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Extracting $|V_{ub}|$ from Omnès Dispersion Relations and Lattice QCD

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Dispersion approaches offer a way to use directly calculated/measured form-factor values from exclusive semileptonic $B \to \pi$ decays to constrain the overall q^2-shape of the form-factor(s). This can make the determination of $|V_{ub}|$ from exclusive decays competitive with that from inclusive decays. Here we consider the use of Omnès dispersion relations for this purpose.

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1. Introduction

To make the determination of $|V_{ub}|$ from exclusive semileptonic $B \to \pi$ decays competitive with that from inclusive decays requires a means of using directly calculated and/or measured form-factor values to constrain the overall q^2 shape of the form factors, f^+ (and f^0). Integrated or partially integrated decay-rates can then be compared to experiment to allow the extraction of $|V_{ub}|$. Dispersion approaches offer a model-independent way to accomplish this task. Dispersive bounds have been applied to $B \to \pi$ decays [1, 2, 3, 4, 5, 6] with recent work [6] combining a constraint on $|V_{ub}|f^+(0)$ from factorisation/SCET with lattice QCD inputs at high q^2 and a ChPT result at $q^2_{\max} = (m_B - m_\pi)^2$. Here we consider an alternative technique, using multiply-subtracted Omnès dispersion relations as a means to incorporate input values for $f^+(q^2)$.

2. Omnès Dispersion Relations

Mandelstam's hypothesis of maximum analyticity and Watson's Theorem relate the phase of the form factor f^+ in $B \to \pi$ decay to the phase shift in the elastic $B\pi \to B\pi$ scattering amplitude in the $J^P = 1^-$ and isospin-1/2 channel. We have

$$\frac{f^+(s+\mathrm{i}\varepsilon)}{f^+(s-\mathrm{i}\varepsilon)} = \frac{T(s+\mathrm{i}\varepsilon)}{T(s-\mathrm{i}\varepsilon)} = e^{2i\delta(s)}, \qquad s > s_{\mathrm{th}}$$

where $s_{\mathrm{th}} = (m_B + m_\pi)^2$ and $T(s)$ is the scattering amplitude,

$$T(s) = \frac{8\pi \mathrm{i} s}{\lambda^{1/2}(s)}(e^{2\mathrm{i}\delta(s)} - 1).$$

With a single subtraction, the Omnès dispersion relation gives

$$f^+(q^2) = f^+(0) \exp\left[\frac{q^2}{\pi}\int_{s_{\mathrm{th}}}^{\infty} \frac{\delta(s)\,ds}{s(s-q^2)}\right].$$

There is a corresponding result for f^0.

In [7] $\delta(s)$ was found from $T(s)$ in an on-shell Bethe-Salpeter scheme with kernel determined by tree-level heavy-meson ChPT (HMChPT), allowing a fit to lattice data for f^+ and f^0 with free parameters $f^+(0) = f^0(0)$ and $g_{BB^*\pi}$ (the coupling of B, B^* and π mesons, which fixes the lowest order HMChPT interaction term). The drawback here was the need to know $\delta(s)$ far above threshold.

To suppress the dependence on $\delta(s)$ at large s, we make additional subtractions. Multiply-subtracted Omnès relations have been used to study final state interactions in $K \to \pi\pi$ decays [8]: here multiple subtractions were made at a single value of s, requiring knowledge of an amplitude and its derivatives at that point. Here we use multiple subtractions at different points [9], allowing the input of $f^+(q^2)$ information at a set of distinct q^2 values. This is ideal for making use of the results of lattice simulations. With multiple subtractions, the Omnès relation reads:

$$f^+(q^2) = \prod_{j=0}^{n} [f^+(q_j^2)]^{\alpha_j(q^2)} \times \exp\left\{ I_\delta(q^2;\{q_j^2\}) \prod_{k=0}^{n} (q^2 - q_k^2) \right\}$$

$$I_\delta(q^2;\{q_j^2\}) = \frac{1}{\pi}\int_{s_{\rm th}}^{+\infty}\frac{ds}{(s-q_0^2)\cdots(s-q_n^2)}\frac{\delta(s)}{s-q^2}$$

$$\text{with} \qquad \alpha_j(q^2) = \prod_{k=0,\,k\neq j}^{n}\frac{q^2-q_k^2}{q_j^2-q_k^2}, \qquad \alpha_j(q_i^2) = \delta_{ij}, \qquad \sum_{k=0}^{n}\alpha_k(q^2) = 1$$

