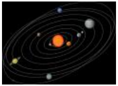
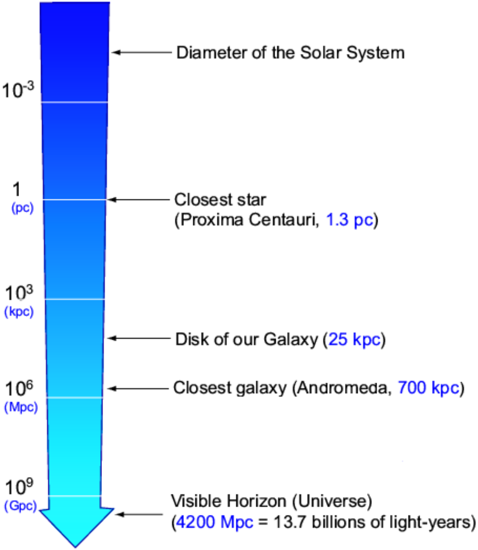
Outline

- Basics of Cosmology
  - Friedmann equations
  - Thermal history
  - Conceptual problems of classical cosmology
- Hot Big Bang
  - Cosmic Microwave Background
  - Big Bang Nucleosynthesis
  - Baryogenesis
  - Phase trnasitions and topological defects
- Missing mass: evidence for new physics
  - Dark matter problem
  - Dark energy problem
  - Particle physics solutions
- Inflationary cosmology
  - How it works and what it does
  - Creation of seeds for structure
  - Comparison with observations

Astronomical Distance Scales

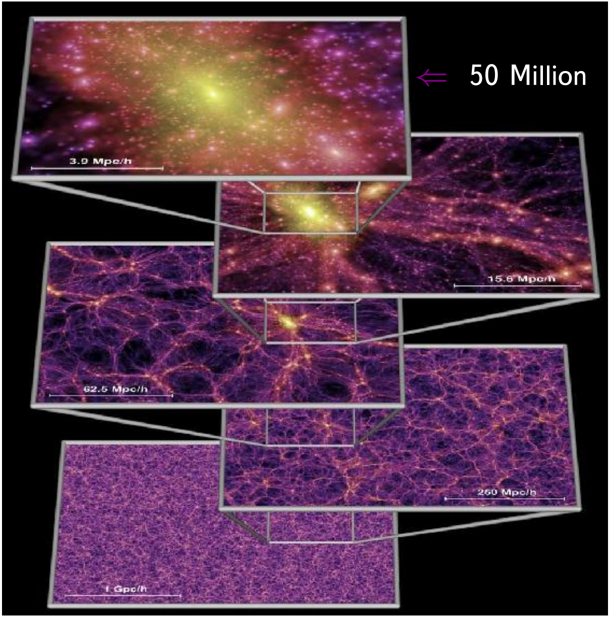




Parsecs (pc)

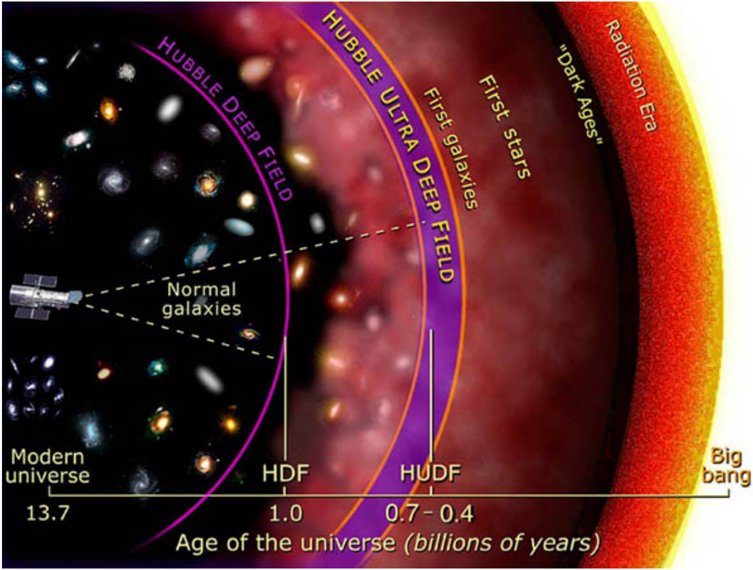
1 parsec = 3.26 light-years ~ 3×10^{13} km

Astronomical Distance Scales

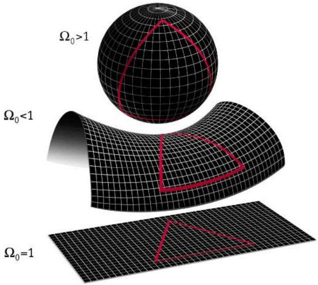


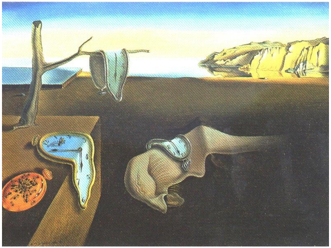
$\Leftarrow$ 50 Million light years

Looking Back in Time



Space and time





- Space is three dimensional and all points are alike. There are three possible realizations of a homogeneous isotropic space.
- Space and time make dynamical framework

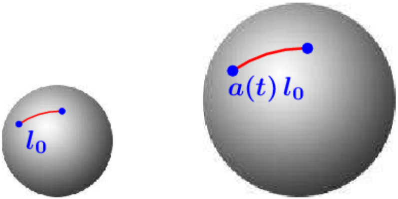
Dynamical Frameworks

Einstein equations

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}}$$

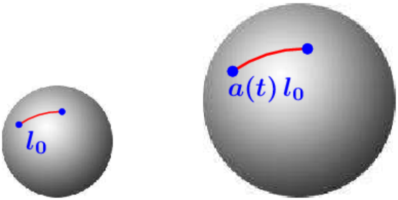
Crucial assumption: The Universe is homogeneous.

Friedmann solution: The Universe should expand.



$$ds^2 = dt^2 - a^2(t)\, dl^2$$

Space-time metric



$$ds^2 = dt^2 - a^2(t)\,dl^2$$

$$dl^2 = \frac{dr^2}{1 - k\,r^2} + r^2\,(d\theta^2 + \sin^2\theta\,d\phi^2)$$

$${}^{(3)}R = \frac{6k}{a^2(t)} \qquad \begin{cases} k = -1 & \text{Open} \\ k = 0 & \text{Flat} \\ k = +1 & \text{Closed} \end{cases}$$

Friedmann equations

Assume ideal fluid for the energy momentum tensor

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Einstein equations written for homogeneous isotropic world give

$$\boxed{\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho - \frac{k}{a^2} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) \end{aligned}}$$

Friedmann equations: the physics behind

Friedmann equation

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

has familiar Newtonian counterpart. Indeed

$$\frac{1}{2}\dot a^2 - \frac{4\pi G}{3}\rho\, a^2 = -\frac{k}{2}$$

With $r = a\, r_0$ and $M = \frac{4\pi}{3}\rho\, r^3$ Friedmann equation takes the form of energy conservation for a test particles bounded in the gravitaional potential

$$\frac{1}{2}\dot r^2 - \frac{G\,M}{r} = -\frac{k\, r_0^2}{2}$$

Geometry versus Universe future

$$\frac{1}{2}\dot{r}^2 - \frac{G\,M}{r} = -\frac{k\,r_0^2}{2}$$

$k = +1$ Binding energy negative
Universe will recollapce

$k = -1$ Binding energy positive
Universe will expand forever

$k = 0$ Critical density $\rho_c \equiv \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Hubble law

To find critical density $\rho_c \equiv \frac{3}{8\pi G} \left(\frac{\dot{a}}{a}\right)^2$ one needs to know

$$H \equiv \frac{\dot{a}}{a}$$ **Hubble "constant"**

Using $r(t) = a(t) r_0$ one can find the velocity with which distance between two points increases

$$v \equiv \dot{a} r_0 = \frac{\dot{a}}{a} a r_0 = H r$$

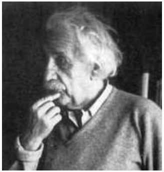
This gives the Hubble law $\boxed{v = H\, r}$

Horizon: $v \sim 1$ at $r \sim H^{-1}$

But at $r \ll H^{-1}$ Newtonian mechanics should be valid

Expansion of the Universe

- Newton did not know that one should worry about horizons, but he worried why the Universe does not collapse under the pull of gravity.



