PROCEEDINGS OF SCIENCE

The Phases of Non-Compact QED_3

Costas Strouthos*
Institute of Chemical Sciences and Engineering, École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland
and
Department of Computer Science, European University Cyprus, 6 Diogenes Str. Engomi, 1516 Nicosia, Cyprus
E-mail: `cstrouth@deas.harvard.edu`

John B. Kogut
Department of Energy, Division of High Energy Physics, Washington, DC 20575 USA
and
Dept. of Physics - TQHN, Univ. of Maryland, 82 Regents Dr., College Park, MD 20742, USA
E-mail: `jbkogut@umd.edu`

Non-compact three-dimensional QED is studied by computer simulations to understand its chiral symmetry breaking features for different values of the number of fermion flavors N_f. We consider the four-component formulation for the fermion fields, which arises naturally as the continuum limit of the staggered fermion construction in $(2+1)$ dimensions. We present preliminary results for the equation of state of the theory in an effort to understand the properties of the chiral phase transition of the theory at a critical number of fermion flavors N_{fc}. Our preliminary results indicate that $N_{fc} \approx 1.5$.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

*Speaker.

PoS(LATTICE 2007)278

1. Introduction

The study of QFTs in which the ground state shows a sensitivity to the number of fermion flavors N_f is intrinsically interesting. According to approximate solutions of continuum Schwinger-Dyson equations (SDEs), QED$_3$ displays this phenomenon. It is believed to be confining and exhibit features such as dynamical mass generation when the number of fermion flavors N_f is smaller than a critical value N_{fc} [1, 2, 3, 4, 5, 6, 7]. Apparently, for $N_f > N_{fc}$, the attactive interaction between a fermion and an antifermion due to photon exchange is overwhelmed by the fermion screening of the theory's electric charge. Furthermore, over the past few years QED$_3$ has attracted a lot of attention, because of potential applications to models of high T_c superconductivity [8, 9, 6, 10, 11] It is also an interesting and challenging model field theory which is being seen as an ideal laboratory to study more complicated gauge field theories.

Initial studies based on SDEs using the photon propagator derived from the leading order $1/N_f$ expansion, where N_f is the number of fermion flavors, suggested that for N_f less than $N_{fc} \simeq 3.2$ chiral symmetry is broken [1]. The model in the limit $N_f \to N_{fc}$ is supposed to undergo an infinite-order phase transition [2]. Other studies taking non-trivial vertex corrections into account predicted chiral symmetry breaking for arbitrary N_f [3]. Studies which treat the vertex consistently in both numerator and denominator of the SDEs have found $N_{fc} < \infty$, with a value either in agreement with the original study [4], or slightly higher $N_{fc} \simeq 4.3$ [5]. An argument based on a thermodynamic inequality predicted $N_{fc} \leq \frac{3}{2}$ [12], a result that was later on challenged in [11]. A gauge invariant determination of N_{fc} based on the divergence of the chiral susceptibility gives $N_{fc} \approx 2.16$ [6]. Recent progress in the direction of gauge covariant solutions for the propagators of QED$_3$ showed that in the Landau gauge a chiral phase transition exists at $N_{fc} \approx 4$ [7]. Furthermore, a perturbative analysis of RG flow equations in the large-N_f limit predicts $N_{fc} \approx 6$ [13].

There have also been numerical attempts to resolve the issue via lattice simulations of QED$_3$. Once again, opinions have divided on whether N_{fc} is finite [14, 15], or whether chiral symmetry is broken for all N_f [16]. A numerical study of the quenched ($N_f = 0$) case has shown that chiral symmetry is broken [17]. More recent numerical results showed that chiral symmetry is also broken for $N_f = 1$, whereas $N_f = 2$ appeared chirally symmetric with an upper bound of 10^{-4} on the dimensionless condensate [18]. The principal obstruction to a definitive answer has been large finite volume effects resulting from the presence of a massless photon in the spectrum, which prevent a reliable extrapolation to the thermodynamic limit. Recent lattice simulations of the three-dimensional Thirring model, which may have the same universal properties as QED$_3$, predicted $N_{fc} = 6.6(1)$ [19]. In this paper we present a study of the QED$_3$ Equation of State based on preliminary results extracted from lattice simulations on large lattices (up to 80^3).

2. The Model

We are considering the four-component formulation of QED$_3$ where the Dirac algebra is represented by the 4×4 matrices γ_0, γ_1 and γ_2. This formulation preserves parity and gives each spinor a global $U(2)$ symmetry generated by $\mathbf{1}$, γ_3, γ_5 and $i\gamma_3\gamma_5$; the full symmetry is then $U(2N_f)$. If the fermions acquire dynamical mass the $U(2N_f)$ symmetry is broken spontaneously to $U(N_f) \times U(N_f)$ and $2N_f^2$ Goldstone bosons appear in the particle spectrum.