With *many* subtractions we need knowledge of $\delta(s)$ only near $s_{\rm th}$. Considering the B^* as a bound state with m_{B^*} not far from $s_{\rm th}$, we can make the approximation $\delta(s) \approx \pi$ in the integral I_δ. The Omnès factor can then be integrated exactly to give an explicit formula for $f^+(q^2)$:

$$f^+(q^2) \approx \frac{1}{s_{\rm th}-q^2}\prod_{j=0}^{n}\left[f^+(q_j^2)(s_{\rm th}-q_j^2)\right]^{\alpha_j(q^2)}, \quad n \gg 1 \tag{2.1}$$

This amounts to constructing an interpolating polynomial for $\ln[f^+(q^2)(s_{\rm th}-q^2)]$ passing through known values at the q_j^2.

3. Fitting $f^+(q^2)$

The multiply-subtracted Omnès formula in equation (2.1) was used in [9] with inputs from a nonrelativistic constituent quark model with encouraging results. Here we will compare our approach with that using dispersive bounds [6], by taking inputs from factorisation/SCET, lattice QCD [10] (as quoted in [6]) and ChPT, together with an experimentally determined value for the semileptonic branching fraction [11]:

$$\mathrm{Br}(B^0 \to \pi^- l^+ \nu_l) = (1.39 \pm 0.12) \times 10^{-4}.$$

We also perform a fit replacing the factorisation result with a lightcone sumrule (LCSR) value for $f^+(0)$. The form factor inputs are listed here:

$\lvert V_{ub}\rvert f^+(0)$	$(7.2 \pm 1.8) \times 10^{-4}$	Factorisation [12]
$f^+(0)$	0.258 ± 0.031	LCSR [13]
$f^+(15.87\,\mathrm{GeV}^2)$	$0.799 \pm 0.058 \pm 0.088$	Lattice QCD [10, 6]
$f^+(18.58\,\mathrm{GeV}^2)$	$1.128 \pm 0.086 \pm 0.124$	Lattice QCD [10, 6]
$f^+(24.09\,\mathrm{GeV}^2)$	$3.262 \pm 0.324 \pm 0.359$	Lattice QCD [10, 6]
$f^+(q^2_{\rm max})$	10.38 ± 3.63	ChPT [6, 14]

In reference [6], the combination of the factorisation/SCET result at $q^2 = 0$ with FNAL–MILC lattice data at high q^2 and ChPT at $q^2_{\rm max}$ led to the result $|V_{ub}| = (3.54 \pm 0.47) \times 10^{-3}$. In our case we find that the ChPT point at $q^2_{\rm max}$ is not really compatible with the factorisation and lattice results and leads to a large value for $|V_{ub}|$. Leaving this point out of the Omnès fit leads to a value for $|V_{ub}|$ compatible with [6] and the fitted $f^+(q^2)$ is compatible within errors with the ChPT result at $q^2_{\rm max}$. This fit is shown in figure 1. Using the LCSR value for $f^+(0)$ instead of the factorisation input also gives a compatible result for $|V_{ub}|$ and has $f^+(q^2_{\rm max})$ compatible within errors with the ChPT point, as shown in figure 2.

The following table shows the values of $|V_{ub}|$ required to fit the branching fraction from [11] and lists the result from [6] for comparison. The fits made here assume completely correlated errors in the input lattice points (a stronger assumption than used in [6]).

