Abundance determination in stellar atmospheres

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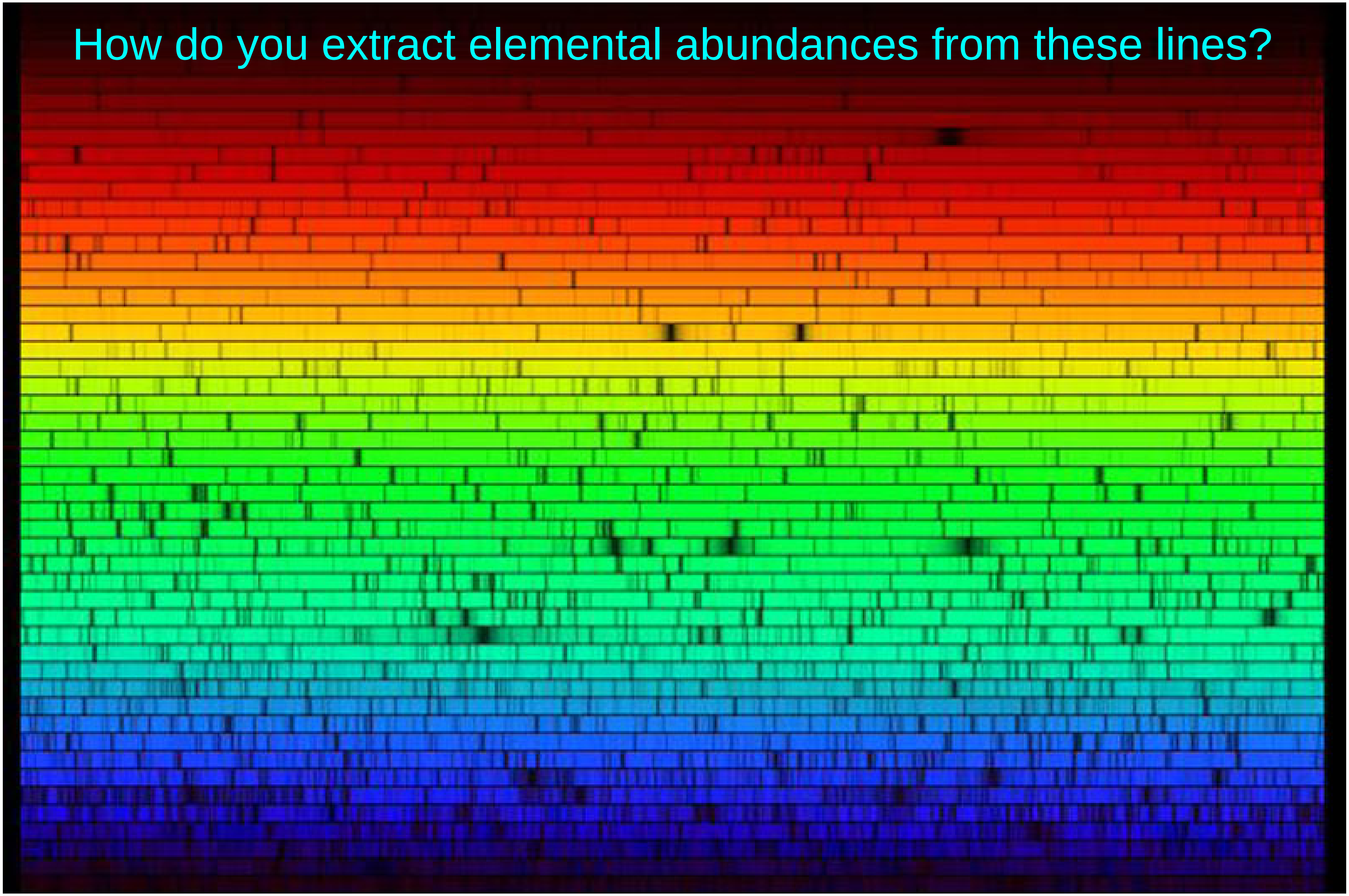
1. Introduction



What you need first:

- The stellar « identity », i.e. spectral type or photometric classification
- Atmospheric properties $\rightarrow$ line formation
  1. Effective temperature
  2. Surface gravity
  3. « Metallicity »

The general scheme you will then follow:

- Interpolate in grids of published model atmospheres or compute your own model
- Compute a theoretical (or « synthetic ») spectrum for assumed abundances (e.g. solar ones)
- Compare with observations and iterate on abundances

2. Relevant stellar parameters:

1. Effective temperature T_{eff}: T of a blackbody with same L and same R as the real star: $L = 4\pi R^2 \sigma T_{eff}^{\ 4}$
2. Surface gravity: $g = \frac{GM}{R^2}$

(usually expressed in cgs units and as a decimal log: logg)

3. Metallicity Z or iron content [Fe/H], and [alpha/Fe]:

$$Z = \frac{mass\_of\_elements\_heavier\_than\_He}{total\_mass\_in\_a\_unit\_volume} \approx 0.018$$ for the ⊙

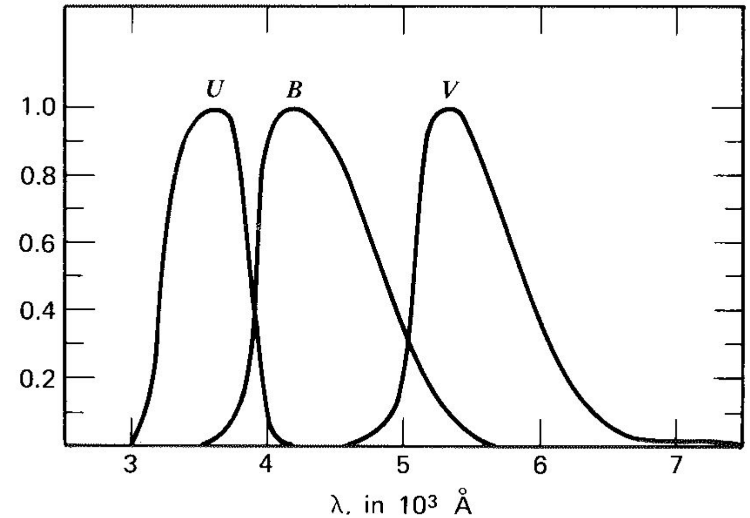
$$\left[\frac{Fe}{H}\right] = \log\left(\frac{N(Fe)}{N(H)}\right)_* - \log\left(\frac{N(Fe)}{N(H)}\right)_{sun}$$

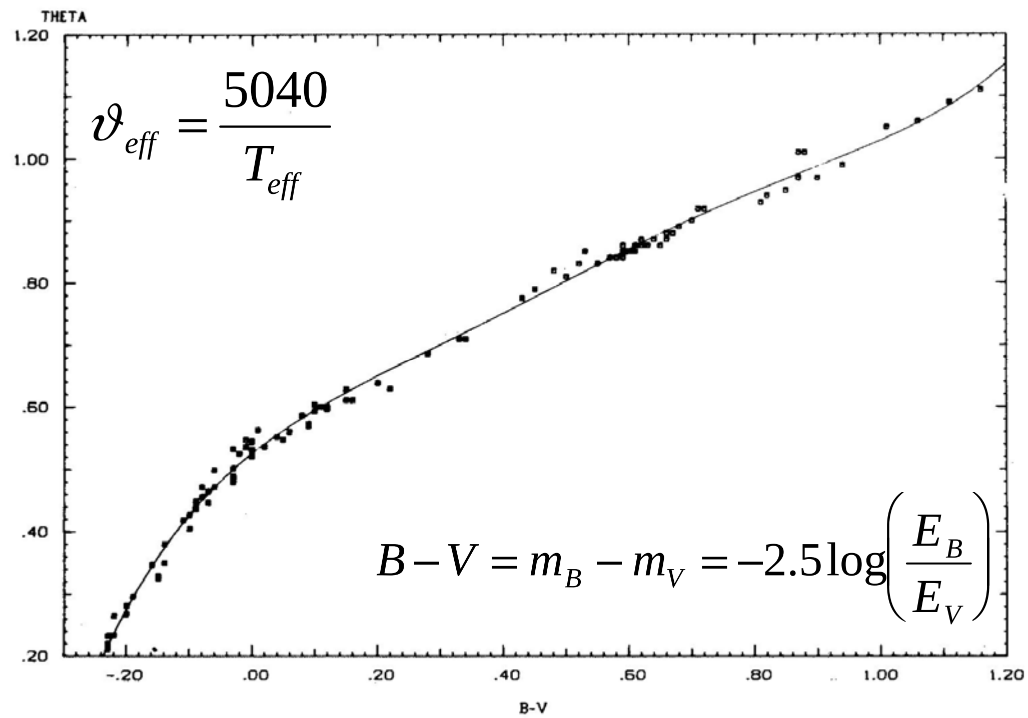
$$\left[\frac{\alpha}{Fe}\right] = \log\left(\frac{N(\alpha)}{N(Fe)}\right)_* - \log\left(\frac{N(\alpha)}{N(Fe)}\right)_{sun}$$ where α is an « α-element »,

