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Observational implications of the bandwidth effects in 70 GHz LFI main beams

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In this work we analyse the observational implications of the bandwidth effects in 70 GHz LFI main beams. Since the beamwidth varies within the bandwidth, the effective beamwidth (i.e. integrated over the bandwidth) will be in principle related to the frequency dependence of the observed source or diffuse component. In the simple approximation of symmetric Gaussian beam, we characterize the effective angular resolutions and window functions derived for various frequency power law (antenna temperature) dependences appropriate to the microwave sky components (cosmic background radiation and foregrounds) and compare them with two reference cases, i.e. the beamwidth at the centre of the frequency bandwidth or integrated over the bandwidth for a frequency independent radiation field.
This work has been done in the framework of Planck LFI acivities.

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1. Introduction

Cosmic microwave background (CMB) anisotropy experiments use receivers with a relatively wide ($\Delta\nu/\nu \sim 20\%$) bandwidth to improve sensitivity (see, e.g., [2], [4], [3]). Optical distortions can affect the beam resolution and shape in a frequency dependent way within the bandwidth. Various astrophysical components, with different spectra, contribute to the global microwave signal. Each of them will be then observed with a different beam resolution and shape integrated over the bandwidth [5]. We quantify here this effect for PLANCK LFI at 70 GHz.

2. Analytical description

Passing from the case of infinitesimal bandwidth to that of a finite bandwidth, the beam response averaged over the bandwidth can be written to a good approximation as:

$$R(\varepsilon,\sigma_m,\sigma_M) \longrightarrow \int d\nu\ \psi(\nu)\ R(\varepsilon,\sigma_m(\nu),\sigma_M(\nu))\,, \tag{2.1}$$

where ε is the angle between the x axis of the reference system of the telescope field of view plane, $x-y$, and the major semi-axis of the beam shape, $\sigma_m(\nu)$ and $\sigma_M(\nu)$ are the minor and major semi-axis of the beam shape respectively. The beam shape in the $x-y$ plane is here described by a bivariate Gaussian (a good approximation up to 20–25 dB below the peak response). We can write R as $R(\varepsilon,\sigma_m,\sigma_M) = \exp[-A_M^2/2\sigma_M^2 - A_m^2/2\sigma_m^2]$, where $A_M = (x-x_0)\cos\varepsilon + (y-y_0)\sin\varepsilon$ and $A_m = -(x-x_0)\sin\varepsilon + (y-y_0)\cos\varepsilon$. The function $\psi(\nu)$ specifies the answer in the bandwidth. It is assumed $\psi(\nu) \neq 0$ only in a small frequency interval $2(\Delta/2)$ around ν_0. To the lowest approximation $\psi(\nu) = 1$ for $\nu_0 - \Delta/2 < \nu < \nu_0 + \Delta/2$ and 0 otherwise (for PLANCK LFI, $\Delta \sim 0.2\nu$). In this study we assume this simple approximation, but our approch can be easily generalized to any choice of this function. In this notation $\sigma_i = FWHM_i/2\sqrt{2\ln 2}$; here i represents the index of the various frequency samples, ν_i, within the bandwidth. The variations within the bandwidth of σ_i along the x and y directions and their average are shown in Fig. 1. Their behaviour can be well fitted by a second order polinomial. In the symmetric Gaussian beam approximation, for the considered beam we have $\sigma_{ave}(\nu) = 16.6440 - 0.299757\cdot\nu + 0.00200582\cdot\nu^2$.

The sky surface brightness in the direction γ can be written as $S_\nu(\gamma) = I(\nu)\cdot S(\gamma)$, where approximately: $I(\nu) \propto \nu^{-0.7}$ and the antenna temperature, $T(\nu) \propto \nu^{-\alpha}$, is $T(\nu) \propto \nu^{-2.7}$ for synchrotron emission; $I(\nu) \propto \nu^{-0.1}$ and $T(\nu) \propto \nu^{-2.1}$ for bremsstrahlung emission; $I(\nu) \propto \nu^4$ and $T(\nu) \propto \nu^2$ for dust emission. For a blackbody at $\simeq 2.7$ K, at frequencies about 70 GHz we have approximately $T(\nu) \propto \nu^{-0.7}$.

