

B_7 , B_8 and chiral Ward identities

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We present recent progress in understanding weak matrix elements on the lattice. We use HYP staggered fermions in quenched QCD to study numerically various properties of the $K^+ \rightarrow \pi^+$ amplitudes of the electroweak penguin operators Q_7 and Q_8 . We check chiral Ward identities to probe the validity of using improved staggered fermions in the calculation of weak matrix elements. We address the issue of mixing with unphysical lower dimension operators, which causes a divergent term in the case of the $\Delta I = 1/2$ amplitudes. We propose a particular subtraction method as the best choice. We also measure the gold-plated ratio R originally suggested by Becirevic and Villadoro.

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1. Introduction to Chiral Ward Identities

By decoupling the heavy particles such as W bosons, Z bosons and heavy quarks from the standard model of electroweak theory, we obtain the low energy effective Hamiltonian relevant to particular processes classified as the $\Delta S = 1$ interaction of primary interest in this paper.

$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu) \quad (1.1)$$

Here, G_F is the Fermi coupling constant and V_{ij} are elements of the CKM matrix. Note that $\tau = -\lambda_t/\lambda_u$ where we define $\lambda_i \equiv V_{id} V_{is}^*$. The $z_i(\mu)$ and $y_i(\mu)$ are the Wilson coefficients at the scale μ . The $Q_i(\mu)$ are the four fermion operators made of u, d, s quark fields. The Wilson coefficients for the current-current operators Q_1 and Q_2 are of $\mathcal{O}(1)$, those for the QCD penguin operators ($Q_i, i = 3, 4, 5, 6$) are of $\mathcal{O}(\alpha_s)$, whereas those for the electroweak penguin operators ($Q_i, i = 7, 8, 9, 10$) are of $\mathcal{O}(\alpha)$.

In this paper, we focus on the electroweak penguin operators Q_7 and Q_8 .

$$Q_7 = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V+A} \quad (1.2)$$

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A} \quad (1.3)$$

According to the group theory analysis, Q_7 and Q_8 belong to the $(8, 8)$ irreducible representation (irrep) of $SU(3)_L \otimes SU(3)_R$. We can further decompose $Q_{(8,8)}$ into $\Delta I = 1/2$ and $\Delta I = 3/2$ irrep.

$$Q_{(8,8)} = \frac{1}{2} (\bar{s}d)_{V-A} (2\bar{u}u - \bar{d}d - \bar{s}s)_{V+A} \quad (1.4)$$

$$Q_{(8,8)}^{\Delta I=3/2} = (\bar{s}d)_{V-A} (\bar{u}u)_{V+A} + (\bar{s}u)_{V-A} (\bar{u}d)_{V+A} - (\bar{s}d)_{V-A} (\bar{d}d)_{V+A} \quad (1.5)$$

$$Q_{(8,8)}^{\Delta I=1/2} = (\bar{s}d)_{V-A} (\bar{u}u)_{V+A} - (\bar{s}u)_{V-A} (\bar{u}d)_{V+A} - (\bar{s}d)_{V-A} (\bar{s}s)_{V+A} \quad (1.6)$$

We can rewrite $Q_{(8,8)}$ as follows:

$$Q_{(8,8)} = \frac{1}{2} [Q_{(8,8)}^{\Delta I=3/2} + Q_{(8,8)}^{\Delta I=1/2}] \quad (1.7)$$

Using chiral perturbation theory, we can analyze the chiral behavior of hadronic matrix elements of the $Q_{(8,8)}$ operators. The details of this analysis will be presented in Ref. [1]. At the lowest order in chiral perturbation theory, the ratio of the $K^+ \rightarrow \pi^+$ amplitudes satisfies the following chiral Ward identity [2, 3, 1].

$$W \equiv \frac{\langle \pi^+ | Q_{(8,8)}^{\Delta I=1/2} | K^+ \rangle_{\text{sub}}}{\langle \pi^+ | Q_{(8,8)}^{\Delta I=3/2} | K^+ \rangle}, \quad \lim_{m_q \rightarrow 0} W = 2 \quad (1.8)$$

In this paper, one of the main goals is to numerically check the above Ward identity, which serves as an important probe to test the validity of using improved staggered fermions in the calculation of weak matrix elements.

parameter	value
gauge action	Wilson Plaquette
fermion action	HYP Staggered
β	6.0 (quenched QCD)
# of confs	218
lattice	$16^3 \times 64$
quark mass	0.01, 0.02, 0.03, 0.04

Table 1: Parameters for the numerical study

2. Numerical Study on Ward identities

We use an ensemble of 218 gauge configurations whose details are summarized in Tab. 1. We use the ρ meson to set the scale $1/a = 1.95$ GeV. We also use the kaon to set the physical strange quark mass. Details of this procedure are presented in Ref. [4]. Throughout this paper, we consider only particles composed of degenerate quarks ($m_s = m_d$). One of the key ingredients of this numerical study is that we use improved staggered fermions made of HYP fat links when we construct the operators and calculate the matrix elements.