The action of the lattice model we study is

$$
\begin{aligned}
S &= \frac{\beta}{2}\sum_{x,\mu<\nu}\Theta_{\mu\nu}(x)\Theta_{\mu\nu}(x)+\sum_{i=1}^{N}\sum_{x,x'}\bar{\chi}_i(x)M(x,x')\chi_i(x') \qquad (2.1)\\
\Theta_{\mu\nu}(x) &\equiv \theta_{x\mu}+\theta_{x+\hat{\mu},\nu}-\theta_{x+\hat{\nu},\mu}-\theta_{x\nu}\\
M(x,x') &\equiv m\delta_{x,x'}+\frac{1}{2}\sum_{\mu}\eta_\mu(x)[\delta_{x',x+\hat{\mu}}U_{x\mu}-\delta_{x',x-\hat{\mu}}U^{\dagger}_{x-\hat{\mu},\mu}].
\end{aligned}
$$

This describes interactions between N flavors of Grassmann-valued staggered fermion fields $\chi,\ \bar{\chi}$ defined on the sites x of a three-dimensional cubic lattice, and real photon fields $\theta_{x\mu}$ defined on the link between nearest neighbour sites x, $x+\hat{\mu}$. Since Θ^2 is unbounded from above, eq.(2.1) defines a non-compact formulation of QED; note however that to ensure local gauge invariance the fermion-photon interaction is encoded via the compact connection $U_{x\mu}\equiv\exp(i\theta_{x\mu})$, with $U_{x+\hat{\mu},-\mu}=U^*_{x\mu}$. In the fermion kinetic matrix M the Kawamoto-Smit phases

$$\eta_\mu(x)=(-1)^{x_1+\cdots+x_{\mu-1}} \qquad (2.2)$$

are designed to ensure relativistic covariance in the continuum limit, and m is the bare fermion mass.

If the physical lattice spacing is denoted a, then in the continuum limit $a\partial\to 0$, eq.(2.1) can be shown to be equivalent up to terms of $O(a^2)$ to

$$S=\sum_{j=1}^{N_f}\bar{\psi}^j[\gamma_\mu(\partial_\mu+igA_\mu)+m]\psi^j+\frac{1}{4}F_{\mu\nu}F_{\mu\nu} \qquad (2.3)$$

ie. to continuum QED in 2+1 euclidean dimensions, with $\psi,\bar{\psi}$ describing N_f flavors of four-component Dirac spinor acted on by 4×4 matrices γ_μ, and $N_f\equiv 2N$. The continuum photon field is related to the lattice field via $\theta_{x\mu}=agA_\mu(x)$, with dimensional coupling strength g given by $g^2=(a\beta)^{-1}$, and the field strength $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\mu A_\nu$. The continuum limit is thus taken when the dimensionless inverse coupling $\beta\to\infty$.

As reviewed in [18], for $a>0$ in the chiral limit the lattice action (2.1) retains only a remnant of the $\mathrm{U}(2N_f)$ global symmetry of (2.3) under global chiral/flavor rotations, namely a $\mathrm{U}(N)\otimes\mathrm{U}(N)$ symmetry which is broken to $\mathrm{U}(N)$ either explicitly by the bare mass $m\neq 0$, or spontaneously by a chiral condensate $\langle\bar{\chi}\chi\rangle\neq 0$, in which case the spectrum contains N^2 exact Goldstone modes. It is expected that the symmetry breaking pattern $\mathrm{U}(2N_f)\to\mathrm{U}(N_f)\otimes\mathrm{U}(N_f)$ is restored in the continuum limit, implying the existence of an additional $7N^2$ approximate Goldstone modes whose masses vanish as $\beta\to\infty$.

3. Numerical Simulations

In this section we present results from numerical simulations performed using the standard Hybrid Molecular Dynamics (HMD) R-algorithm. In order to ensure that the $O(N^2dt^2)$ systematic errors (dt is the time step of the HMD trajectory) are negligible, we performed two sets of simulations one with $dt=0.010$ and another with $dt=0.005$ on 80^3 lattices with $m=0.005$ and

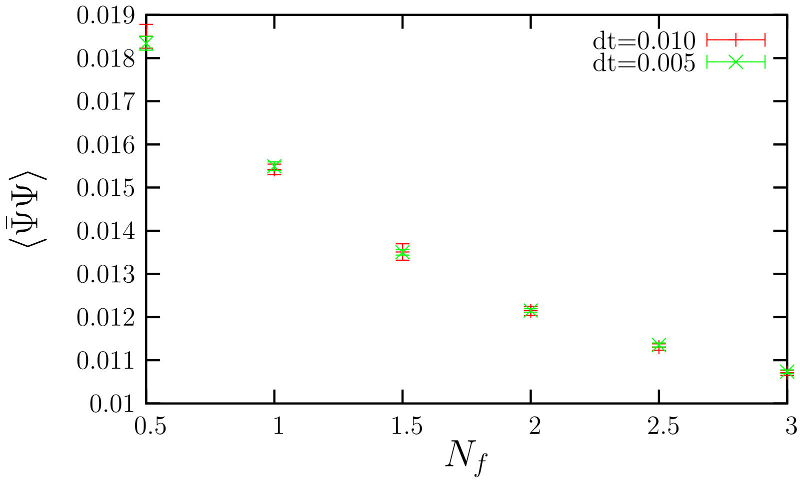

Figure 1: Chiral condensate versus N_f from simulations with $dt = 0.010$ and $dt = 0.005$; $\beta = 0.90$, $m = 0.005$, and $L = 80$.

