.

For $\beta = 5.29$ and $\kappa_{\mathrm{sea}} = 0.13550, 0.13590$ we determine the decay constants for three different volumes $L^3 \times T = 12 \times 32$, $16 \times 32$ and $24 \times 48$, which corresponds to $L \approx 1.0, 1.3, 2.0$ fm. We find the pseudo-scalar decay constant $f_{\mathrm{PS}}$ to be shifted by $2-6\%$ when increasing the spatial extent from 1.3 to 2.0 fm. This seems to be larger than one would expect from the results of chiral perturbation theory [9]. No significant finite size effects have been observed for the vector couplings $f_{\mathrm{V}}$ and $f_{\mathrm{V}}^{\perp}$.

In the chiral limit, our result for the pseudo-scalar decay constant is $131(7)$ MeV, where only the statistical error has been taken into account. An extrapolation to the physical pion mass gives a slightly larger value of 133(7) MeV, which still agrees very well with the experimental result $f_\pi^{\mathrm{exp}} = 130.7$ MeV. Chiral logarithms predicted by chiral perturbation theory, which have not

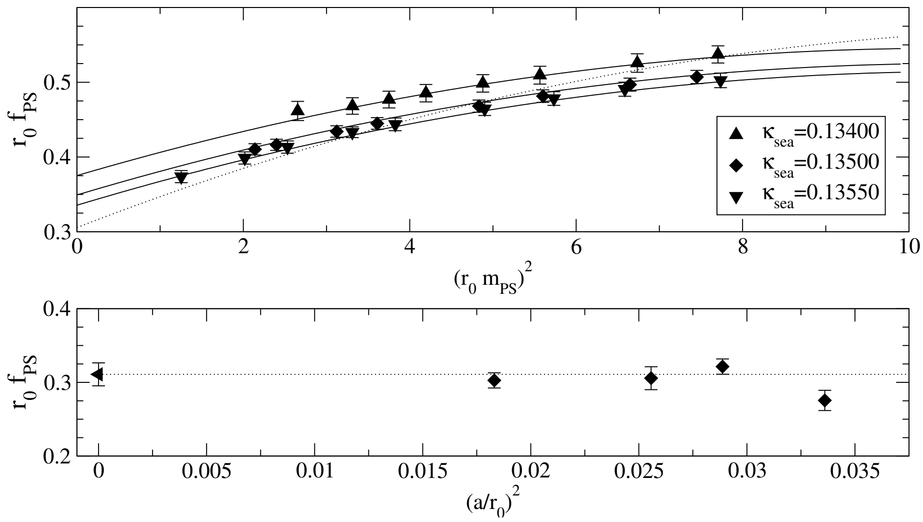


**Figure 2:** The upper plot displays the result for $r_0 f_{\mathrm{PS}}$ at $\beta = 5.29$. The lines show the result of a fit to Eq. (3.1). Solid lines are used for the valence quark mass dependency, a dotted for the sea quark mass dependency. The lower plot shows $r_0 f_{\mathrm{PS}}$ as a function of $(a/r_0)^2$ together with a fit to a constant. Both $r_0/a$ and $a f_{\mathrm{PS}}$ have been extrapolated to the chiral limit.

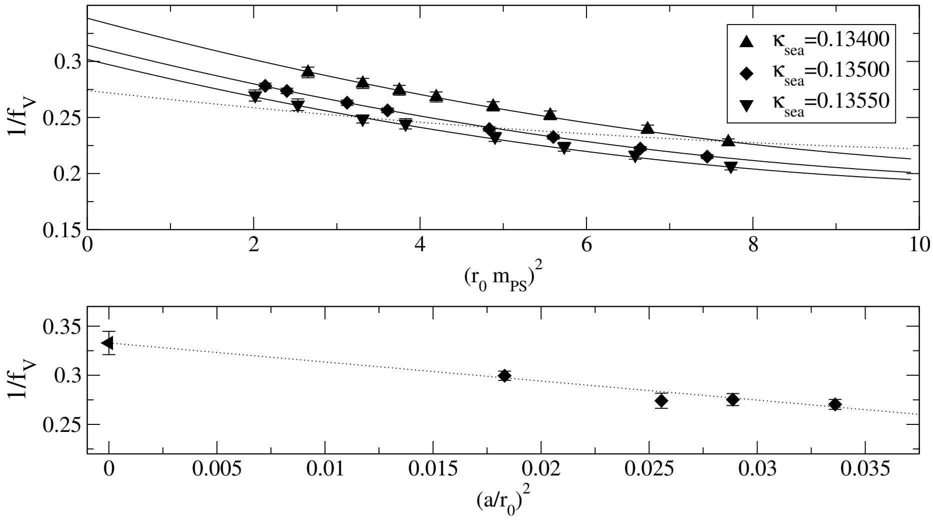


**Figure 3:** Similar to Fig. 2, but for $1/f_V$. An ansatz linear in $a^2$ was used for the continuum extrapolation.

been taken into account here, would shift this result down. Extrapolating our results to the kaon mass gives us an estimate of $f_K \approx 155$ MeV. This again agrees well with the experimental result $f_K^{\text{exp}} = 159.8$ MeV. It should be noted however that our estimate for $f_K$ is based on simulations with degenerate quark masses. For the rho we find $1/f_\rho = 0.333(12)$. This is slightly larger than the experimental estimate $1/f_\rho^{\text{exp}} \approx 0.27$ obtained from the branching ratio $\text{BR}(\tau \to \rho^- \nu_\tau)$. Extrapolating our results for the tensor coupling gives $f_V^{\perp\,\text{RGI}} = 183(4)$ MeV or, after conversion to the $\overline{\text{MS}}$-scheme, $f_V^{\perp\,\overline{MS}}(2\,\text{GeV}) = 168(3)$ MeV. This result is larger compared to previous results from QCD sum rules [10] and from lattice simulations employing the quenched approximation [1, 11], which are in the range of $140-150$ MeV.

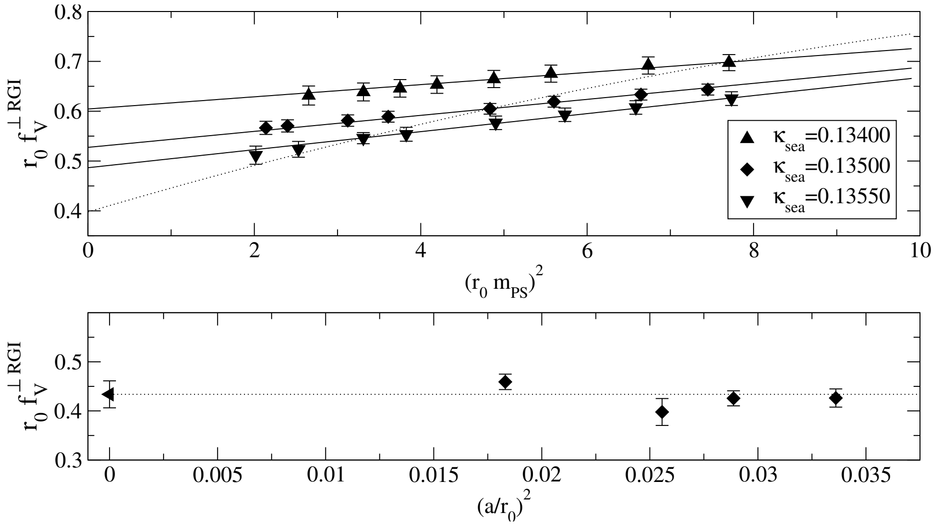


**Figure 4:** Similar to Fig. 2, but for $f_V^{\perp\,\text{RGI}}$.

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