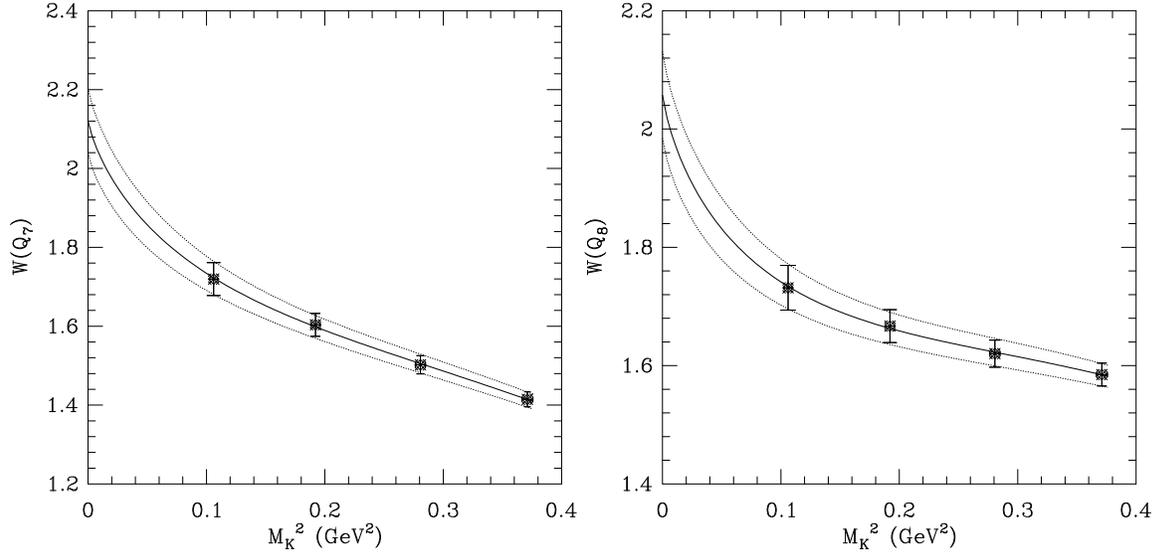


Figure 1: W for Q_7 (left) and Q_8 (right)

We measure W of Eq. (1.8) for four different quark masses. The results for Q_7 and Q_8 are presented in Fig. 1. In Fig. 1 we plot the ratio W as a function of m_K^2 . We fit the data to the following form suggested by quenched chiral perturbation theory [3, 1]:

$$f(m_K^2) = c_0 \left[1 + 3 \frac{m_K^2}{(4\pi f)^2} \log \left(\frac{m_K^2}{(4\pi f)^2} \right) \right] + c_1(m_K^2) + c_2(m_K^2)^2 \quad (2.1)$$

Here, even though we know that $c_0 = 2$ from chiral perturbation theory, we fit the data by considering c_0 a free parameter to see how well it agrees with the prediction of chiral perturbation

	Q_7	Q_8
c_0	2.1206 ± 0.0820	2.0574 ± 0.0734
c_1	4.1970 ± 0.2298	4.6681 ± 0.2194
c_2	-3.9582 ± 0.4876	-3.9037 ± 0.4538

Table 2: Fit parameters of the Ward identity for Q_7 and Q_8 .

theory. The fitting results are summarized in Tab. 2. Note that the c_0 values are consistent with the theoretical prediction of $c_0 = 2$ within a statistical uncertainty of 1.5σ for Q_7 and 0.8σ for Q_8 .

Taking into account the fact that we neglect finite volume effects, higher order corrections in chiral perturbation theory and two-loop corrections in the matching, the agreement is quite good. This suggests that we should be able to obtain a solid result for the Q_7 and Q_8 matrix elements in the chiral limit using improved staggered fermions.