i.e. with a nucleus made of an integer number of α particles

2.1. How to determine T_{eff}?

- Multicolour photometry calibrated with stars having fundamentally determined T_{eff}:
  - Johnson's UBV
  - Stromgren's uvby
  - Geneva UBV B1 B2 V1 G
- Low resolution spectroscopy: ratios of suitable strong lines $\rightarrow$ T_{eff} or spectral type + calibration T_{eff} vs spectral type



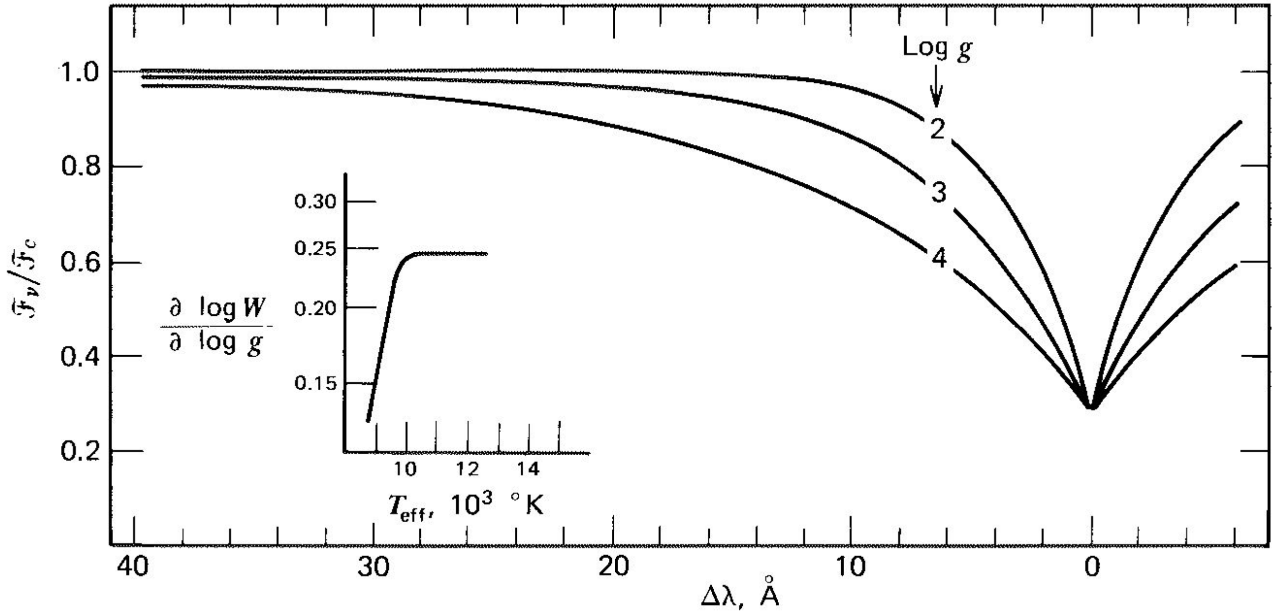


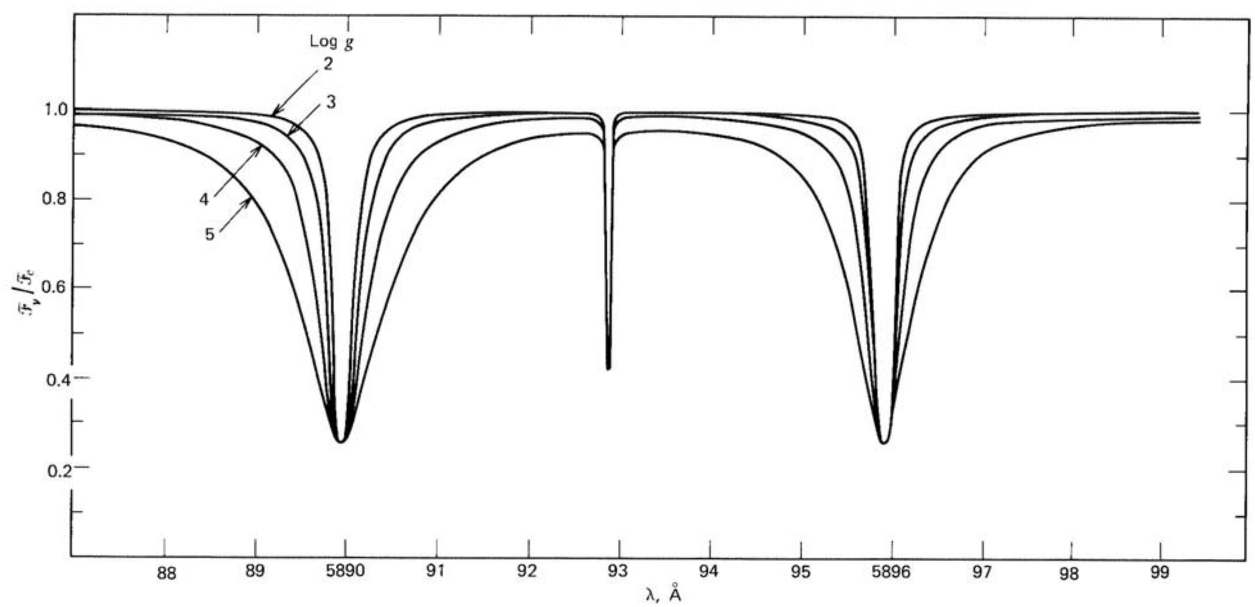
$$\vartheta_{eff} = \frac{5040}{T_{eff}}$$