- In 1917 Einstein added cosmological constant to his equations thinking it will provide static solutions.

- In 1922 Friedmann had shown that the Universe **must** expand as a whole. After some debate Einstein admitted mistake and called the introduction of a cosmological constant "the greatest blunder of my life".

Why the Universe did not collapsed under the pull of gravity ?

Resolution is in awkward initial conditions called Big Bang

$$\frac{1}{2}\,\dot{r}^2 \;-\; \frac{G\,M}{r} \;=\; -\frac{k\,r_0^2}{2}$$

which imply enormous fine-tuning and which are hard to accept.

We will see how the modern inflationary cosmology solves this problem of initial conditions.

Cosmological Parameters

Introduced to parametize Fiedmann equation and its solution $a(t)$

t	Age
$H = \dot{a}/a$	Hubble "constant" at time t
$\rho_c = 3H^2/8\pi G$	Critical density
$\Omega = \rho/\rho_c$	$\Omega_B = \rho_B/\rho_c, \quad \Omega_r = \rho_r/\rho_c, \ \ldots$
$q = -\ddot{a}a/\dot{a}^2$	Decceleration parameter

Present values $t_0, \ H_0, \ \ldots$ are called cosmological parameters

$t_0 = (13.7 \pm 0.2)$ Gyr

$H_0 \equiv 100 \ h$ km s^{-1} Mpc^{-1}, where $h = 0.71 \pm 0.04$

$\rho_c = 10 \ h^2$ GeV m^{-3}

$\Omega_0 = 1.02 \pm 0.02$

$q_0 = -0.6 \pm 0.3$

Matter content in the Universe

- Light. (Relativistic degrees of freedom)
  Major energy fraction at early times.
- Baryonic matter. (Stars)
  Observable world today.
- Dark matter. (Should be there)
  Major matter fraction today.
- Dark energy. (Vacuum)
  Major energy fraction today.

Equation of state

Definition: $$\boxed{w \equiv \frac{p}{\rho}}$$

If $w = \text{const}$, conservation of energy gives $\rho = a^{-3(1+w)}\,\rho_0$

Ordinary forms of matter

- Radiation .... $w = \frac{1}{3}$ .... $\rho = a^{-4}\,\rho_0$
- Matter .... $w = 0$ .... $\rho = a^{-3}\,\rho_0$

Hypothetical matter

- Cosmic strings .... $w = -\frac{1}{3}$ ..... $\rho = a^{-2}\,\rho_0$
- Domain walls .... $w = -\frac{2}{3}$ ..... $\rho = a^{-1}\,\rho_0$
- Cosmological constant ... $w = -1$ ..... $\rho = \text{const}$

Law of expansion

Friedmann equation in spatially flat Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

for $w = \text{const}$ gives

$$a = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

In particular

- Radiation ..... $w = \frac{1}{3}$ ..... $a = (t/t_0)^{1/2}$
- Matter .... $w = 0$ ..... $a = (t/t_0)^{2/3}$

Hot Big Bang

In the past the Universe was dense and therefore hot

Friedmann equations: the physics behind

One of the two Friedmann equations can be excluded in favour of

$$\boxed{\frac{d\rho}{dt} + 3\,\frac{\dot{a}}{a}\,(\rho + p) = 0}$$

which is nothing but energy-momentum conseravtion
$T^{\mu\nu}{}_{;\nu} = 0$

And this is nothing but the First Law of thermodynamics

$$\boxed{dE + p\,dV = T\,dS}$$

Here $E = \rho\,V = \rho\,a^3$ is energy and S is entropy.

Isentropic expansion

Friedmann expansion driven by an ideal fluid is isentropic, $dS = 0$. Dissipation is negligible usually.
Entropy:

$$S = \frac{2\pi^2}{45} g_* \, T^3 \, a^3 = \text{const} \, ,$$

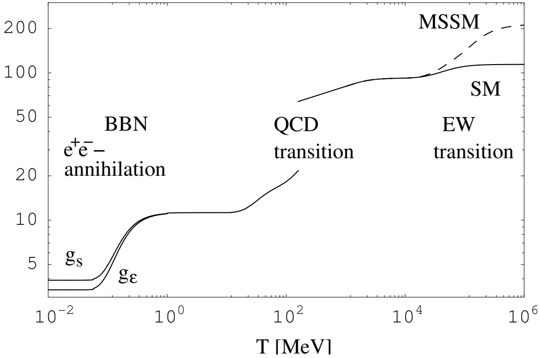
$$g_* = \sum_{i=\text{bosons}} g_i + \frac{7}{8} \sum_{j=\text{fermions}} g_j \, .$$

Useful relation

$$\boxed{a \propto \frac{1}{T}}$$

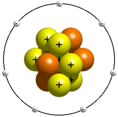
Relativistic degrees of freedom

Particles with $m \ll T$ should be counted only, i. e. g_* is a function of temperature



Georgy Antonovich Gamov

Nuclear physicist and cosmologist



- Explained nuclear alpha decay by quantum mechanical tunneling (1928).
- His model of atomic nuclei (1929) served as the basis for the modern theories of nuclear fission and fusion.
- He developed, with Edward Teller, a theory of nuclear beta decay (1936).

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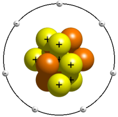
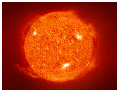
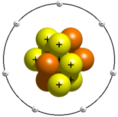
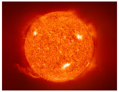




- Later he used his knowledge of nuclear reactions to interpret stellar evolution and element formation in stars (1938).
- Modeled red giants, supernovae, and neutron stars (1939).

Georgy Antonovich Gamov

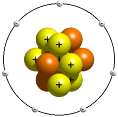
Nuclear physicist and cosmologist

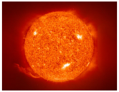



- In 1954 he proposed the concept of a genetic code and first suggested how the genetic code might be transcribed...

Georgy Antonovich Gamov

Nuclear physicist and cosmologist





- In 1946 G. Gamov realized that ^{4}He could not have been produced in stars. He suggested, as a way out, that the early Universe itself was the Oven in which light elements were cooked up.
- He also calculated the left-over heat which should be measured today as 5° K CMBR.

The Hot Big Bang theory was born.

Cosmic Microwave Background Radiation

Predicted by G. Gamov in 1946: $\mathbf{5}\ K^{\circ}$

Measured by A. Penzias and R. Wilson in 1965: $\mathbf{3.5}\ K^{\circ}$

- 2.725 K° above absolute zero
- mm-cm wavelength
- 410 photons per cubic centimeter
- 10 trillion photons per second per squared centimeter

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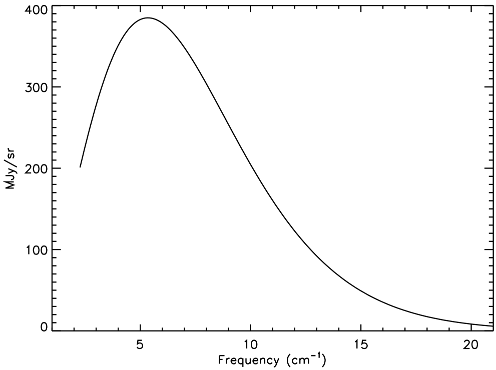
- 2.725 K° above absolute zero
- mm-cm wavelength
- 410 photons per cubic centimeter
- 10 trillion photons per second per squared centimeter
- Few percent of TV "snow"

Perfect Blackbody

The CMBR spectrum is strictly blackbody, $T = 2.725$ K



Bose-Einstein distribution

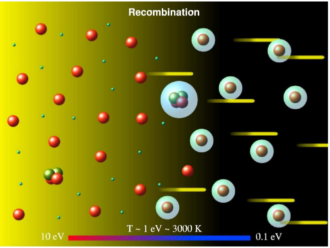
$$n = \frac{1}{\exp(E/T) - 1}$$

COBE (1994)

There is no explanation to it but the hot Big Bang

Last scattering of light

Matter is ionized at temperatures higher than hydrogen ionization energy $E_{\text{ion}} = 13.6$ eV. At lower T neutral atoms start to form.



$$\frac{n_e\, n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} \mathrm{e}^{-E_{\text{ion}}/T}$$

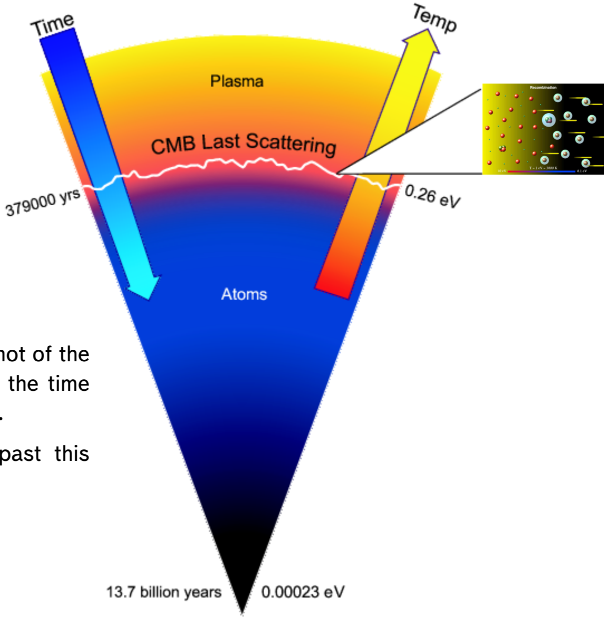
Univerese became transparent for radiation when

$$\sigma_{\gamma e}\, n_e \sim t$$

Here $\sigma_{\gamma e} = 8\pi\alpha^2/3m_e^2$ is the Compton cross-section.