3. Numerical Study on Individual Amplitudes

In Ref. [3], Laiho and Soni presented the results of the chiral log corrections to the individual matrix elements of the $Q_{(8,8)}$ operator using (partially) quenched chiral perturbation theory. For the $\Delta I = 3/2$ amplitude, the chiral behavior is

$$\frac{\langle \pi^+ | Q_{(8,8)}^{(3/2)} | K^+ \rangle}{f_\pi^2} = b_0 \left[1 - 2 \frac{m_K^2}{(4\pi f)^2} \log \left(\frac{m_K^2}{(4\pi f)^2} \right) \right] + b_1(m_K^2) + b_2(m_K^2)^2 \quad (3.1)$$

For the $\Delta I = 1/2$ amplitude, the chiral behavior is

$$\frac{\langle \pi^+ | Q_{(8,8)}^{(1/2)} | K^+ \rangle_{\text{sub}}}{f_\pi^2} = d_0 \left[1 + \frac{m_K^2}{(4\pi f)^2} \log \left(\frac{m_K^2}{(4\pi f)^2} \right) \right] + d_1(m_K^2) + d_2(m_K^2)^2 \quad (3.2)$$

where the subscript sub means a kind of subtraction method, which will be explained later. Here, note that the b_i and d_i parameters cannot be determined from the chiral perturbation theory except for the ratio $d_0/b_0 = 2$.

In the case of the $\Delta I = 3/2$ amplitudes, there is no mixing with unphysical lower dimension operators. However, this is not the case for the $\Delta I = 1/2$ amplitudes. There is mixing with unphysical lower dimension operators in the next to leading order $\mathcal{O}(m_K^2)$. The mixing coefficients are power divergent of $\mathcal{O}(1/a^2)$. Hence, even though they are suppressed by m_K^2 , the size of this mixing dominates the physical signal due to the divergent coefficient in the range of quark masses that we use in this numerical study. Therefore it is essential that the mixing with unphysical lower dimension operators should be subtracted non-perturbatively. There are many ways to handle this. The details for different options will be discussed in Ref. [1]. Here, we focus on the best method that we choose for this numerical study. Regardless of the alternative methods to calculate $\langle 0 | Q_{(8,8)}^{(1/2)} | K^0 \rangle$, if we take the derivative of the $K^0 \rightarrow 0$ amplitude with respect to the strange quark mass and take the limit of $m_s \rightarrow m_d$, we can select only the divergent mixing coefficient up to the next leading order of the quenched chiral perturbation theory. In other words,

$$\lim_{m_s \rightarrow m_d} \frac{\partial}{\partial m_s} \langle 0 | Q_{(8,8)}^{(1/2)} | K^0 \rangle = -\frac{8i}{f} c_4^r B_0. \quad (3.3)$$

where B_0 is defined as

$$m_K^2 = B_0(m_s + m_d) \quad (3.4)$$

Here, c_4^r is the divergent mixing coefficient in the notation of Ref. [3]. The details on the derivation of Eq. (3.3) will be explained in Ref. [1]. At any rate, we use Eq. (3.3) to determine c_4^r . Using this c_4^r , we can subtract the divergent mixing away. Therefore, the final results of the matrix element $\langle \pi^+ | Q_{(8,8)}^{(1/2)} | K^+ \rangle_{\text{sub}}$ do not contain any contribution from the mixing with unphysical lower dimension operators.

