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Gauge Extensions of the Minimal Supersymmetric Standard Model

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The minimal supersymmetric standard model, and extensions, have stringent upper bounds on the mass of the lightest Higgs boson if perturbativity up to the Planck scale is assumed. These bounds are softened tremendously if the Higgs is charged under an asymptotically free gauge group: A model with an additional $SU(2)$ gauge group can easily produce Higgs masses above 200 GeV while avoiding electroweak constraints. If one allows some fine-tuning of the high-scale value of the gauge coupling, Higgs masses greater than 350 GeV are achieved. Unification of couplings is predicted to similar accuracy as in the minimal supersymmetric standard model with only small deviations at the two-loop level.

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1. Introduction

Once LEP-II failed to observe a light Standard Model-like Higgs, the Minimal Supersymmetric Standard Model (MSSM) has been faced with an awkward question: how much fine-tuning is too much fine-tuning? It's an awkward question for a theory whose central pillar of theoretical success, compared to the Standard Model (SM), is the *absence* of fine-tuning. (See [1] for a review of the MSSM.)

The challenge posed by the LEP-II bound is a bit surprising. Though there are approximately 100 free parameters in the MSSM, the Higgs quartic couplings are not tuneable and are fixed to be of order g^2 by the strict requirements of supersymmetry (SUSY). These couplings predict, at tree-level, that the lightest CP-even Higgs state is lighter than the Z-boson, $m_{h^0} < m_Z$. SUSY breaking effects modify this bound, especially those from the stop/top sector due to the large top Yukawa coupling.

Briefly, in the effective field theory below the mass of the two stops, SUSY breaking contributions to the quartic come from two sources: threshold corrections from integrating out the two stops, and logarithmic renormalization group running of the quartic below the scale of the stops. The first depends on the trilinear mixing term, A_t, and the average mass of the stops, $M_{\tilde{t}}$. The second second depends on $\log m_{\tilde{t}}^2/m_t^2$. To increase m_{h^0} as much as possible, one wants $A_t/M_{\tilde{t}} \gtrsim 1$, and $m_{\tilde{t}} \gg m_t$ [2]. Typically, one needs $m_{\tilde{t}} \sim 500 \rightarrow 1000$ GeV to escape the LEP-II bound, depending on the size of A_t. Since this stop mass feeds back into the Higgs soft mass parameter, such large stop masses reintroduce fine-tuning back into the theory at the level of a few percent [3].

In fact, the situation is a bit more subtle. The mediation scale of SUSY breaking also plays a role in the amount of fine-tuning, by controlling the amount of renormalization group running in the Higgs soft mass. A true solution to the SUSY hierarchy problem should have both a low mediation scale and a large value of A_t. Within the MSSM, I am aware of only one proposal that solves the SUSY little hierarchy problem: the recently discovered 'Mirage Mediation', discussed by K.Choi elsewhere in this volume.

Given that it appears so difficult to make the MSSM natural, perhaps it's time to consider supersymmetric alternatives to the MSSM, with the goal of explaining the LEP-II results while maintaining the overall naturalness of the theory. There are two distinct means of attack. One can extend the MSSM via new fields whose sole purpose is to increase the tree-level value of the Higgs quartic, and lift the physical Higgs mass above the LEP-II bound: The physical Higgs escaped detection because LEP-II couldn't probe high enough energies. Alternatively, the physical Higgs can be disguised through non-standard couplings, e.g. from a large singlet-admixture which doesn't couple to the SM: the physical Higgs escaped detection because LEP-II was unable to probe the proper decay channels with sufficient sensitivity. Here, I will focus on models of the first type, which increase the value of the physical Higgs mass above the LEP-II bound.

In fact, even among models which lift the physical Higgs mass through new fields, there are two mechanisms: D-terms or F-terms. Recently, models incorporating D-term contributions have been used [4, 5], which can reach physical Higgs mass as high as ~ 350 GeV. A good review of F-term models is in [6], and many more recent models exists in the literature, including the unnatural version presented by K.S. Babu elsewhere in these proceedings. In both scenarios, large physical Higgs masses can only be achieved with large low energy couplings, which require some form of asymptotic freedom in the UV to maintain perturbative control. D-terms present a simple method of UV completing these large low-energy couplings while maintaining naturalness.