Numerically $T_{\text{ls}} = 0.26$ eV

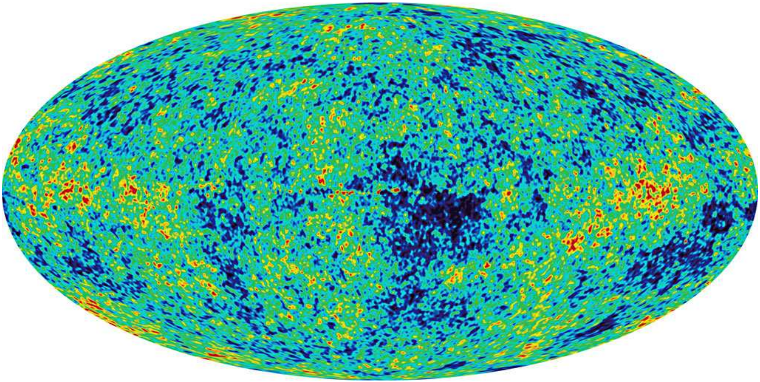
Last scattering of light



CMBR is a snapshot of the baby Universe at the time of last scattering.

We cannot see past this surface.

Temperature map of the sky



- Temperature slightly different in different patches of the sky - 1 part in 100,000.

CMB power spectrum

The temperature anisotropy, $T(\mathbf{n})$, is expanded in a spherical harmonics

$$T(\mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}).$$

The angular power spectrum, C_l, is defined as

$$C_l^{sky} = \frac{1}{2l+1} \sum_m |a_{lm}|^2.$$

Assuming random phases, the temperature anisotropy for each multipole moment, ΔT_l, can be associated with the angular spectrum

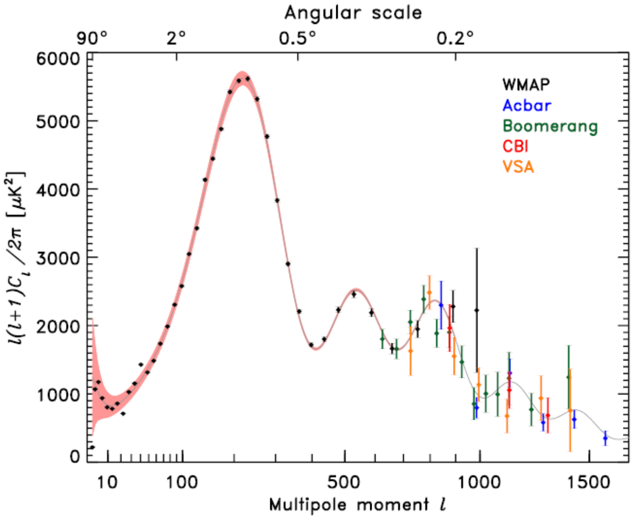
$$\Delta T_l = \sqrt{C_l^{sky}\ l(l+1)/2\pi}.$$

The correlation function is

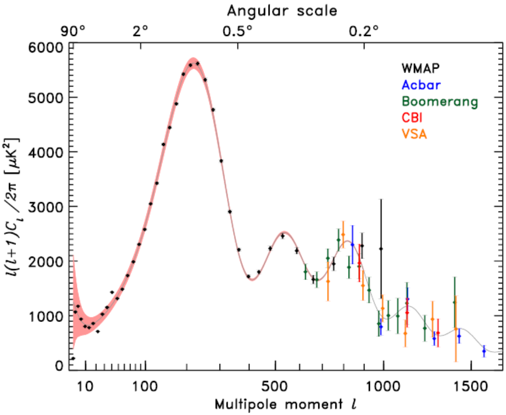
$$C(\theta) = \frac{1}{4\pi} \sum_l (2l+1) C_l\ P_l(cos\ \theta)$$

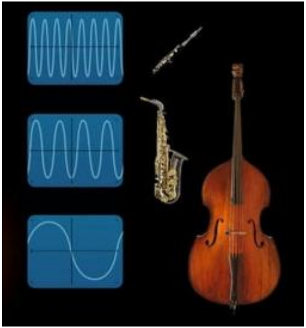
where P_l is the Legendre polynomial or order l.

CMB power spectrum: tool of Precision Cosmology



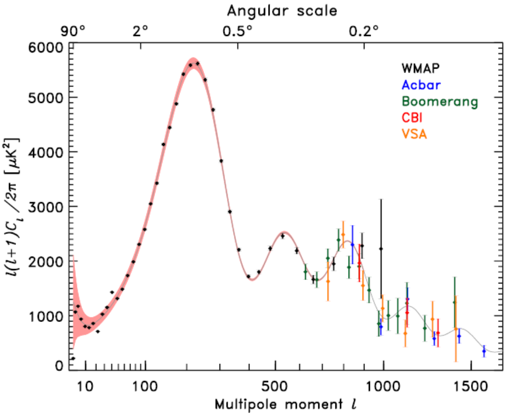
Tool of Precision Cosmology





Soundscape of the sky

Tool of Precision Cosmology



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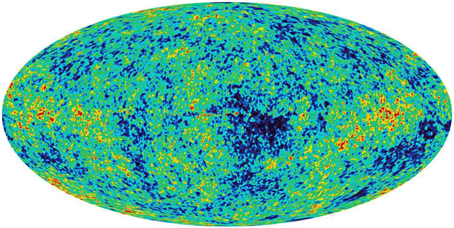
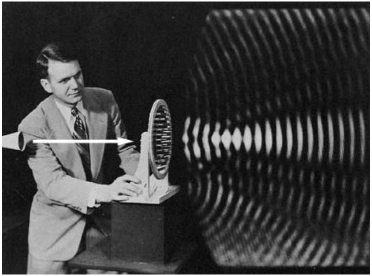