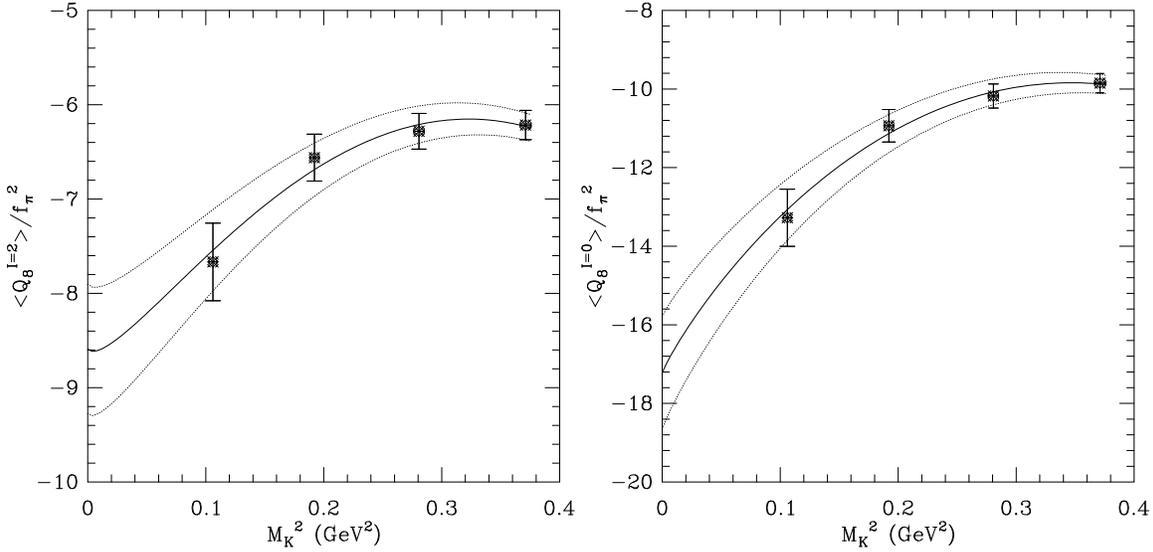


Figure 2: $\langle \pi^+ | Q_{(8,8)}^A | K^+ \rangle / f_\pi^2$ matrix elements for Q_8 (left: $\Delta I = 3/2$) and (right: $\Delta I = 1/2$)

We measure the $K^+ \rightarrow \pi^+$ amplitudes of the Q_8 operator for four different quark masses. The results are presented in Fig. 2. Here, we normalize the amplitudes by f_π^2 for the analysis convenience without loss of generality.¹ We fit the data to the functional form suggested by the quenched chiral perturbation theory which is given in Eq. (3.1) and Eq. (3.2). As you can see in Fig. 2, the intercepts of the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes respect the Ward identity ($d_0/b_0 = 2$) within statistical uncertainty.

4. Gold-plated Ratio

In Ref. [5], Becirevic and Villadoro proposed that it is possible to classify ratios of the $K^+ \rightarrow \pi^+$ amplitudes as gold-plated and silver-plated. Here, the gold-plated ratio means that it does not contain any chiral log correction at the one loop level and correspondingly the leading finite volume effects also cancel off. This gold-plated ratio (R) is defined as

$$R \equiv \frac{\langle \pi^+ | Q_8^{(3/2)} | K^+ \rangle}{\langle \pi^+ | Q_7^{(3/2)} | K^+ \rangle} \quad (4.1)$$

¹Note that f_π^2 by itself cannot possess chiral logs in the limit of $m_s = m_d$ in quenched QCD.

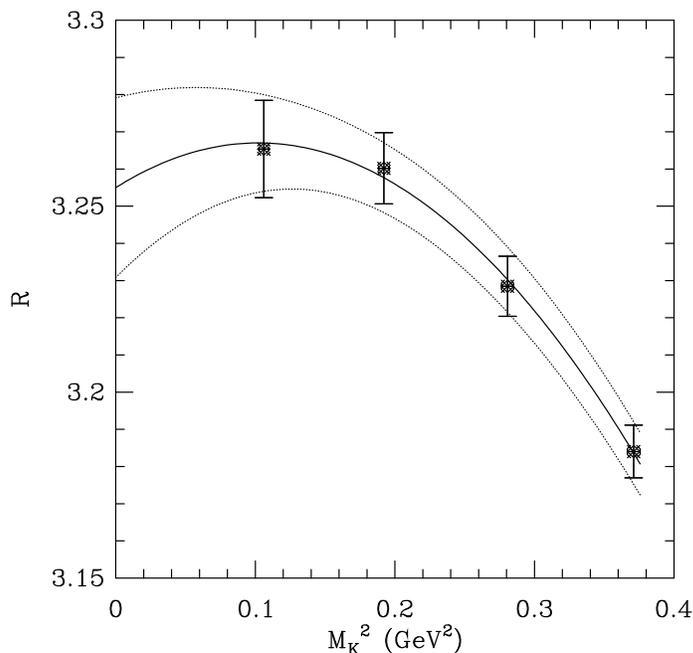


Figure 3: R vs. M_K^2

We measure R over four quark masses and the results are presented in Fig. 3. The quadratic fitting results in Fig. 3 are

$$R(m_K^2) = 3.255(24) + 0.235(154)(m_K^2) - 1.153(247)(m_K^2)^2 \quad (4.2)$$

Here, note that the coefficient of the quadratic term is reasonable (of $\mathcal{O}(1)$) whereas the linear term has a relatively tiny coefficient. This weak dependence on the linear term might be interpreted as a signal for the gold-plated ratio. However, the noticeable dependence on the quadratic term gives us some doubt on whether we call R gold-plated literally.

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