The rest of this contribution is an updated review of [4]: Section 2 describes the mechanism of producing the large-low energy quartic and Section 3 has updated constraints on the ultimate size of the physical Higgs mass.

2. Non-Decoupling D-terms

In the MSSM, the only Higgs quartic couplings come from the D-terms of $SU(2)_W \times U(1)_Y$ gauge groups:

$$\frac{g_W^2}{8}\left(H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d + \dots\right)^2 + \frac{g_Y^2}{8}\left(|H_u|^2 - |H_d|^2 + \dots\right)^2, \tag{2.1}$$

where ... indicate the remainder of the MSSM scalars. The physical Higgs mass is so small simply because these couplings are proportional to $(g_W^2 + g_Y^2)/8$.

To enhance the quartic coupling, deconstruct the $SU(2)_W$ group into two separate $SU(2)_W$ groups, $SU(2)_1 \times SU(2)_2$, which are broken to the diagonal subgroup $SU(2)_W$ at the TeV scale by a bi-fundamental scalar VEV, $<\mathbf{\Sigma}>= \mathbf{1}u$. The low energy gauge coupling for the unbroken $SU(2)_W$ subgroup is

$$\frac{1}{g_W^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}. \tag{2.2}$$

Any doublet under $SU(2)_W$ must be embedded as a $(1/2, 0)$ or $(0, 1/2)$ under $SU(2)_1 \times SU(2)_2$.

What's most interesting is that the low energy coefficient of the $SU(2)_W$ D-term is not necessarily given by g_W^2. In particular, if the field Σ has a SUSY breaking soft-mass, m_Σ^2 which is of order its VEV, u, then 'memory' of both quartics will filter down to the low-energy theory through a modification of the D-term coefficient. Of course, as $m_\Sigma^2 \to 0$ this memory is erased entirely and out pops the low energy MSSM, with the standard ineffectual MSSM quartic. The opposite limit, which looks like a hard-breaking of SUSY, leads to a substantial gain in the physical Higgs mass bound.

Specifically, let's charge the Higgs fields under $SU(2)_1$. Above the scale of diagonal symmetry breaking, the $SU(2)_1 \times SU(2)_2$ D-term is

$$\frac{g_1^2}{8}\left(\mathrm{Tr}\left[\Sigma^\dagger \sigma^a \Sigma\right] + H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d\right)^2 + \frac{g_2^2}{8}\left(\mathrm{Tr}\left[\Sigma \sigma^a \Sigma^\dagger\right]\right)^2. \tag{2.3}$$

I'll specify where to add the remaining MSSM fields in a bit. Add the superpotential $\mathscr{W} = \lambda S\left(\frac{1}{2}\Sigma\Sigma + w^2\right)$ with an additional soft-mass m_Σ^2 for Σ, leading to the scalar potential

$$V_\Sigma = \frac{1}{2}B\Sigma\Sigma + h.c. + m_\Sigma^2|\Sigma|^2 + \frac{\lambda^2}{4}|\Sigma\Sigma|^2. \tag{2.4}$$

Here, $\Sigma\Sigma$ is contracted with two epsilon tensors and $B = \lambda w^2$. For sufficiently large B, Σ acquires a VEV, $\langle\Sigma\rangle = u\mathbf{1}$, with $u^2 = (B - m_\Sigma^2)/\lambda^2$, which breaks $SU(2)_1 \times SU(2)_2$ to the diagonal subgroup. The minimum lies in a D-flat direction, leaving both Higgs fields massless at tree-level.