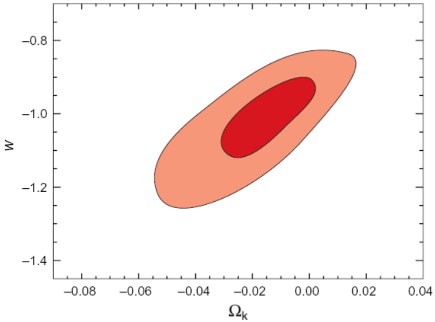


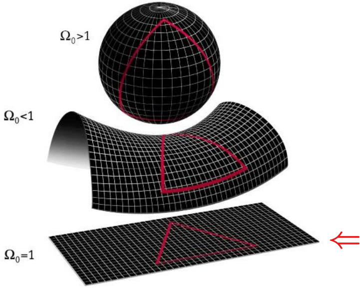
Instant photo of sound waves

Cosmological parameters

Description	Symbol	Value	+ uncertainty	− uncertainty
Total density	Ω_{tot}	1.02	0.02	0.02
Equation of state of quintessence	w	< -0.78	95% CL	—
Dark energy density	Ω_Λ	0.73	0.04	0.04
Baryon density	$\Omega_b h^2$	0.0224	0.0009	0.0009
Baryon density	Ω_b	0.044	0.004	0.004
Baryon density (cm^{-3})	n_b	2.5×10^{-7}	0.1×10^{-7}	0.1×10^{-7}
Matter density	$\Omega_m h^2$	0.135	0.008	0.009
Matter density	Ω_m	0.27	0.04	0.04
Light neutrino density	$\Omega_\nu h^2$	< 0.0076	95% CL	—
CMB temperature (K)[a]	T_{cmb}	2.725	0.002	0.002
CMB photon density (cm^{-3})[b]	n_γ	410.4	0.9	0.9
Baryon-to-photon ratio	η	6.1×10^{-10}	0.3×10^{-10}	0.2×10^{-10}
Baryon-to-matter ratio	$\Omega_b \Omega_m^{-1}$	0.17	0.01	0.01
Fluctuation amplitude in $8h^{-1}$ Mpc spheres	σ_8	0.84	0.04	0.04
Low-z cluster abundance scaling	$\sigma_8 \Omega_m^{0.5}$	0.44	0.04	0.05
Power spectrum normalization (at $k_0 = 0.05$ Mpc^{-1})[c]	A	0.833	0.086	0.083
Scalar spectral index (at $k_0 = 0.05$ Mpc^{-1})[c]	n_s	0.93	0.03	0.03
Running index slope (at $k_0 = 0.05$ Mpc^{-1})[c]	$dn_s/d\ln k$	-0.031	0.016	0.018
Tensor-to-scalar ratio (at $k_0 = 0.002$ Mpc^{-1})	r	< 0.90	95% CL	—
Redshift of decoupling	z_{dec}	1089	1	1
Thickness of decoupling (FWHM)	Δz_{dec}	195	2	2
Hubble constant	h	0.71	0.04	0.03
Age of universe (Gyr)	t_0	13.7	0.2	0.2
Age at decoupling (kyr)	t_{dec}	379	8	7
Age at reionization (Myr, 95% CL))	t_r	180	220	80
Decoupling time interval (kyr)	Δt_{dec}	118	3	2
Redshift of matter-energy equality	z_{eq}	3233	194	210
Reionization optical depth	τ	0.17	0.04	0.04
Redshift of reionization (95% CL)	z_r	20	10	9
Sound horizon at decoupling (°)	θ_A	0.598	0.002	0.002
Angular size distance to decoupling (Gpc)	d_A	14.0	0.2	0.3
Acoustic scale[d]	ℓ_A	301	1	1
Sound horizon at decoupling (Mpc)[d]	r_s	147	2	2

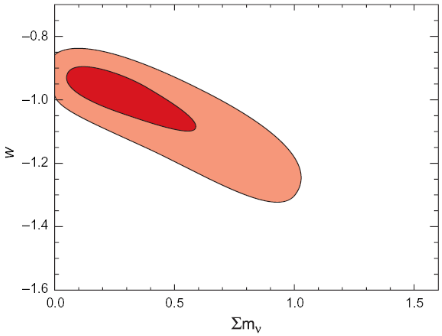
Geometry of the Universe





The Universe is spatially flat and is dominated by the dark energy with the equation of state $w = -1$

Constraints on the neutrino mass



Neutrino temperature

$$T_\nu = 1.947\ K$$

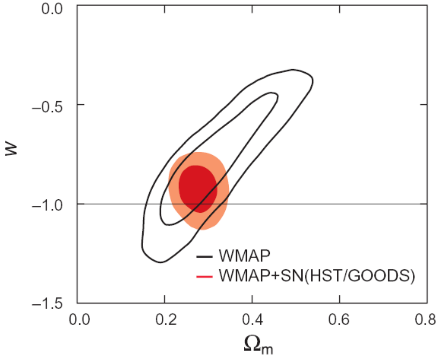
This gives $n_{\nu i} = 115\ \mathrm{cm}^{-3}$

Since $\rho_\nu = \sum_i m_{\nu i}\, n_{\nu i}$

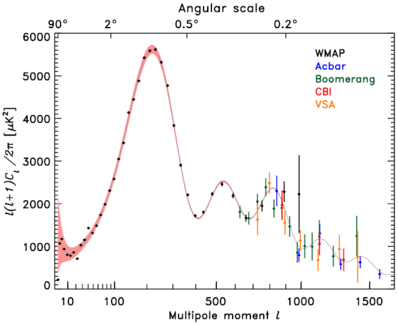
we also have a constraint

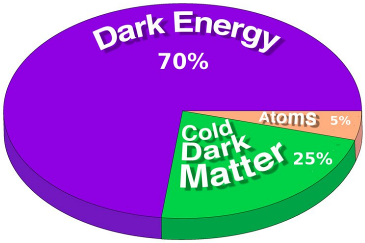
$$\Omega_\nu < 10^{-2}\, \Omega_m$$

Constraints on the matter abundance



Matter content in the Universe



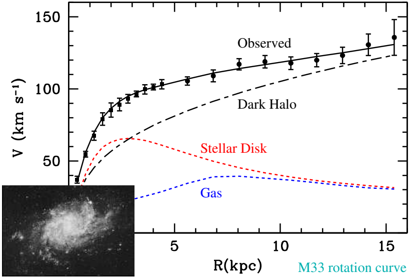


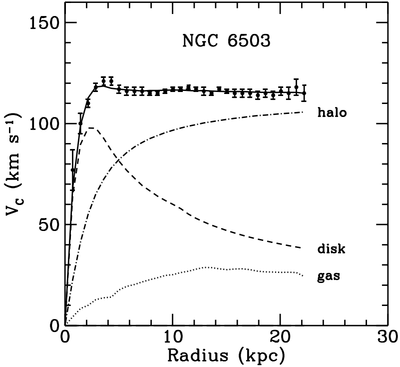
96% of the Universe is made of unknown substance

Missing mass is seen on all cosmological scales and reveals itself via

- Flat rotational curves in galaxies
- Gravitational potential which confines galaxies and hot gas in clusters
- Gravitational lenses in clusters
- Gravitational potential which allows structure formation from tiny primeval perturbations
- Gravitational potential which creates CMBR anisotropies
- ...

Dark matter in galaxies





Newtonian Dynamics:

$$\frac{v_{\rm rot}^2}{r} = \frac{G\,M(r)}{r^2} \quad \longrightarrow \quad v_{\rm rot} = \sqrt{\frac{G\,M(r)}{r}}$$

Halo structure

- Simplest self-gravitating stationary solution which gives flat rotational curves - isothermal sphere

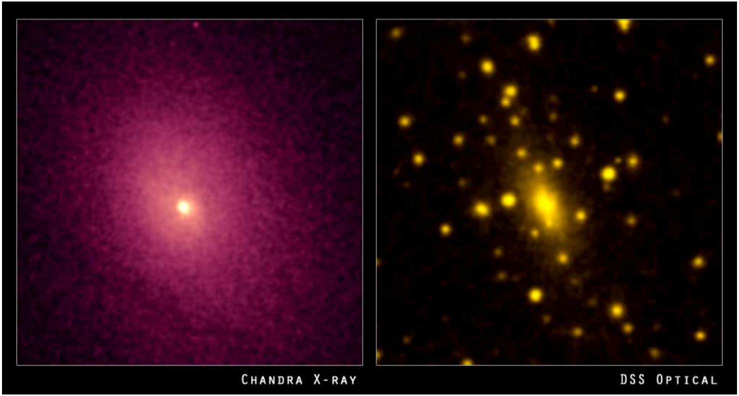
$$\rho(r) = \frac{\rho_0}{(1+x^2)}, \quad \text{where} \quad x \equiv r/r_c$$

- Empirical fit to numerical simulations, Navarro, Frenk & White (NFW) profile

$$\rho(r) = \frac{\rho_0}{x\,(1+x)^2}$$

- $\ldots$

Abell 2029



X-ray and optical data produce similar density profiles dominated by single DM component, consistent with NFW profile

Mass estimate. I.

Virial theorem $2\langle E_{\text{kin}}\rangle + \langle E_{\text{pot}}\rangle = 0$

where $\langle E_{\text{kin}}\rangle = \frac{1}{2}Nm\langle v^2\rangle, \quad \langle E_{\text{pot}}\rangle = -\frac{1}{2}\frac{G\,N^2 m^2}{\langle r\rangle}$

gives for the total mass estimate, $M \equiv Nm$

$$M \sim \frac{2\langle r\rangle\langle v^2\rangle}{G}$$

E.g. for Coma cluster $\frac{M}{L} \sim 300\,h\,\frac{M_\odot}{L_\odot}$ *Zwicky (1933)*

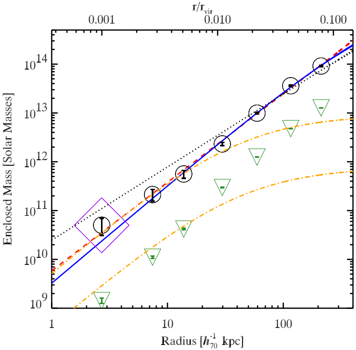
Mass of visible galaxies 1% - 7%
Hot gas contributes 10% - 40%
The rest should be dark matter.