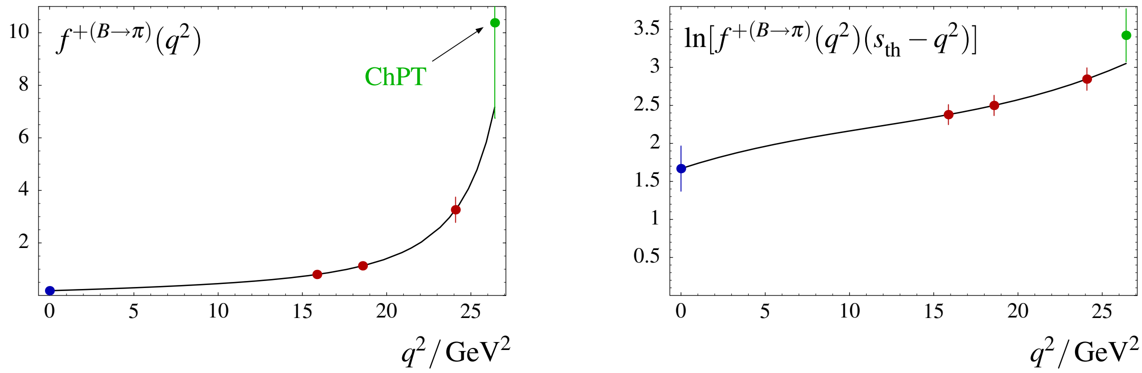

Figure 1: Omnès dispersion relation fit using factorisation/SCET input (blue) at $q^2 = 0$, together with lattice QCD inputs (red) at high q^2. The ChPT point (green) at $q^2_{\rm max}$ is not part of the fit.

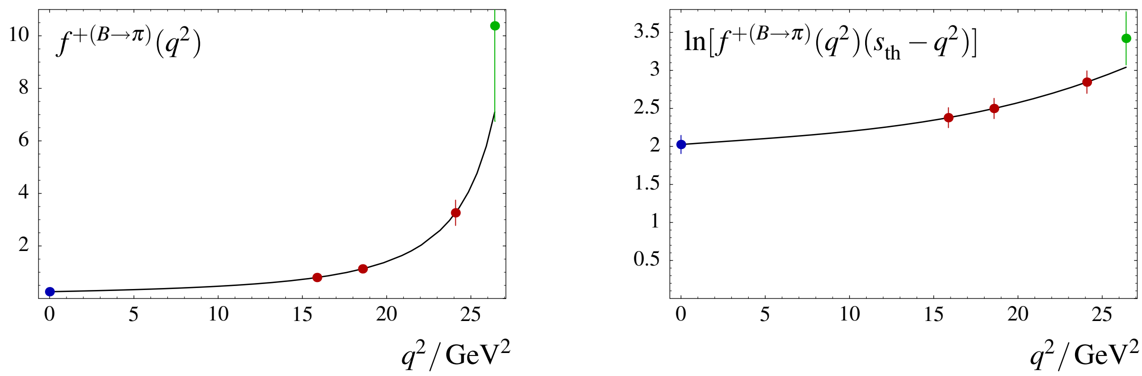

Figure 2: Omnès dispersion relation fit using $f^+(0)$ from LCSR (blue), together with lattice QCD inputs (red) at high q^2. The ChPT point (green) at $q^2_{\rm max}$ is not part of the fit.

	$\|V_{ub}\|/10^{-3}$
Factorisation + LQCD + ChPT	5.3 ± 1.8
Factorisation + LQCD	4.0 ± 0.7
LCSR + LQCD	3.7 ± 0.5
AGRS [6] (Factorisation + LQCD + ChPT)	3.5 ± 0.5

4. Conclusion

The multiply-subtracted Omnès relation gives a model-independent method for extending $f^+(q^2)$ over the whole range of q^2 using measured values at a given set of q^2. With the approximation made here, an explicit formula results. This is ideal for combining lattice results with other known information at specific values of q^2. Both the Omnès method and techniques using dispersive bounds [6] show the importance of having knowledge at widely spaced q^2. The combination of an input at $q^2 = 0$ with lattice results is especially constraining.

Watson's theorem relates the phases of the form factor and the elastic scattering amplitude only below the inelastic threshold, so inelastic scattering effects are not taken into account here. Moreover, with many subtractions the input values at the subtraction points and the phase should satisfy constraints in order to avoid bad asymptotic behaviour of the form factor [15]: this too is also not accounted for in the simple formula presented here.

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