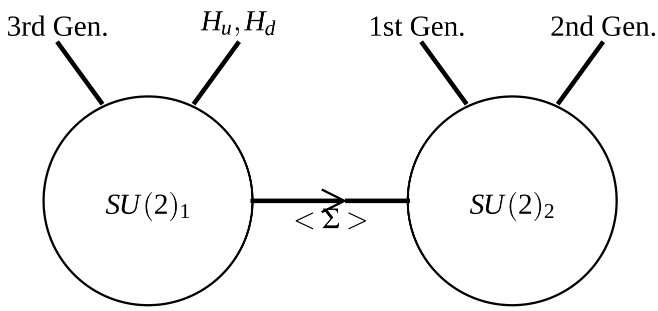

Figure 1: Modified $SU(2)^2$ gauge setup. The Higgs doublets and third generation superfields are charged under $SU(2)_1$. The first two generation superfields are charged under $SU(2)_2$. A bi-fundamental, Σ, breaks $SU(2)^2$ down to $SU(2)_W$.

Under the remaining $SU(2)_W$, Σ contains a complex triplet, T, along with a complex singlet. Integrating out the real part of the heavy triplet at tree-level gives the effective Higgs potential below the triplet mass,

$$\frac{g_W^2}{8}\Delta\left(H_u^\dagger\vec{\sigma}H_u + H_d^\dagger\vec{\sigma}H_d\right)^2 + \frac{g_Y^2}{8}\left(|H_u|^2 - |H_d|^2\right)$$
$$\text{with } \Delta = \frac{1+\frac{2m^2}{u^2}\frac{1}{g_2^2}}{1+\frac{2m^2}{u^2}\frac{1}{g_1^2+g_2^2}}. \tag{2.5}$$

The MSSM $SU(2)_W$ D-term is recovered in the limit $u^2 \gg m_\Sigma^2$, because SUSY protects the D-term below the gauge-breaking scale.

The tree-level Higgs mass now is enhanced by the non-decoupling D-term, and satisfies

$$m_{h^o}^2 < \frac{1}{2}\left(g_W^2\Delta + g_Y^2\right)v^2\cos^2 2\beta. \tag{2.6}$$

To maximize the upper bound, Δ should be made as large as possible by sending $g_1 \to \infty$, $g_2 \to g$ and $m_\Sigma^2 \gg u^2$ by as much as possible without introducing fine-tuning.

3. Maximizing the Physical Higgs Mass

Naturalness, perturbativity, and electroweak precision constraints prevent us from pushing the physical Higgs mass to arbitrary large values. Naturalness puts an upper bound on the scale of the heavy vectors, who cutoff the hard-breaking effects from the modified low-energy D-term. Naturalness also puts an upper-bound on the mass of m_Σ^2, which feeds in at two loops into the Higgs soft mass. Choosing $M_V \sim$ TeV and $m_\Sigma \sim 10$ TeV generate fine-tuning no worse than 10%. There is no fine-tuning from the rest of the MSSM soft-sector since the remaining soft-masses are set to the weak-scale.

To push g_1 as large as possible, place as little matter as possible in g_1, so that g_1 runs asymptotically free in the UV. The moose which describes this deconstruction is shown in Figure 1. Yukawa couplings for the first two generations can be generated by adding a massive Higgs-like pair of doublets $\overline{H}', H'$, that are charged under $SU(2)_2$. They couple to the first two generations via Yukawa-type couplings and mix with the regular Higgses via superpotential operators such as