$$B - V = m_B - m_V = -2.5 \log\left(\frac{E_B}{E_V}\right)$$

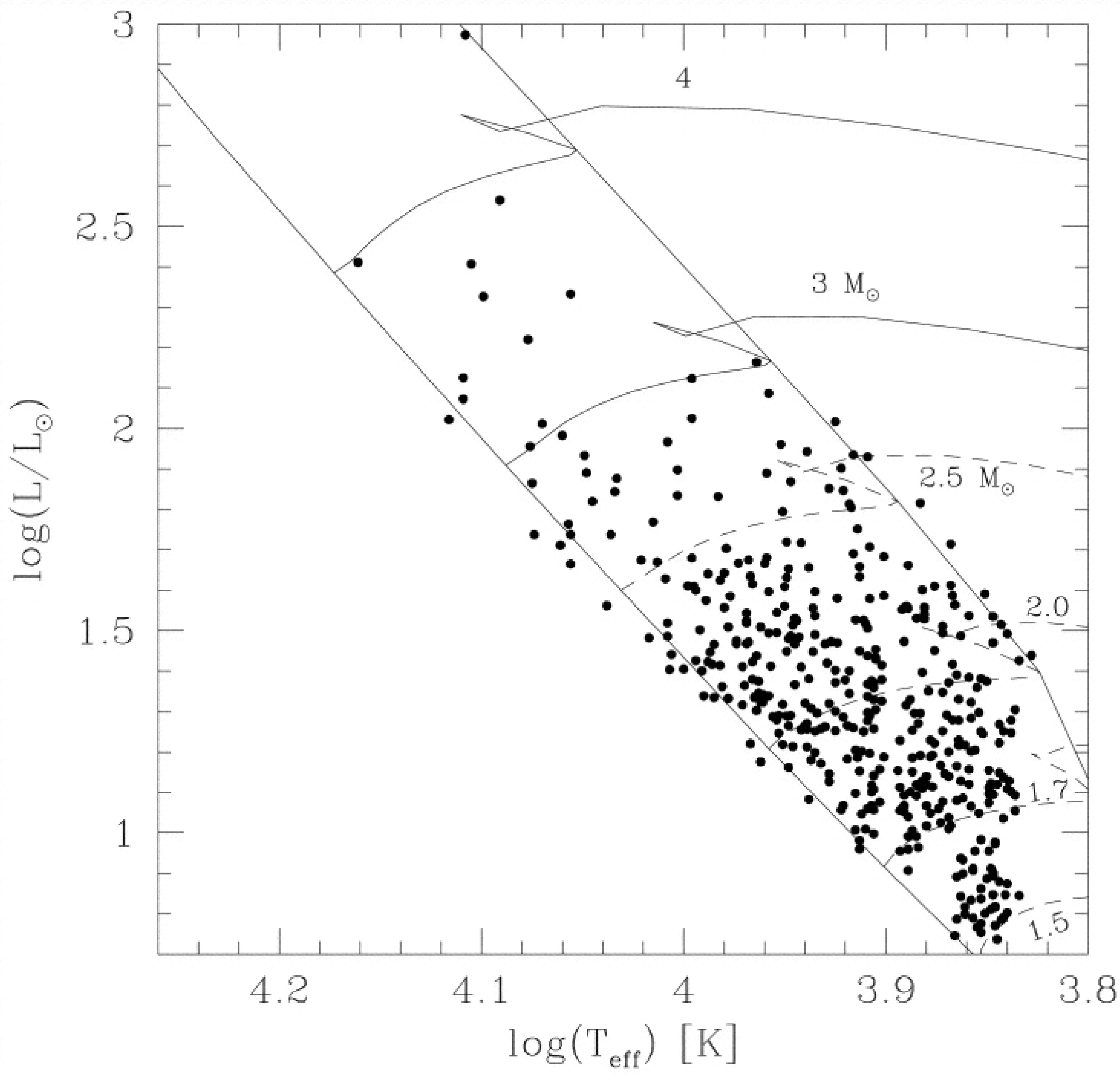
2.2. How to determine logg?

- Spectroscopy:
  - Ratio of Fe II to Fe I lines
  - B-type stars: profile of H Balmer lines
  - Later type stars: profile of strong metallici lines (e.g. Na I D doublet, Ca II infrared triplet)
- Calibrated photometry
- Parallaxes $\Rightarrow$ L, R, (M) $\rightarrow$ g



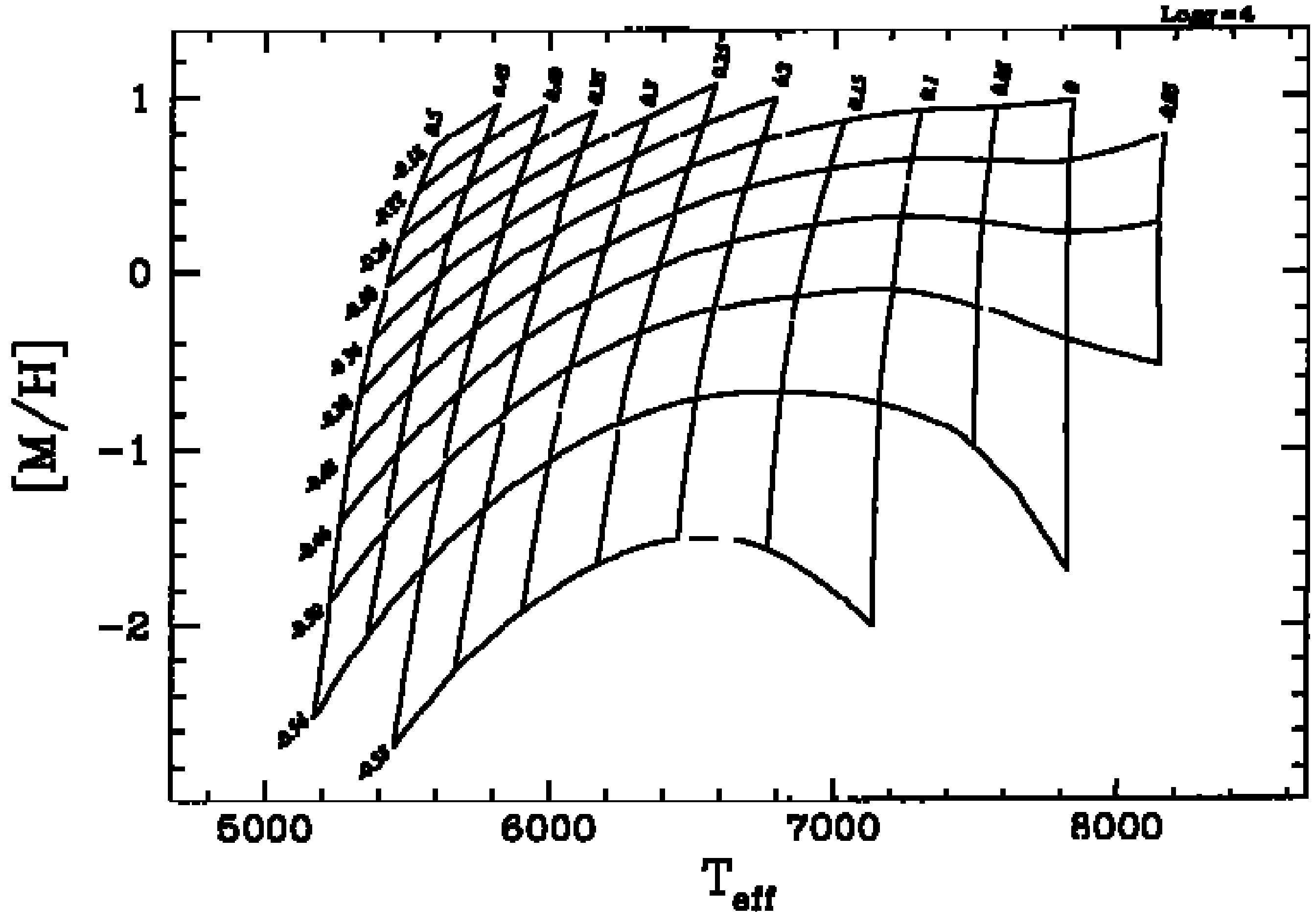


Parallaxes+photometry+Theoretical evolutionary tracks



2.3. How to determine (a first guess of) [Fe/H]?

- Calibrated photometry
- Low resolution spectroscopy, e.g. equivalent width of some strong lines
- N.B. for [α/Fe], only spectroscopy will work