Dark matter in clusters of galaxies

Mass estimate. II. Assume hot gas is in thermal equilibrium in gravitational well created by cluster. Measuring temperature profile one can reconstruct gas pressure and density and hence the gravitational potential.

Detailed modeling of Abell 2029 gives

triangles: gas contribution
dot-dashed curves: stellar mass with M/L of 1 and 12
solid line: NFW dark matter profile
$\rho \propto \frac{1}{x(1+x^2)}$,
where $x \equiv r/r_s$ and $r_s = 540$ kpc



Baryonic fraction **14%**. This also gives $\Omega_m \approx 029$.

Lewis et al (2003)

Dark matter in clusters of galaxies

Abell 2218

Acts in this example as a gravitational lens

By today the structure is formed already, $\delta\rho/\rho \sim 1$.
Initial perturbations were small, $\delta\rho/\rho \sim 10^{-5}$.
Perturbations do not grow in the radiation dominated epoch, during matter domination $\delta\rho/\rho \sim a$. Morover, perturbations in baryons can grow only after recombination. But

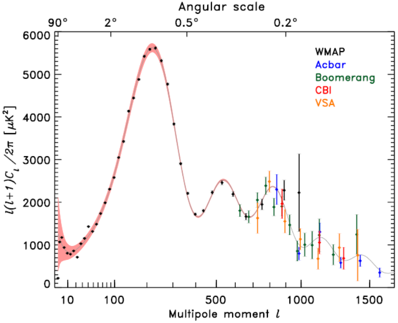
$$\frac{a_{\text{today}}}{a_{\text{dec}}} = 1 + z_{\text{dec}} = 1090$$

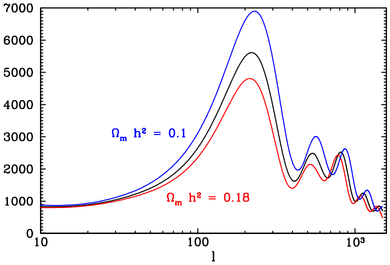
Therefore, in a baryonic universe structure can grow only by a factor

$$10^3$$

One needs non-baryonic dark matter to facilitate structure growth

CMBR and Dark matter





Best fit values (CMB data only): $h = 0.73 \pm 0.05$

$$\Omega_m h^2 = 0.13 \pm 0.01, \qquad \Omega_B h^2 = 0.023 \pm 0.001$$

Or

$$\boxed{\Omega_{\mathrm{CDM}} = 0.2 \pm 0.02}$$

Non-baryonic Dark Matter candidates

name	mass
Graviton	10^{-21} eV
Axion	10^{-5} eV
Sterile Neutrino	10 keV
Mirror matter	1 GeV
WIMP	100 GeV
WIMPZILLA	10^{13} GeV

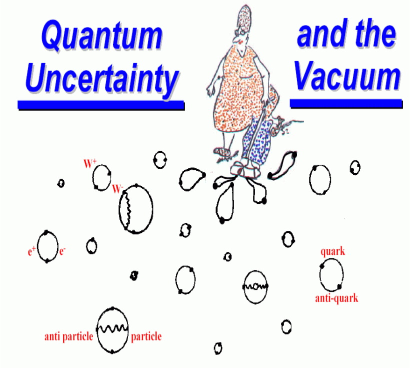
Dark matter and Dark energy: what's the difference?

"Sure it's beautiful, but I can't help thinking about all that interstellar dust out there."

DARK MATTER:

- From the fluid dynamics point of view it behaves like a dust
- After identification of dark matter a new member in the the "zoo" of elementary particles will appear,
- and we will learn about the underlying particle physics theory, its symmetries and will single out the relevant model.

Dark matter and Dark energy: what's the difference?



DARK ENERGY:

- Dark energy is the energy of the vacuum. It remains constant with expansion.
- We do not understand it on the fundamental level yet. (Why it is non-vanishing and at present is relevant cosmologically?)
- Understanding of it has great promise.

Age of the Universe

During mater dominated expansion $a \propto t^{2/3}$. Therefore

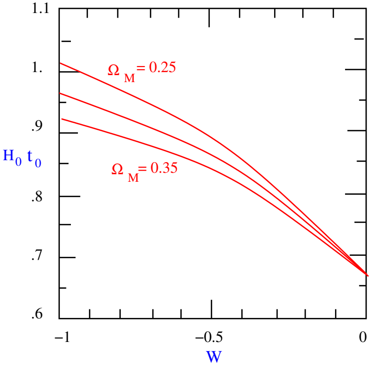
$$H_0\ t_0 = 2/3$$

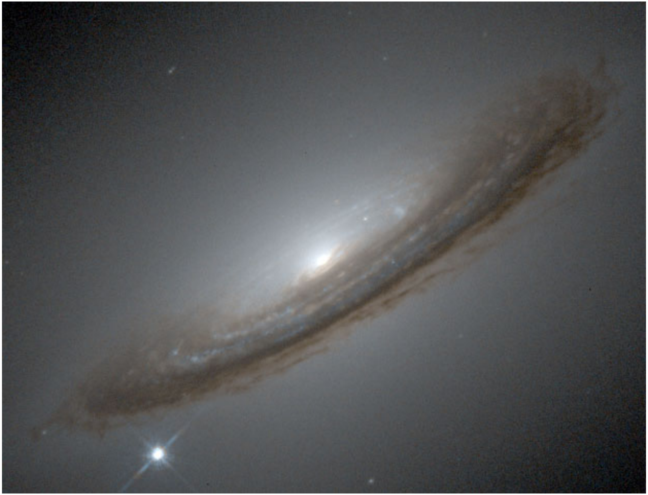
Pre-WMAP measurments:

$$H_0 = 70 \pm 7 \text{ km sec}^{-1} \text{ Mpc}^{-1}, \qquad t_0 = 13 \pm 1.5 \text{ Gyr}$$

and $\quad H_0\ t_0 = 0.93 \pm 0.15$

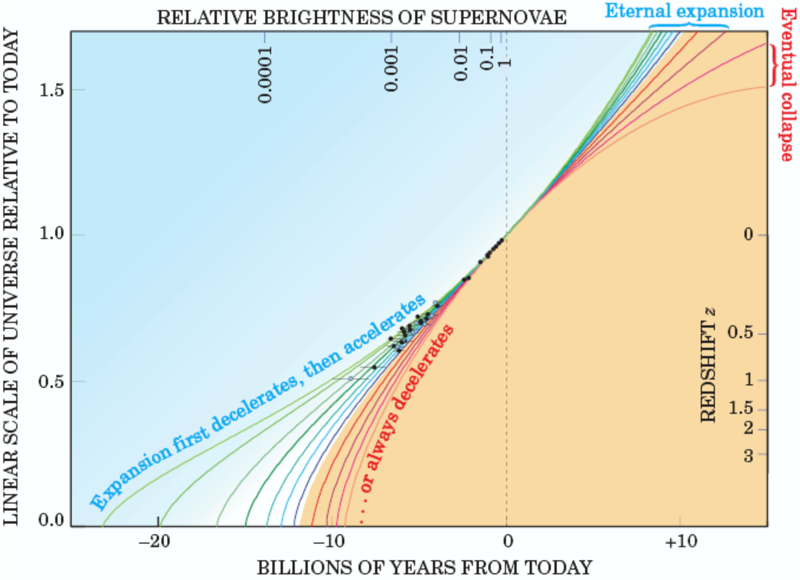
For two components, usual matter and dark energy:



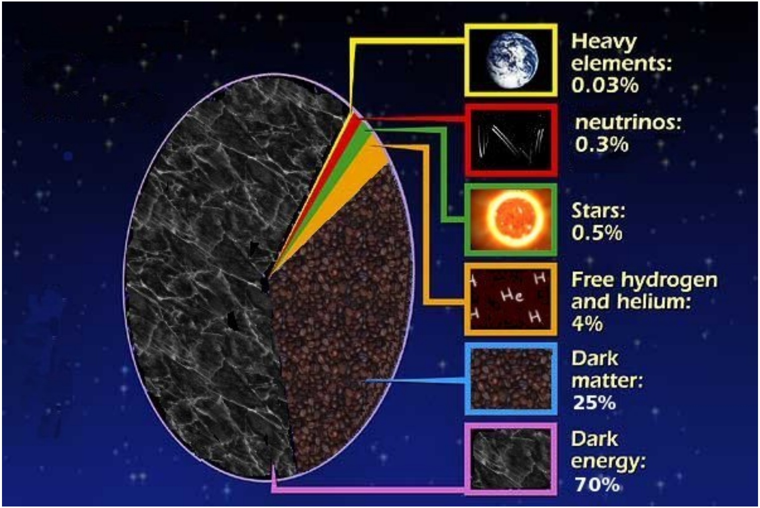


Supernovae: direct probe of the expansion history

Expansion history



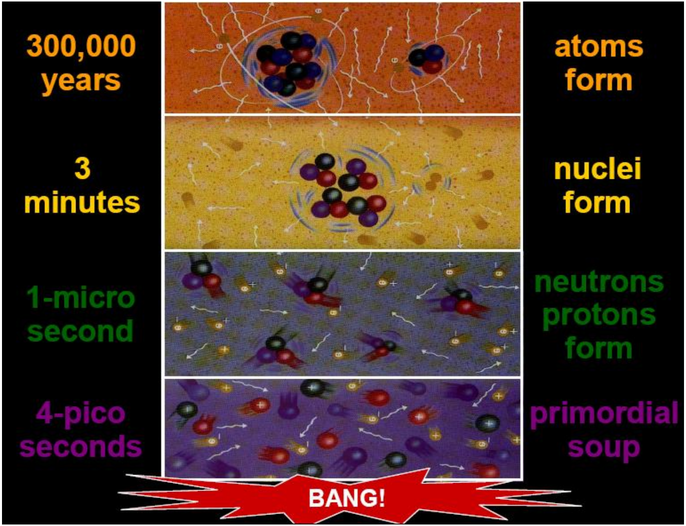
Detailed Matter content



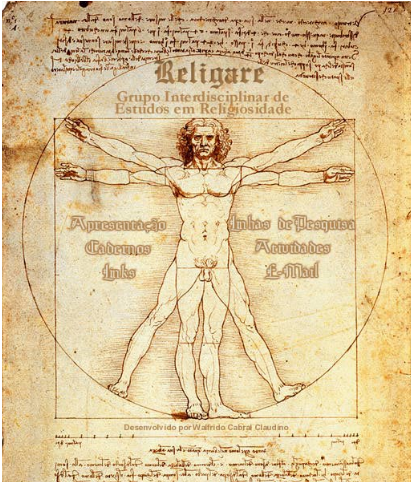
Cooked in stars

Cooked in Big Bang

Thermal History



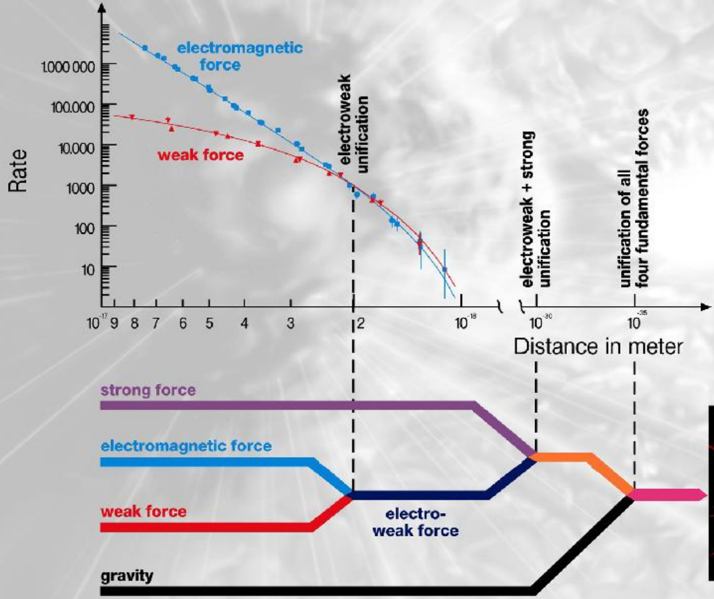
Symmetry



- Laws of physics do not depend upon observer or choice of coordinates
- Different interactions (or particles) can be unified in one entity
- True ultimate theory is maximally symmetric
- The symmetries are broken in our world
  (aka our vacuum)

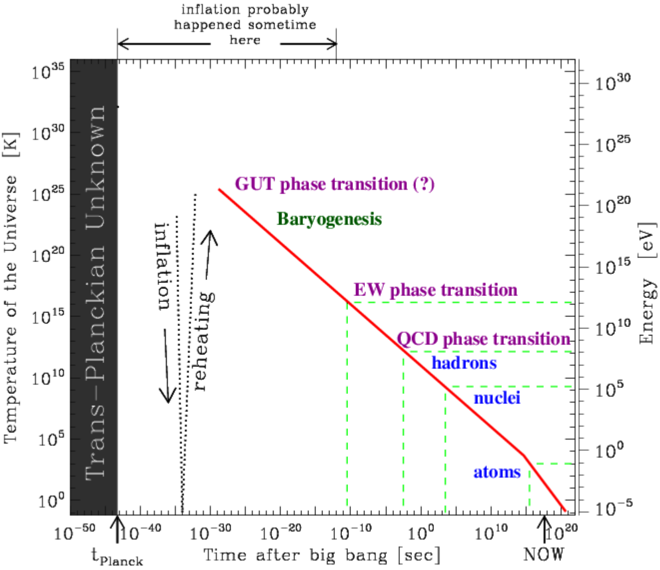
Symmetry: the guiding principle of physics

Unification of forces



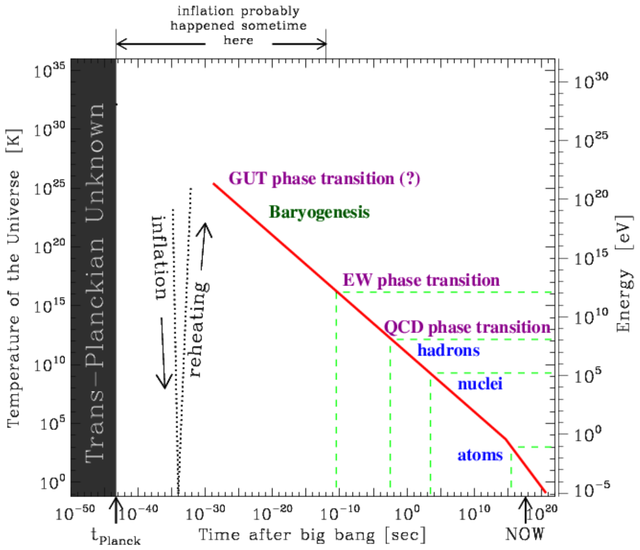
big bang

Thermal History



Thermal History

Universe is hot and expands: it was even hotter in the past.



Big Bang Nucleosynthesis: Helium abundace

^{4}**He** is the second most abundant element, constitutes about 25%. Chemical equilibrium between protons and neutrons is maintained by weak interactions

$$n + \nu \rightleftarrows p + e^-, \qquad n + e^+ \rightleftarrows p + \overline{\nu},$$

which get out of thermal equilibrium at $T_f \sim$ **1 MeV**
On the other hand

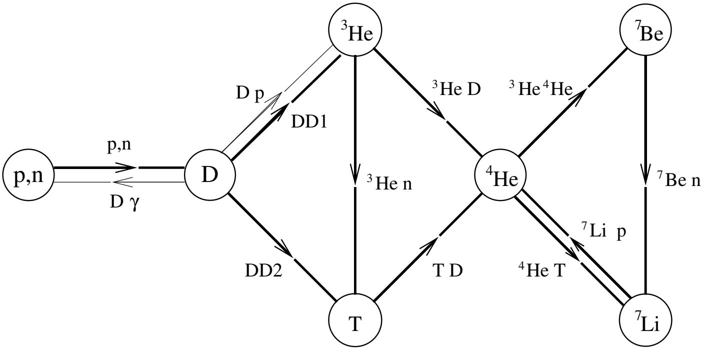
$$\Delta m \equiv m_n - m_p = 1.29 \text{ MeV}$$

Therefore, at freeze-out

$$\frac{n}{p} \sim \mathrm{e}^{-\Delta m / T_f} \approx 0.27$$

It is important also that neutron lifetime (980 s) is much longer than the age of the Universe at this time (1 s). ^{4}**He** is the most bound among the light elements, $E_{\text{bind}} \approx 28$ MeV. Therefore, almost all neutrons produced in the early universe should end up in ^{4}**He**.