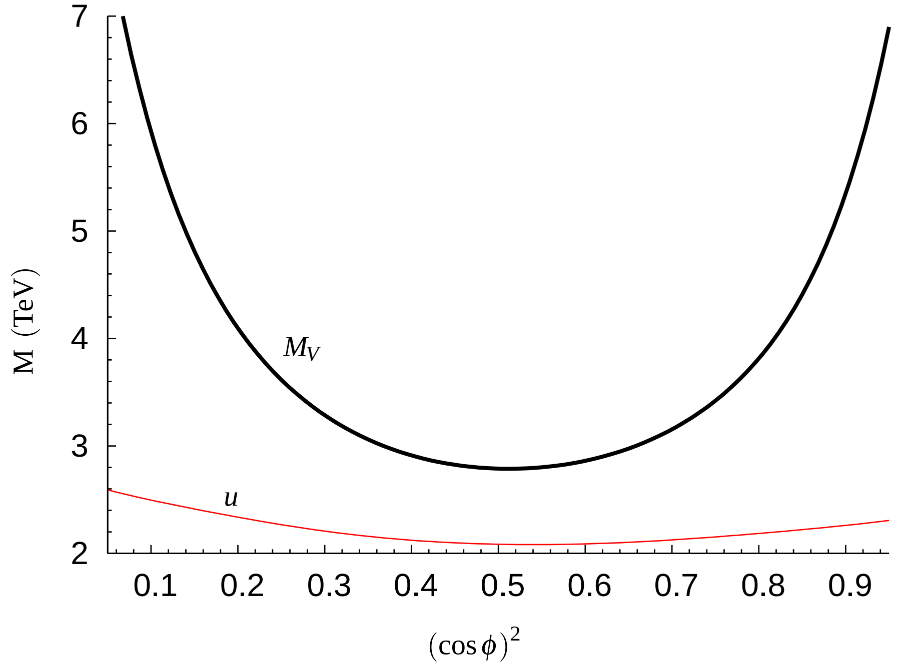

Figure 2: 95% CL bounds on the mass of the heavy vector bosons M_V and the diagonal breaking scale u as a function of $\cos\phi$ (with $g_1 = g_w/\sin\phi$ and $g_2 = g_w/\cos\phi$). Large values of $\cos\phi$ corresponds to large values of g_1, which is where corrections to the light generation couplings are negligible, and corrections to the third generation couplings can be substantial.

$\lambda'\overline{H}\Sigma H'$. A supersymmetric mass $\mu_{H'} > \langle\Sigma\rangle$ for the new doublets generates naturally small Yukawa couplings for the first two generations at low energies.

The deconstructed $SU(2)$ setup shifts the tree-level W and Z mass due to heavy Z', W' mixing, but this only occurs at order v^4/u^4. However, due to the non-universal flavor setup, there are important corrections to the other electroweak precision measurements. These were first analyzed in the context of extended technicolor models [7]. There are tree-level corrections to G_F and non-oblique corrections to fermion couplings. Constraints on the model are computed from the corrections to the leading dimension six operators, as described in [8]. The constraints are most severe when one of the gauge couplings start to become non-perturbative, and the 95% confidence level bounds are plotted in Figure 2.

The model also contributes a shift in the ρ parameter from a neutral triplet VEV, as well as shifts to the oblique parameters from the physical Higgs itself. Model dependent shifts to the oblique parameters come from the remainder of the two-Higgs doublet sector, and also from the mass splitting between scalar superpartners. The large effect of Δ enhances splitting between scalar members of any $SU(2)_W$ doublet at the same time that it raises the Higgs mass. These effects are not included in the above constraints.

Using the minimum value for u shown in Figure 2, there is a new physical Higgs mass bound as a function of g_1, as shown in Figure 3. As can be seen, the physical Higgs mass can be pushed to as large as 350 GeV or larger if the non-perturbative scale of g_1 is tuned to be close to a few TeV.