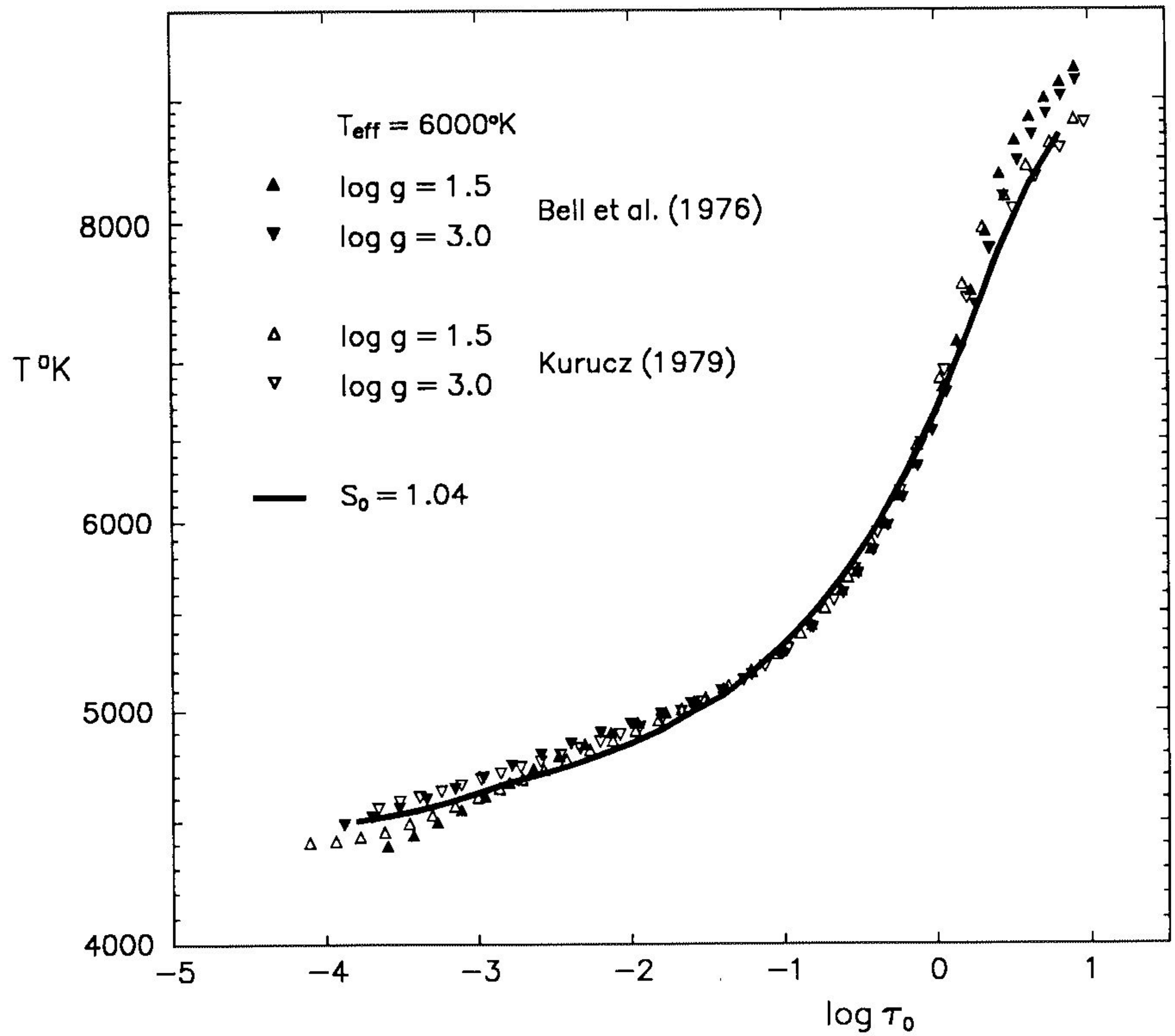


Lines iso-(B2-V1) and iso-m_2 of the Geneva photometric system

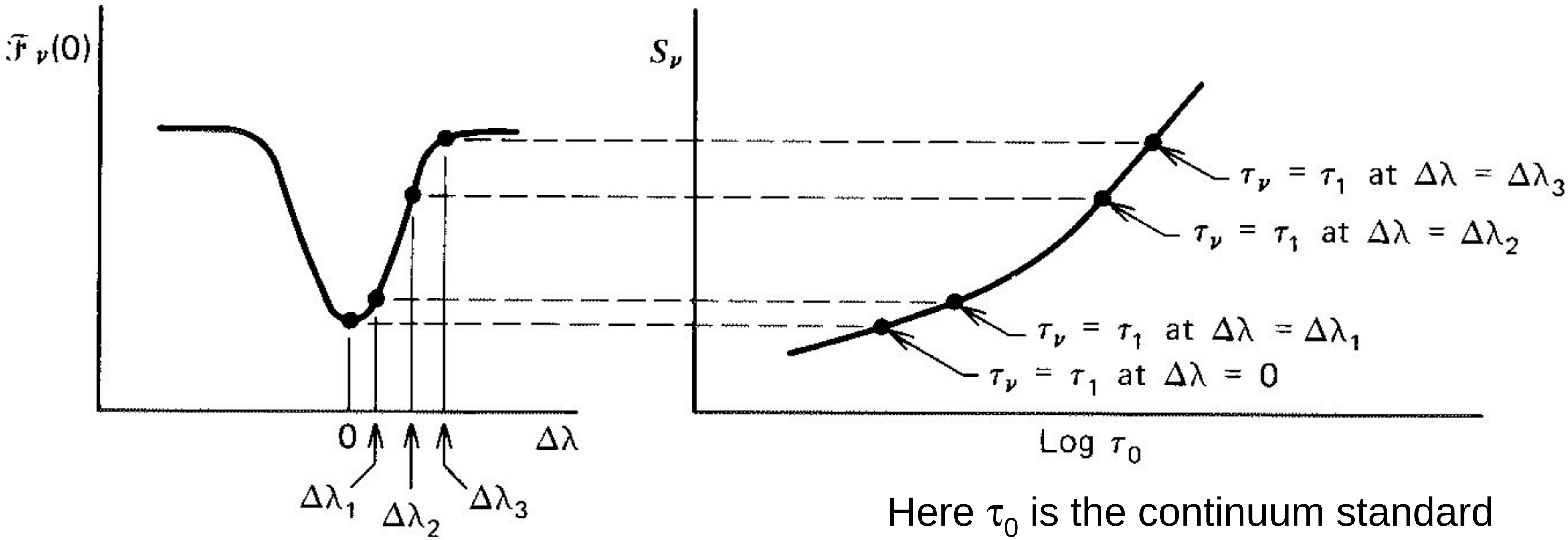
3. Spectral line formation & profile

3.1. Why are « photospheric » lines in absorption?

a) atmospheric structure



b) Negative T gradient → source function ↘ outwards ⇒photospheric lines in absorption (similar argument than for limb darkening)



Here τ_0 is the continuum standard optical depth linked with geometric depth (e.g. τ_{5000})

S_ν^l = line source function:
It reduces to the Planck function for Boltzman's statistics

$$S_\nu^l = \frac{j_\nu}{\kappa_\nu} = \frac{2h\nu^3}{c^2}\frac{1}{\frac{g_2 N_1}{g_1 N_2} - 1}$$

c) Detailed line profile obtained by solution of the Transfer equation

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Where $S_\nu = \dfrac{\dot{j}^l_\nu + \dot{j}^c_\nu}{\kappa^l_\nu + \kappa^c_\nu}$ is the source function

And $d\tau_\nu = (\kappa^l_\nu + \kappa^c_\nu)\rho dx$ the infinitesimal optical depth.