Big Bang Nucleosynthesis

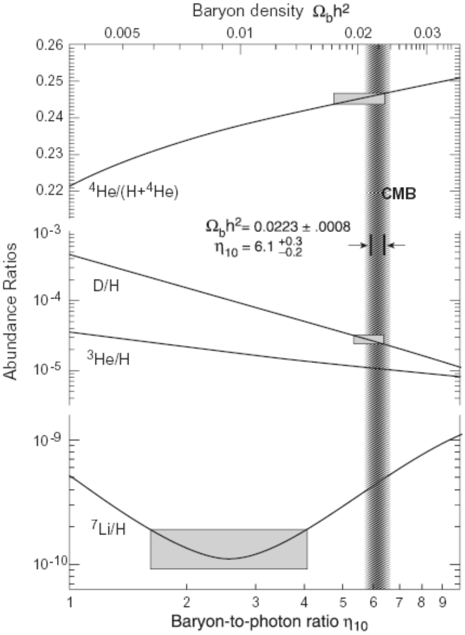


Involves numerical solution of coupled kinetic equations

$$\frac{dn_i}{dt} = I(n_1, n_2, \dots)$$

for the concentrations of elements in the expanding Universe

BBN light element abundances



Baryon asymmetry η_{10} is determined from CMB with a precision of 4%

Baryon asymmetry

In a comoving volume, at late times, entropy and the number of baryons are conserved. This gives important cosmological parameter, baryon asymmetry:

$$\eta = \frac{n_B}{n_\gamma}$$

Observationally $\quad \eta = (6.1 \pm 0.25) \times 10^{-10}$
This quantity should and can be understood dynamically within frameworks of the Big Bang.

Baryon asymmetry can be generated if

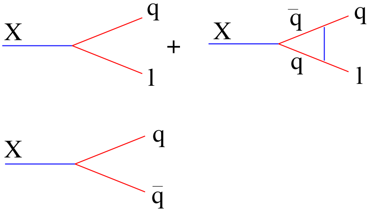
- Baryon number is not conserved
- C- and CP- are violated
- There are deviations from thermal equilibrium

Sakharov (1967), Kuzmin 1970)

Baryogenesis

Mechanisms

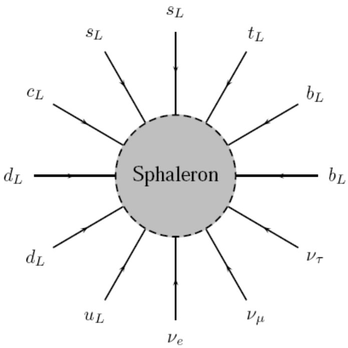
- Grand Unified Baryogenesis
- Leptogenesis
- ...



Grand Unified Baryogenesis.
Out of equilibrium decays of heavy leptoquarks.

If C- and CP- are violated
$\Gamma(X \to q, l) \neq \Gamma(\bar{X} \to \bar{q}, \bar{l})$

Sphalerons



It converts three baryons to three antileptons.

At $T \gg M_W$ this process washes out any $B + L$

Kuzmin, Rubakov, Shaposhnikov (1985)

Leptogenesis.

Lepton asymmetry can be generated in decays or oscillations of ν_R

Lepton asymmetry $\Rightarrow$ Sphalerons $\Rightarrow$ Baryon asymmetry

Puzzles of classical cosmology

WHY THE UNIVERSE

- is so old, big and flat ?
  $t > 10^{10}$ years
- homogeneous and isotropic?
  $\delta T/T \sim 10^{-5}$
- contains so much entropy?
  $S > 10^{90}$
- does not contain unwanted relics?

  (e.g. magnetic monopoles)

can be solved with hypothesis of Inflation

Basics of inflation

Definition

"Inflation" is a period of accelerated universe expansion

$$\ddot{a} > 0$$

Friedmann equations

$$\ddot{a} = -\frac{4\pi}{3} G a (\rho + 3p)$$

We have inflation whenever $p < -\rho/3$

Getting something for nothing

$$T_\mu^{\ \nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Energy-momentum conservation $T^{\mu\nu}{}_{;\nu} = 0$ can be written as

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

Consider $T_{\mu\nu}$ for a vacuum. Vacuum has to be Lorentz invariant, hence $T_\mu^{\ \nu} = V\,\delta_\mu^{\ \nu}$ and we find $p = -\rho \quad \Rightarrow \quad \dot\rho = 0$

Energy of the vacuum stays constant despite the expansion !

The Inflaton field

Consider $T_{\mu\nu}$ for a scalar field φ

$$T_{\mu\nu} = \partial_\mu \varphi \, \partial_\nu \varphi - g_{\mu\nu} \, \mathcal{L}$$

with the Lagrangian :

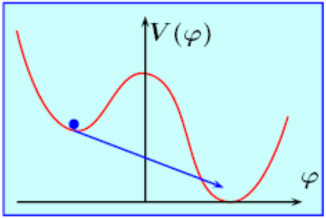
$$\mathcal{L} = \partial_\mu \varphi \, \partial^\mu \varphi - V(\varphi)$$

In a state when all derivatives of φ are zero, the stress-energy tensor of a scalar field is that of a vacuum, $p \approx -\rho$

$$T_{\mu\nu} = V(\varphi) \, g_{\mu\nu}$$

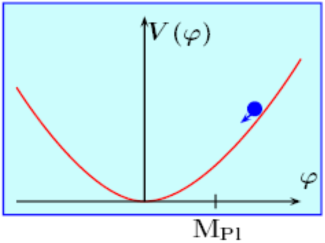
There are two basic ways to arrange $\varphi \approx \text{const}$ and hence to imitate the vacuum-like state.

1. A. Guth: consider potential with two minima



2. A. Linde: consider the simplest potential

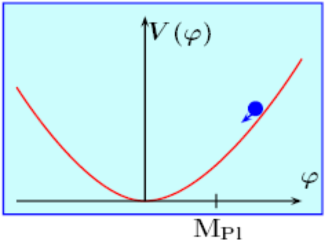
$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$



Equation of motion

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$$

If $H \gg m$ the field rolls down slowly ("slow-roll")



$$H \sim m\,\varphi / M_{\mathrm{Pl}}$$

$\varphi > M_{\mathrm{Pl}}$ Inflation
$\varphi < M_{\mathrm{Pl}}$ Reheating

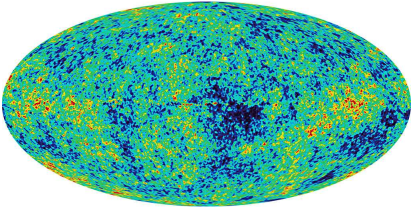
During Inflation the Universe is empty, in a vacuum state.

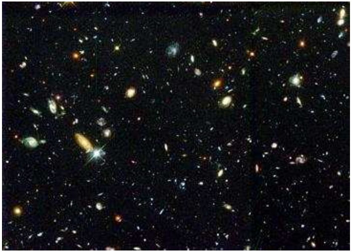
Particle physicist

Cosmologist

Where all matter and seeds for structure formation came from ?