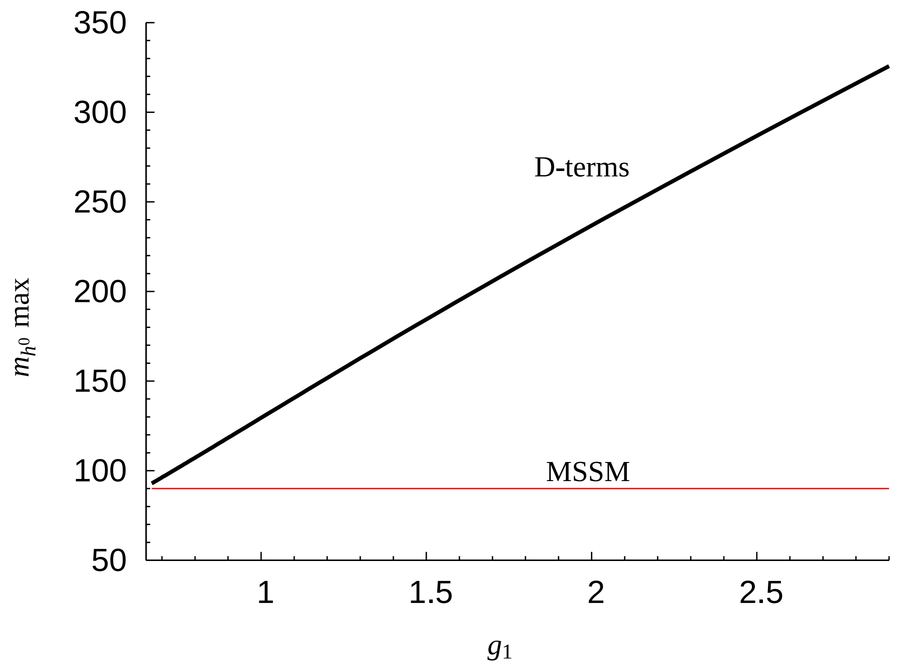

Figure 3: Tree-level bounds on the lightest CP-even state, h^0, in the MSSM andin the $SU(2)$-extended model.

Here are a typical set of parameters:

- $g_1(u) = 1.80$, $g_2(u) = .70$, inspired by a GUT with $g_1(\Lambda_{GUT}) = .97$. Additional spectator fields (see the full description at the end of the section for details) are included in the running to aid in unification.
- $u = 2.2$ TeV, above the lower limit from electroweak constraints, giving $M_{W'}, M_{Z'} \sim 4.5$ TeV.
- $m = 10$ TeV. One-loop finite corrections to the Higgs mass parameter from supersymmetry breaking are < 300 GeV whereas two-loop RGE contributions can be somewhat larger if one assumes high-scale supersymmetry breaking.

For this reasonable set of parameters, $m_h \sim 220$ GeV at tree-level in the large $\tan\beta$ and decoupling limits. Loop corrections to the effective potential from the top sector and the additional physics will make a relatively small shift in the tree level result.

One interesting feature of this model is that because there is a gauge coupling larger than that of $SU(3)$ color, the top Yukawa "fixed point" has a much larger value than in the MSSM. In this sense, a favorable region of parameter space includes some of $\tan\beta < 1$ which can both be consistent with the Higgs mass bound and avoid a Landau pole for the top Yukawa.

This model can also be made consistent with gauge coupling unification. The full group $SU(3)_c \times SU(2)_1 \times SU(2)_2 \times U(1)_Y$ can be embedded in $SU(5) \times SU(5)$ [9] broken by a bi-fundamental field at the GUT scale with a VEV $\langle \Xi \rangle = diag\{M, M, M, 0, 0\}$. Gauge coupling unification is predicted (with theoretical uncertainty beyond one-loop) because the standard model gauge couplings are only a function of the diagonal gauge coupling. At one loop, one can track the diagonal $SU(2)$ through its beta-function coefficient b as it is the sum of those of the two $SU(2)_i$. It receives an extra -6 from the additional triplet of gauge bosons. There are also two triplets charged under $SU(2)_2$ which, with the diagonal-breaking Σ field, contribute +6 to the diagonal beta function, and an additional vector-like pair of triplets to effectively complete a 5 and $\overline{5}$ with the extra pair of Higgs-like fields (however, they should be from a split multiplet as they must not share the Yukawa couplings with the doublets due to proton decay). With these additions, the $SU(2)$ model achieves the same unification accuracy as in the MSSM at one loop. Though there is a gauge coupling that gets relatively strong, its two-loop effect is still small as g_1 is quite perturbative for nearly all of the running.

4. Conclusion

Whatever one's take on the little hierarchy problem in the MSSM, it seems important to understand the physical Higgs mass bound in SUSY theories excruciatingly well before the LHC turns on. Though many supersymmetric extensions are possible, it still seems very difficult for models which have weak-scale superpartners to exceed a physical Higgs mass of $\sim$ 300GeV.

Since this work was first completed, many models using F-terms and D-terms have appeared in the literature, in an attempt to evade the LEP-II bound. The model based on D-terms presented here has a rich phenomenology of new states, heavy vectors and scalar triplets, that appear near a TeV. Additional uses of D-terms have also been found: As a means of enhancing F-term contributions [10] and for producing viable baryogenesis [11]. Further work waits, particular in exploring the full space of beyond-the-MSSM phenomenology at the LHC.

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