- I_ν: specific intensity (erg s^{-1} cm^{-2} $sterad^{-1}$ Hz^{-1})
- j_ν: emissivity (erg s^{-1} g^{-1} $sterad^{-1}$ Hz^{-1})
- k_ν: absorption coefficient (cm^2 g^{-1})

S_ν can be split into line and continuum source functions:

$$S_\nu = \frac{S_l + (\kappa^c_\nu / \kappa^l_\nu) S_c}{1 + \kappa^c_\nu / \kappa^l_\nu}$$

3.2. Wanted: appropriate ionization state & energy level

- Enough atoms in the right ionization state (LTE assumption): Saha's equation

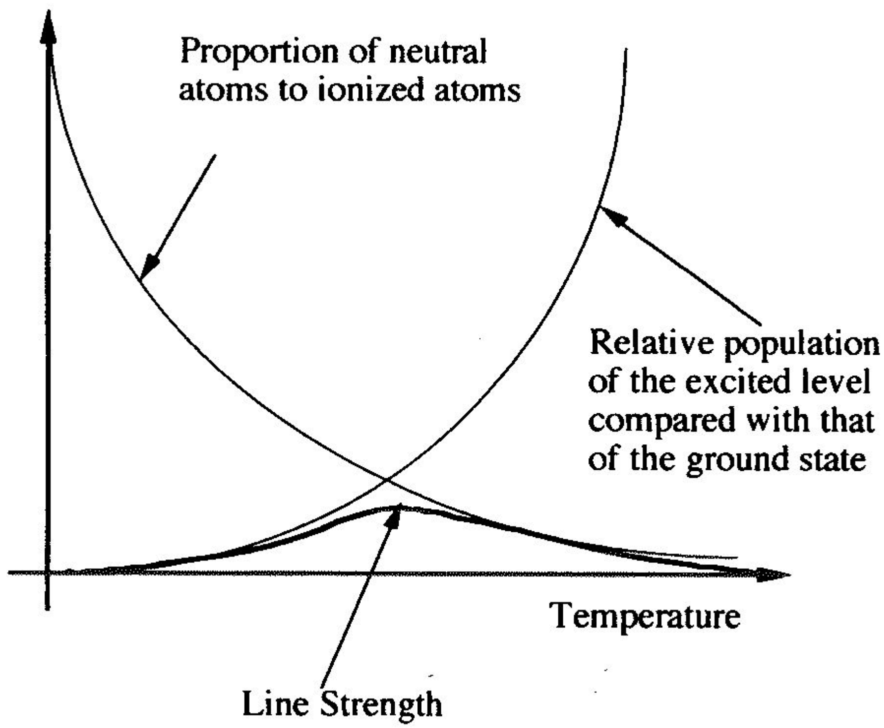
$$\frac{N_{i+1}N_e}{N_i} = 2\frac{U_{i+1}}{U_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-I_i/kT}$$ with $$U_i = \sum_n g_{i,n} \exp\left(-\chi_{i,n}/kT\right)$$ $$\chi_n = E_n - E_1$$

- Enough atoms or ions excited in the right energy level (LTE assumption):

Boltzmann's statistics

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT}$$

Combination of both →



3.3. Line width: microscopic broadening mechanisms

- **Natural width: Lorentz' profile**
  - γ: damping constant
  - f: oscillator strength
  - ν_0: line center
  - N: number density of absorbing atoms/ions
  - Typical FWHM: $\Delta\lambda \approx 5\ 10^{-4}$ A

$$\kappa_\nu \rho = N \frac{\pi e^2}{m_e c} f \frac{\gamma / 4\pi^2}{(\nu - \nu_0)^2 + (\gamma / 4\pi)^2}$$

- **Thermal (or Doppler) width: gaussian profile**
  - Mean abs. coefficient per atom:

$$\alpha_\nu d\nu = \frac{\sqrt{\pi} e^2}{mc} f \frac{1}{\Delta \nu_D} \exp\left[-\left(\frac{\Delta \nu}{\Delta \nu_D} \right)^2 \right] d\nu$$

with $\Delta \nu = \nu - \nu_0$

and $$\Delta \nu_D = \frac{\upsilon_0}{c} \nu_0 = \frac{\nu_0}{c} \left(\frac{2kT}{m} \right)^{\frac{1}{2}}$$

  - In practice, one has to introduce a microturbulent velocity:

$$\Delta \nu_D = \frac{\nu_0}{c} \left(\frac{2kT}{m} + \upsilon_{turb}{}^2 \right)^{\frac{1}{2}}$$

  - Typical FWHM: $\Delta\lambda \approx 0.4$ A

- **Collisional broadening (van der Waals):** damping profile, γ_6 (cool stars)
- **Statistical Stark effect:** damping profile γ_4 (hot stars, esp. H lines)

Total absorption coefficient per atom:

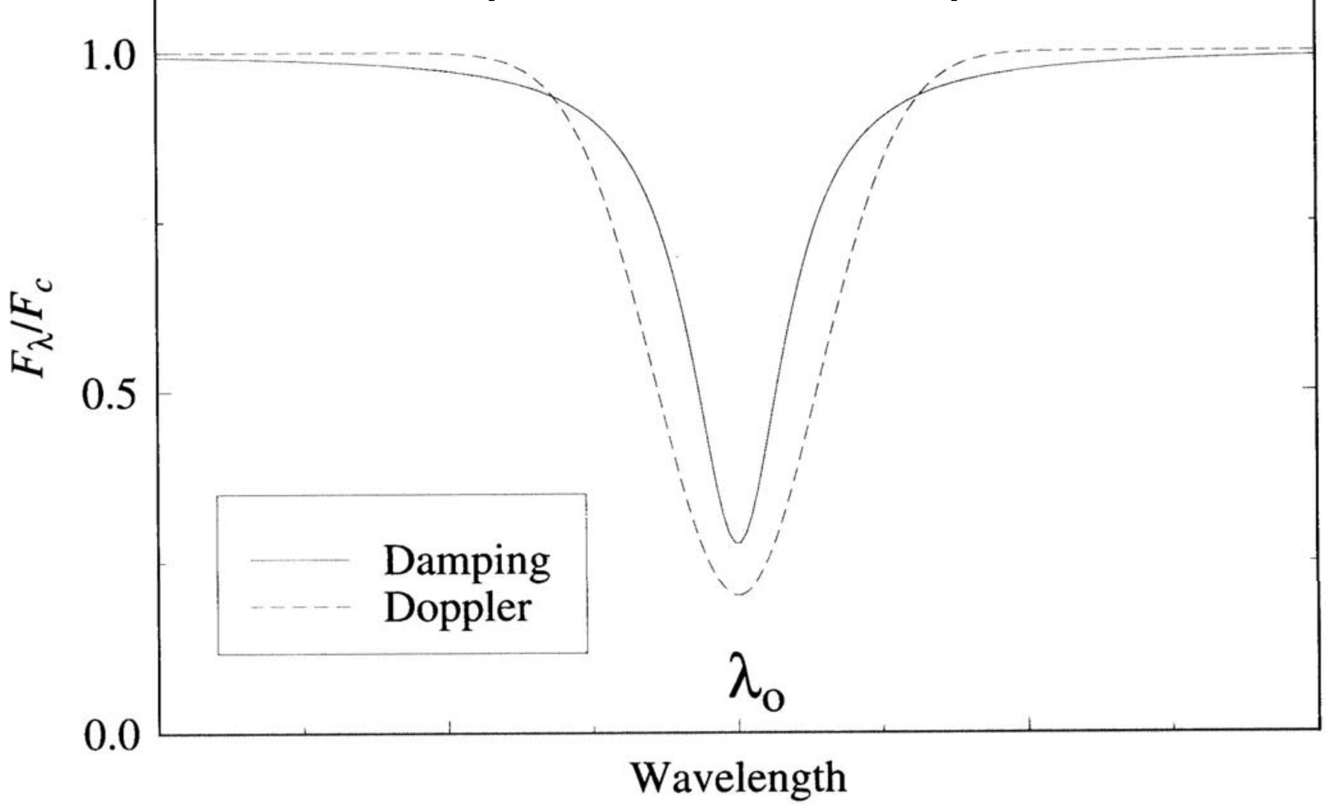
$$\alpha(total) = \alpha(nat.) \otimes \alpha(v.d.Waals) \otimes \alpha(Stark) \otimes \alpha(thermal)$$

where

$$\gamma = \gamma_{nat} + \gamma_6 + \gamma_4$$

Convolution of damping and thermal profiles: Voigt function

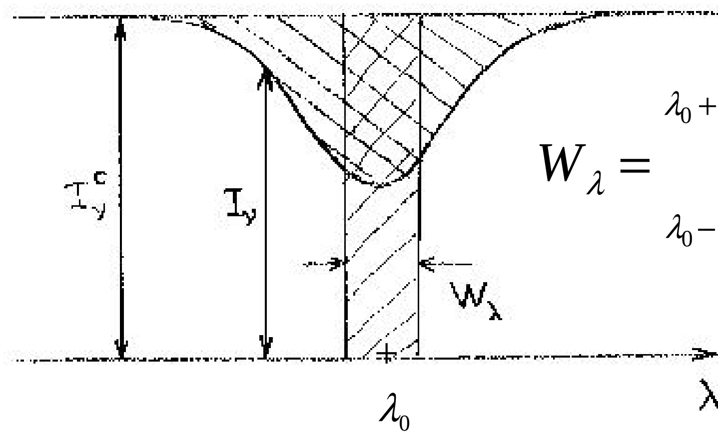
Total absorption coefficient per atom:



4. Curve of growth and abundances

4.1. Equivalent width:

$$W_\lambda = \int_0^\infty \frac{I_c - I_\lambda}{I_c} d\lambda$$



$$W_\lambda = \int_{\lambda_0 - W_\lambda/2}^{\lambda_0 + W_\lambda/2} d\lambda$$

Geometric interpretation: W_λ =width of a rectangular, completely opaque line

For a 2-component atmosphere (continuum source+cool layer of depth h), let the rectified line profile be

$$r(\lambda) = \frac{I_\lambda(0) e^{-[\kappa_l(\lambda) + \kappa_c(\lambda)]h}}{I_\lambda(0) e^{-\kappa_c(\lambda)h}} = e^{-\kappa_l(\lambda)h}$$

then $$W_\lambda = \int_0^\infty [1 - r(\lambda)] d\lambda$$

For faint lines, $$W_\lambda \approx \int_0^\infty \kappa_l(\lambda) h \cdot d\lambda = Nh \int_0^\infty \alpha_\lambda d\lambda = Nhf \frac{\pi e^2}{m_e c} \frac{\lambda^2}{c} \propto Nhf\lambda^2$$

where N is the number density of atoms able to absorb the incident radiation

4.2. Shape of the curve of growth

- Weak lines: linear part $W \propto A$
- Stronger lines: «plateau», saturation of Doppler core

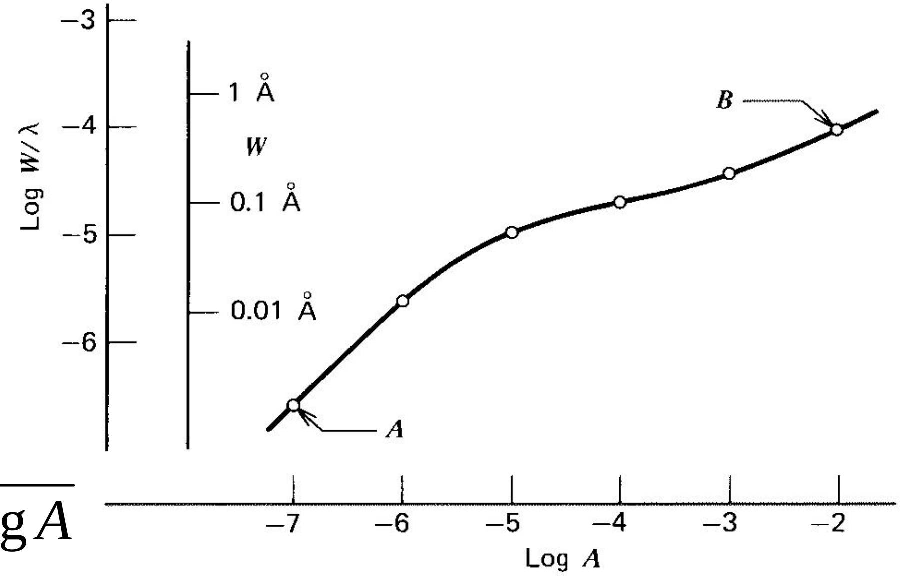
$$W \propto \sqrt{\log A}$$

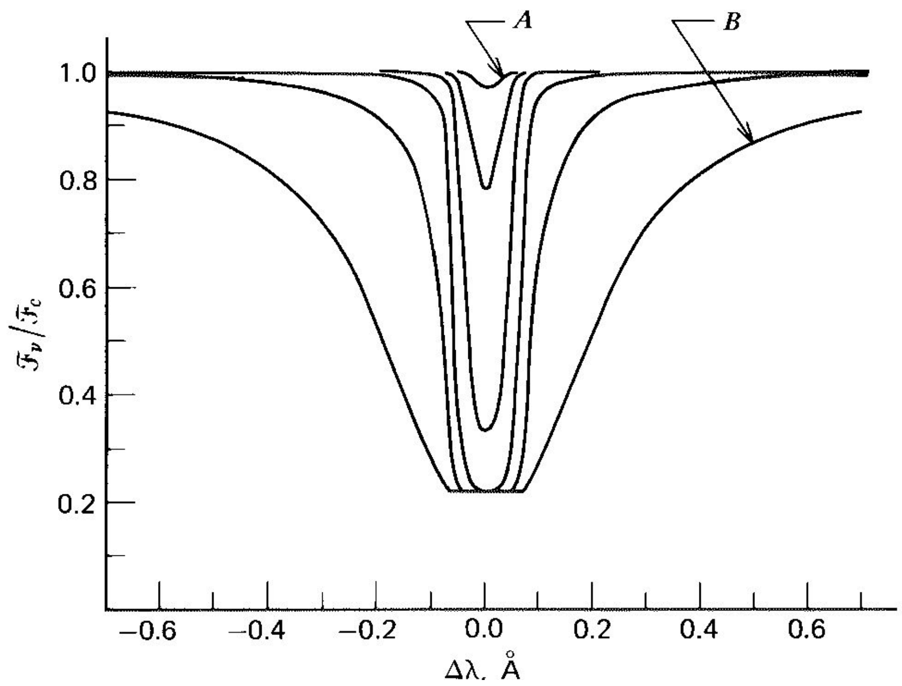
- Very strong lines: damping wings dominate

$$W \propto \sqrt{A}$$

➢ Small lines are best for abundance determination

➢ Average-strong lines are worst!





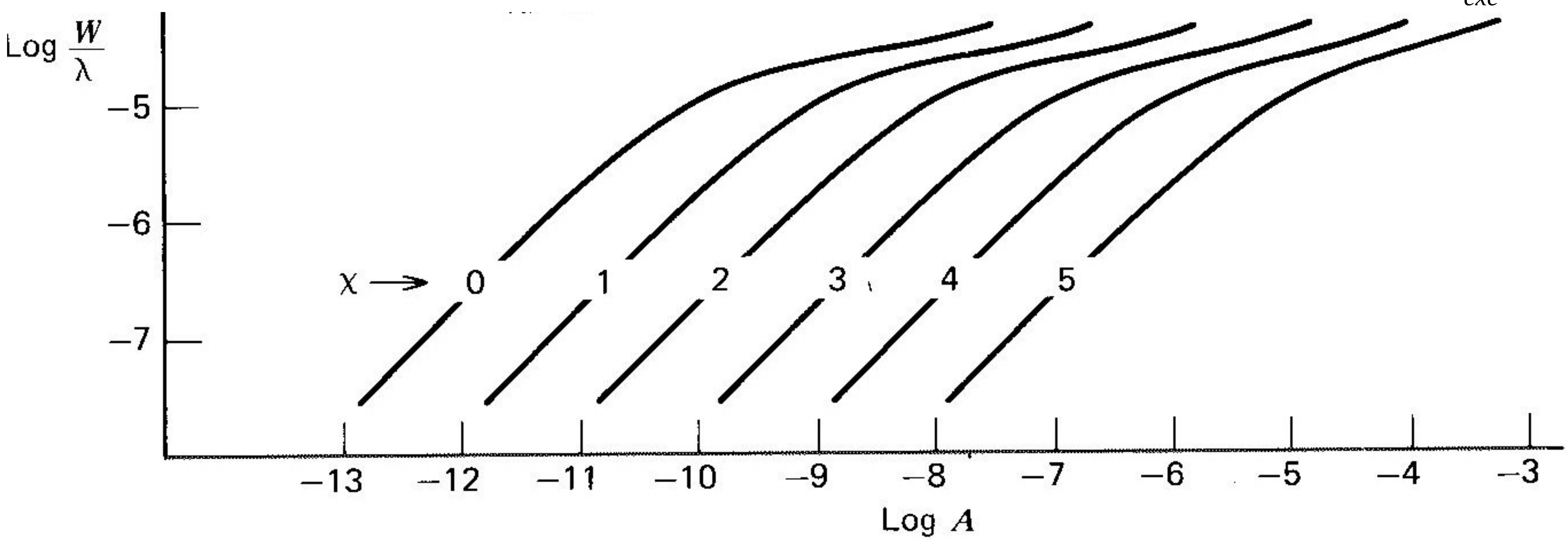
4.3. Towards abundances: the information content of the c.o.g.