Integrating over the beamwidth, the total power can be written as: $W_{tot} = \frac{1}{2}\int A_e^{bb}(\Omega) S^{bb}(\Omega)\, d\Omega$, where $A_e^{bb}(\Omega) = \int_\nu \sigma(\nu) A_e(\nu) P_n(\nu,\theta)\, d\nu / \int \sigma(\nu)\, d\nu$ defines the antenna efficient area in the bandwidth and the function describing the source, weighted on the bandwidth, can be written as $\int_\nu S_\nu(\Omega)\, d\nu \equiv S^{bb}(\Omega) \equiv S(\Omega)\int \sigma(\nu)\, d\nu \quad [\mathrm{W}m^{-2}\mathrm{sr}^{-1}]$.

Differently, if we observe a point source (i.e. of angular extension much smaller than the beamwidth) at the (monochromatic) frequency ν with a symmetric Gaussian beam, the beam response is $P_n(\nu,\theta) = \exp(-\theta^2/2\sigma_b^2(\nu))$, where θ is the angular distance from the direction of the antenna pattern maximum (i.e. beam centre). The temperature dependence on the frequency ν can

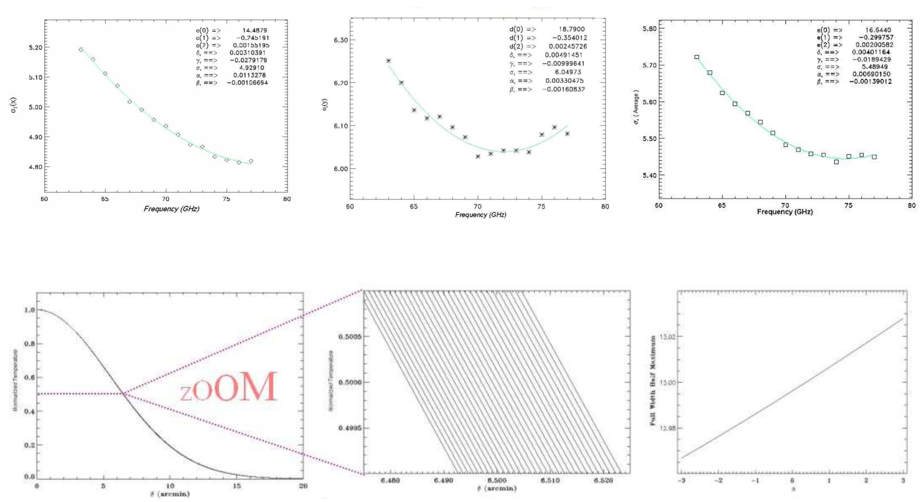

Figure 1: Top panels: beamwidth as function of the frequency along the x (left panel) and y (middle panel) directions, and their average (right panel). Bottom panels: the observed temperature at various angles θ from the beam centre for different values of α from -3 to 3 (left panel); zoom of left panel close the half power value to able to see the lines at different α (middle panel); FWHM (in arcmin) averaged over the bandwidth as function of the spectral index α (right panel). Note that the maximum variation is equal to $0.06126'$. Clearly, for spectra decreasing (increasing) with frequency the effective beamwidth is larger (smaller).

be expressed by $\delta T_\nu^{obs} = \int_{src} P_n T_\nu \, d\Omega / \int P_n \, d\Omega$, where the integral $\int_{src} P_n T_\nu \, d\Omega$ is over the source extent. Integrating over the bandwidhth, we have:

$$T^{obs} = \int T_\nu^{obs} \, d\nu \quad \text{equivalent to} \quad T^{obs} = \int \frac{\int_{src} P_n T_\nu \, d\Omega}{\int P_n \, d\Omega} \, d\nu \quad . \tag{2.2}$$

3. Observational implications

The case of a point source is of particular interest since it allows to map the effective beam [1] integrated over the bandwidth and then to define the effective beam resolution integrated over the bandwidth as function of the radiation spectral behaviour.