Predictive power of Inflation





Fluctuations in inflaton field

$\Downarrow$

CMBR anisotropy
379,000 years after

$\Downarrow$

Large-scale structure
13.7 billions years after

Unified Theory of Creation

During Inflation the Universe is “empty”. But small fluctuations obey

$$\ddot{u}_k + [k^2 + m^2_{\text{eff}}(\tau)]\, u_k = 0$$

and it is not possible to keep fluctuations in vacuum if m_{eff} is time dependent

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The source for $m_{\text{eff}} = m_{\text{eff}}(\tau)$ is time-dependence of classical backgrounds:

- Expansion of space-time, $a(\tau)$
- Evolution of the inflaton field, $\phi(\tau)$

$$\ddot{u}_k + [k^2 + m_{\text{eff}}^2(\tau)]\, u_k = 0$$

Technical remarks:

- This equation holds for all metric perturbations and for all particle species
- Equations look that simple in conformal reference frame $ds^2 = a(\eta)^2\,(d\eta^2 - dx^2)$
- For conformally coupled, but massive scalar $m_{\text{eff}} = m_0\, a(\eta)$
- m_{eff} may be non-zero even for massless fields, graviton is the simplest example $m_{\text{eff}}^2 = -\ddot{a}/a$
- Of particular interest are ripples of space-time itself
    - curvature fluctuations (scalar)
    - gravitons (tensor)

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Power spectrum

Decompose a scalar field over creation and annihilation operators

$$\phi(\mathrm{x}, t) = \frac{1}{a} \int \frac{d^3k}{(2\pi)^{3/2}} \left[u_k(t)\, a_{\mathrm{k}}\, \mathrm{e}^{i\mathrm{kx}} + u_k^*(t)\, a_{\mathrm{k}}^\dagger\, \mathrm{e}^{-i\mathrm{kx}} \right]$$

Regularized variance

$$\langle 0|\varphi^2(x)|0\rangle_{\mathrm{reg}} = \frac{1}{a^2} \int \frac{d^3k}{(2\pi)^3} \left(|u_k|^2 - \frac{1}{2\omega_k} \right) \equiv \int P_\varphi(k)\, \frac{dk}{k}$$

Therefore, the power spectrum of field fluctuations is

$$\boxed{P_\varphi(k) = \frac{k^3}{2\pi^2 a^2} \left(|u_k|^2 - \frac{1}{2\omega_k} \right)}$$

Inflationary perturbations

Assume Hubble parameter during inflation is constant, $a(\eta) = -\frac{1}{H\eta}$

Mode functions of massles field $(\xi = 0)$ obey

$$\ddot{u}_k + \left[k^2 - \frac{2}{\eta^2}\right] u_k = 0$$

Solution with proper initial conditions at $\eta \to -\infty$

$$u_k = \frac{\mathrm{e}^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right)$$

After horizon crossing $(k\eta \ll 1)$: $u_k \to -\frac{1}{\sqrt{2}} \frac{i}{k^{3/2}\,\eta}$

and

$$\boxed{P_\varphi(k) \to \frac{H^2}{(2\pi)^2}}$$

Curvature perturbations

Spatial Curvature $\quad {}^{(3)}R \; \propto \; \frac{1}{a^2}$

Its perturbation ζ :

$$\zeta = \frac{\delta a}{a} = H \delta t = H \, \frac{\delta \varphi}{\dot{\varphi}}$$

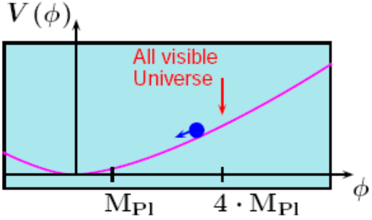
Therefore

$$P_\zeta(k) \;=\; \frac{H^2}{\dot{\varphi}^2} \; P_\varphi(k)$$

and we find

$$\boxed{P_\zeta(k) \;=\; \frac{1}{4\pi^2} \, \frac{H^4}{\dot{\varphi}^2}}$$

Slow-roll



- Cosmological scales encompass small $\Delta\phi$ interval
- Potentail should be flat over this range of $\Delta\phi$

Therefore, observables essentially depend on a first few derivatives of H (or V) defined at some scale ϕ_0.

Slow-roll parameters:

$$H(\phi_0)$$

$$\epsilon \equiv \frac{M_{Pl}^2}{4\pi}\left(\frac{H'}{H}\right)^2$$

$$\eta \equiv \frac{M_{Pl}^2}{4\pi}\frac{H''}{H}$$

Slow-roll

In the slow-roll approximation $\ddot{\varphi}$ can be neglected and equation of moton

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'$$

gives

$$\dot{\varphi} = -\frac{V'}{3H}$$

Since $\rho \approx V$ we also can use

$$H^2 = \frac{8\pi G}{3} V$$

This gives for curvature perturbations

$$P_\zeta(k) = \frac{H^4}{4\pi^2\,\dot{\varphi}^2} = \frac{1}{\pi\epsilon}\frac{H^2}{M_{\mathrm{Pl}}^2}$$

Normalizing to CMBR

Let us consider the simple model $V = \frac{1}{2}m^2\varphi^2$. We have

$$H = \sqrt{\frac{4\pi}{3}\frac{m\varphi}{M_{\rm Pl}}}, \qquad \epsilon = \frac{M_{\rm Pl}^2}{4\pi\varphi^2}$$

and

$$\zeta_k \equiv P_\zeta(k)^{1/2} = \sqrt{\frac{16\pi}{3}}\,\frac{m\,\varphi^2}{M_{\rm Pl}^3}$$

Since $\delta T/T = 2\zeta_k/3$ we find

$$m \approx \frac{\delta T}{T}\frac{M_{\rm Pl}}{30} \approx 10^{13}\ \text{GeV}$$

Power spectra and consistency relation

Power spectra of Scalar (curvature) and Tensor (gravity waves) perturbations

$$P(k)_S = \frac{1}{\pi\epsilon}\frac{H^2}{M_{\rm Pl}^2} \qquad \Rightarrow \qquad \frac{P(k)_T}{P(k)_S} = 16\epsilon$$
$$P(k)_T = \frac{16}{\pi}\frac{H^2}{M_{\rm Pl}^2}$$

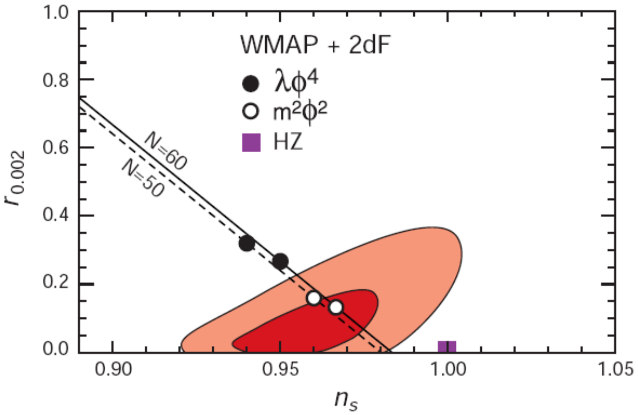
Spectra can be approximated as power law functions

$$P(k)_S = P(k_0)_S \left(\frac{k}{k_0}\right)^{n-1}$$
$$P(k)_T = P(k_0)_T \left(\frac{k}{k_0}\right)^{n_T}$$

In slow roll parameters one finds $n - 1 = 2\eta - 4\epsilon, \;\; n_T = -2\epsilon$

3 year WMAP data

Reconstruction of inflaton potential, $V(\phi) \propto \phi^n$



$m \sim$ $\mathbf{10^{13}}$ **GeV**

Puzzles of classical cosmology

Curvature problem and the solution

Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

can be re-written as

$$k = \dot{a}^2\,(\Omega - 1) = \text{const}$$

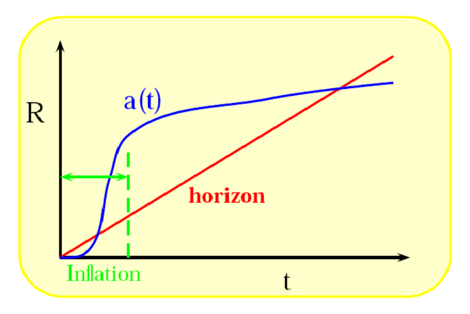
- Problem: During matter or radiation dominated stages $\dot{a}^2$ decreses (since $\ddot{a} < 0$), therefore Ω is driven away from unity. To get $\Omega \sim 1$ today, one needs enormous initial fine-tuning, say at BBN epoch $|\Omega(t_{\text{NS}}) - 1| < 10^{-15}$
- Solution: Accelerated expansion drives Ω to unity. Therefore, the problem can be solved if at early times $\ddot{a} > 0$

Puzzles of classical cosmology

Horizon problem and the solution

Horison: $\propto t$

Physical size: $\propto \quad a(t) \propto t^{\gamma}$



During matter or radiation dominated stages $\gamma < 1$, therefore the visible universe at early times contains many causially disconnected regions.

Problem is solved if there was a period with $\gamma > 1$ or $\ddot{a} > 0$

Basics of inflation

Puzzles of classical cosmology

WHY THE UNIVERSE

- is so old, big and flat ?
  $t > 10^{10}$ years
- homogeneous and isotropic?
  $\delta T/T \sim 10^{-5}$
- contains so much entropy?
  $S > 10^{90}$
- does not contain unwanted relics?

  (e.g. magnetic monopoles)

can be solved with hypothesis of Inflation