$$W_\lambda \propto \frac{\pi e^2}{m_e c^2} \lambda^2 N_{kl} f \qquad \text{(weak lines)}$$

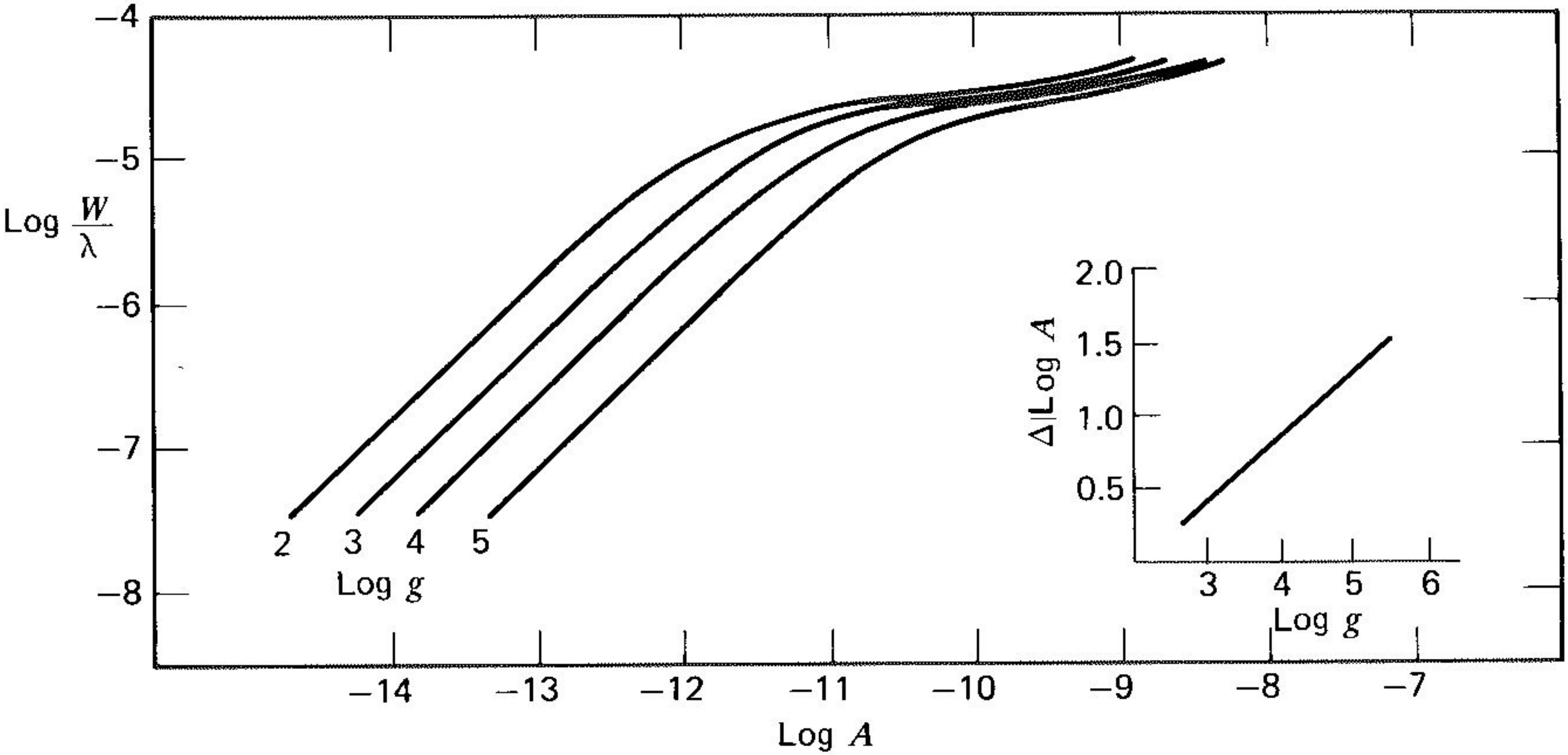
$$N_{kl} = N_k \frac{g_{kl}}{U_k(T)} \exp\left(-\frac{\chi_{kl}}{kT}\right) = A \frac{N_k}{N_E} N_H \frac{g_{kl}}{U_k(T)} \exp\left(-\frac{\chi_{kl}}{kT}\right) \qquad \text{with} \quad A = \frac{N_E}{N_H}$$

$$\log\left(\frac{W}{\lambda}\right) = \log\left(\frac{\pi e^2}{m_e c^2} \frac{N_k / N_E}{U_k(T)} N_H\right) + \log A + \log(g_{kl} f \lambda) - \chi_{kl} \theta_{exc} + const$$