Of course, if $\sigma_b(\nu)$ were independent of ν then the observed temperature can be separated as: $T^{obs}(\theta) = \mathfrak{Espr}(\theta) \times \int T_\nu \, d\nu$, where $\mathfrak{Espr}$ depends only on θ and the integral depends only on the source spectral behaviour. The angular resolution will be then independent of the source spectrum. In the real case $\sigma(\nu)$ depends on the frequency and the angular resolution will depend on the spectral behaviour of considered radiation field [5] according to $T^{obs}(\theta) = \int T_\nu^{obs} d\nu$.

Adopting the $\sigma_{ave}(\nu)$ parabolic law of previous section, we calculated the observed temperature for $-3 \le \alpha \le 3$ (at steps of 0.2) in order to cover the large variety of foreground spectra and for angles of view from the beam centre $0 \le \theta \le 4\sigma_{ave}$. Explicitely, the integral used for the computation is:

$$T = \int_{\nu_0-\Delta/2}^{\nu_0+\Delta/2} \frac{\left(\frac{\nu_{GHz}}{70}\right)^{-\alpha} \exp\left(\frac{-\theta^2}{2\sigma_b^2(\nu)}\right)}{\sigma_b^2(\nu)} \, d\nu \, . \tag{3.1}$$

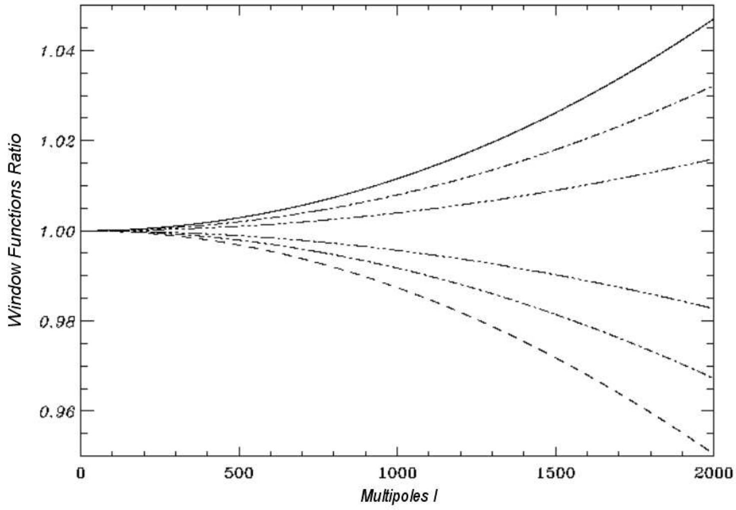

Figure 2: Ratio between the window function averaged on the bandwidth computed with various values of α $(-3,-2,-1,1,2,3$ from top to bottom) and that computed for α=0. Note that for $l \lesssim 900(1500)$ this ratio differs from 1 less than 1%(2.5%).

The results of this analysis are shown in Fig. 1.

Given the beamwidth averaged over the bandwidth, $\sigma(\alpha)$, we computed the corresponding window function, $\exp[-(\ell(\ell+1)\sigma^2(\alpha))]$ for the various considered values of the spectral index α [5]. The result is shown in Fig. 2. For $l \lesssim 900(1500)$ we find a window function modification less than 1%(2.5%).

Since, in general, the impact of a frequency dependent beam distortion on beam effective resolution and window function can be written as a function of frequency dependent parameters and direction dependent parameters and the expected distortions are small, the approach presented here can be used to identify to first order the frequency dependence of main beam distortion effect for various radiation field spectral behaviours.

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