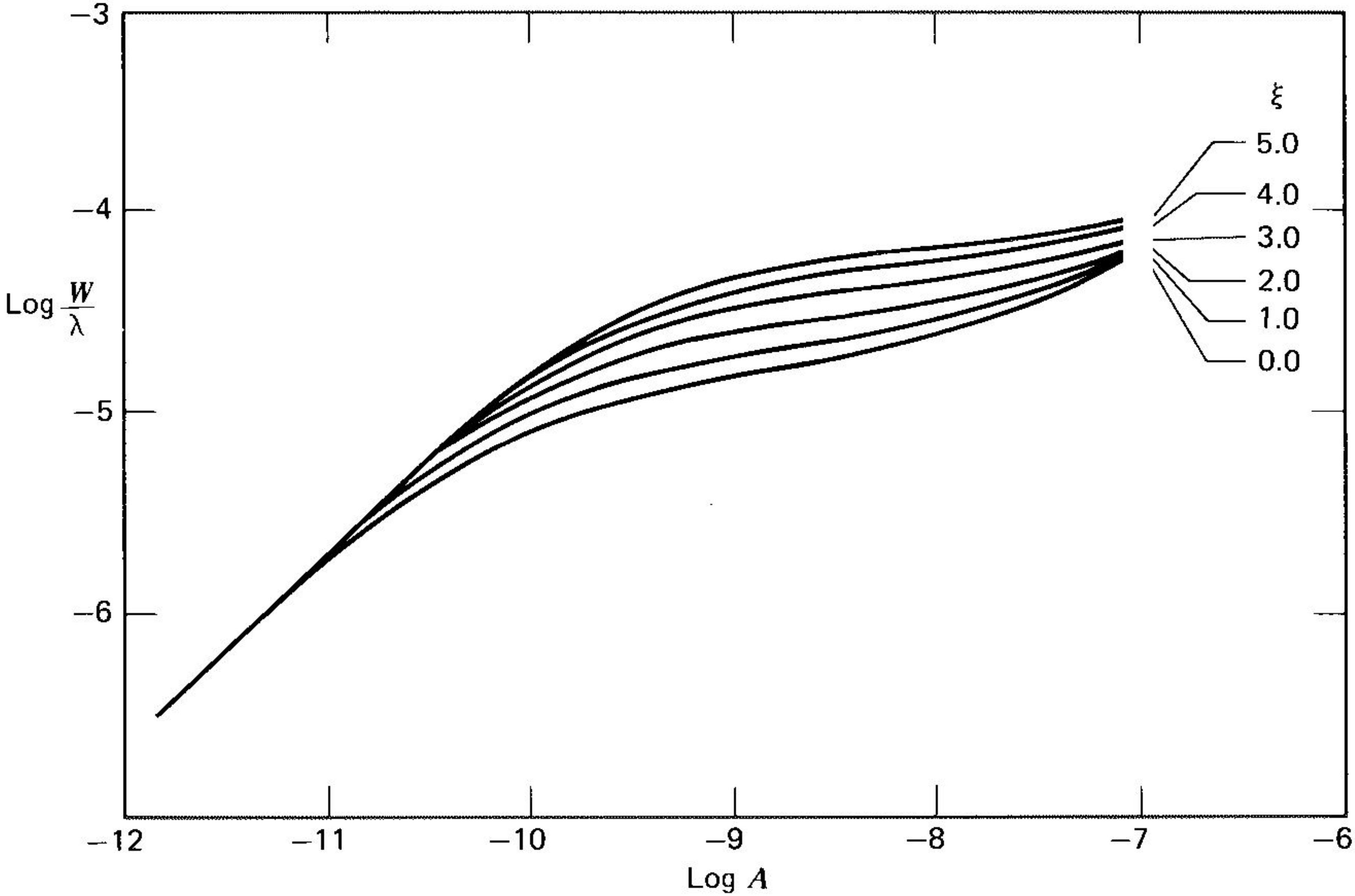
$$\theta_{exc} = \frac{5040}{T_{exc}}$$



Effect of surface gravity on lines of ionized species: due to the 1st term: if ionization N_k/N_E ↗, A ↘ for a given W/λ



Effect of microturbulence: increase of Doppler width desaturates the lines ⇒ strong lines useful to get V_{turb}



Principle of abundance estimate

- Compute a theoretical c.o.g. for a given line of a given atom/ion
- Observe W of this line $\rightarrow$ log A_0

That is what codes do implicitly for each line.

If one doesn't want to compute more than 1 c.o.g.:

- Observed W of other lines

$\downarrow$

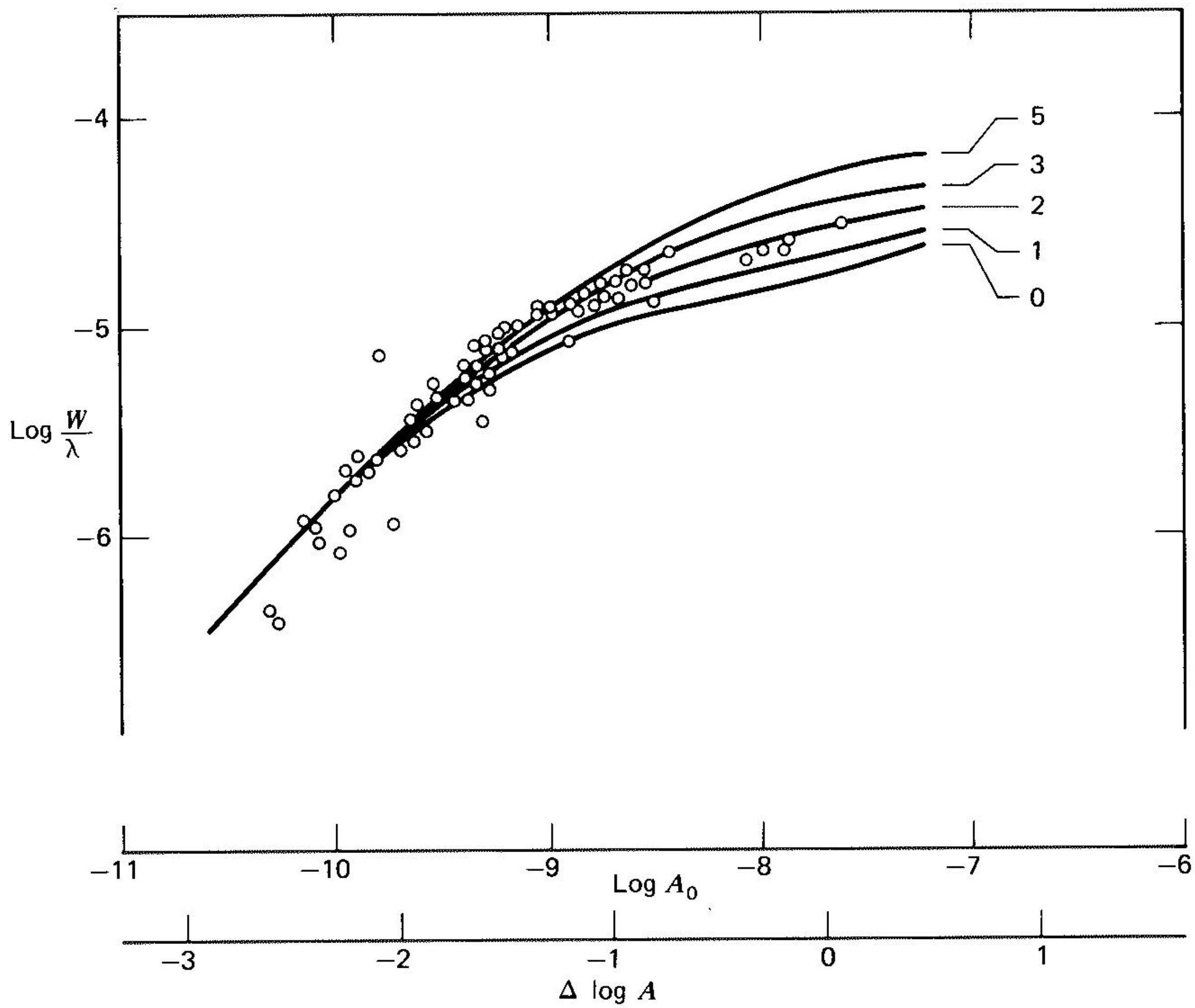
plot log(W/λ) vs Δlog A

$\downarrow$

empirical curve of growth

$\downarrow$

shift that curve onto the theoretical one $\rightarrow$ shift=log A_0



$$\Delta \log A = \log\left(\frac{gf\lambda}{g_0 f_0 \lambda_0}\right) - \theta_{exc}(\chi - \chi_0)$$

5. Stellar abundances in practice: data and tools needed

- Data needed:
  - Model atmospheres (esp. T-τ relation) for various T_{eff}, logg and chemical compositions
    - Kurucz models (ETL, plane parallel): very extended grid,
      http://kurucz.harvard.edu/grids.html
    - MARCS models (ETL, plane parallel): for cool stars (4000 to 8000 K)
      http://marcs.astro.uu.se/
    - TLUSTY models (NLTE, plane parallel): for hot stars (27500 to 55000 K)
      http://nova.astro.umd.edu/Tlusty2002/tlusty-grids.html

  - Line data: wavelengths, excitation potentials, oscillator strengths, broadening parameters

    http://ams.astro.univie.ac.at/vald/
  - Observed spectrum normalized to the continuum, and equivalent widths

- Tools needed:
  - Data reduction software!
    - Iraf, MIDAS and/or instrument-specific pipelines
  - Good procedure for continuum normalization
    - (not so easy!)
  - Code of spectral synthesis, e.g.:
    - Synspec (Hubeny & Lanz) for hot stars
      http://nova.astro.umd.edu/Synspec43/synspec.html
    - Moog (Sneden) for average and cool stars
      http://verdi.as.utexas.edu/moog.html
- Expected precision:
  - Differential abundances (rel. to the sun): ≈ 0.04-0.05 dex
  - Absolute abundances: $>$ 0.1-0.2 dex!

  Problems:

  - ➢ NLTE effects
  - ➢ Uncertain or wrong log(gf) values
  - ➢ 3D hydrodynamic models $\rightarrow Z_{\odot}$=0.012 instead of 0.018!

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