$\beta = 0.90$. As shown in Fig. 1 the values of the chiral condensate from the two sets of simulations with $N_f = 0.5,...,3.0$ agree within statistical error, implying that the systematic effects are negligible.

Next we present results from simulations with $N_f = 1.5$ and $N_f = 2.0$ (Figs. 2 and 3). More specifically, we study the behavior of the dimensionless chiral condensate $\beta^2\langle\bar{\psi}\psi\rangle$ versus the dimensionless fermion bare mass βm on different lattice sizes. In the $N_f = 1.5$ case the finite size discrepancy between the $L = 54$ and $L = 80$ results is $\sim 7\%$, implying that the finite size effects for $L = 80$ are small. In the $N_f = 2.0$ case the discrepancy is $\sim 1\%$, implying that in this case the values of the condensate extracted from the 80^3 simulations should be even closer to the thermodynamic limit values.

For $N_f = 2$, we fitted the 54^3 data to

$$\beta^2\langle\bar{\psi}\psi\rangle = a_0 + a_1\cdot(\beta m) + a_2\cdot(\beta m)^2. \tag{3.1}$$

The extracted value of $a_0 = -8\times10^{-8}$ with a statistical error 5×10^{-7} and fit quality $\chi^2/DOF = 0.9$ is consistent with zero with relatively high accuracy. For $N_f = 1.5$, we fitted the 80^3 data to (3.1) and got $a_0 = -2\times10^{-6}$ with a statistical error 10^{-6} and fit quality $\chi^2/DOF = 4.1$. This result is also very close to zero although the fit quality is lower than in the $N_f = 2.0$ case. As we will se below the low fit quality could be attributed to the fact that $N_f = 1.5$ may be close to the chiral phase transition.

We also checked the effects of lattice discretization on the values of the chiral condensate by comparing data extracted from simulations with $\beta = 0.90$ and $\beta = 1.20$ for $N_f = 0.5,...,2.0$. We achieved this by fixing for the two sets of simulations the physical volume $(L/\beta)^3$ and the physical mass βm. The results presented in Fig.4 show that the lattice discretization effects at $\beta = 0.90$ are small for $N_f > 0.5$, whereas for $N_f = 0.5$ there is an $\sim 8\%$ discrepancy between the values of the condensate at $\beta = 0.90$ and $\beta = 1.20$.

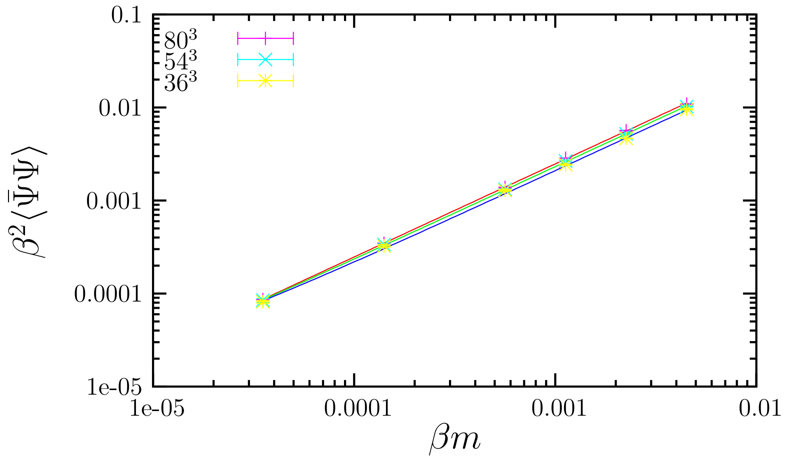

Figure 2: Dimensionless condensate versus dimensionless bare mass for $N_f = 1.5$.

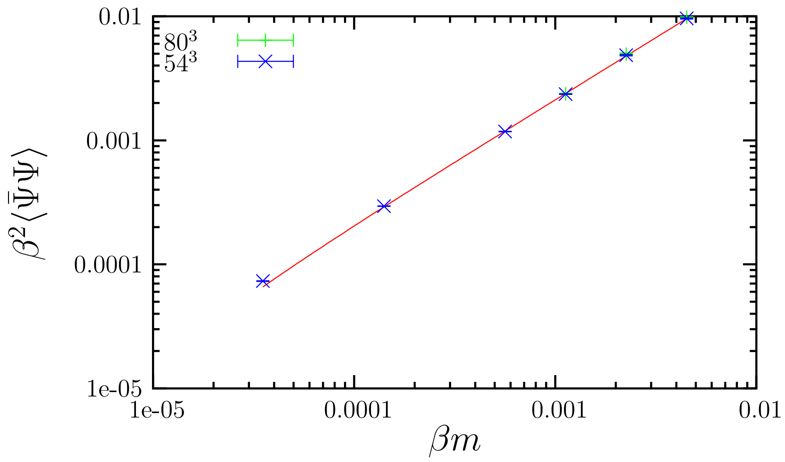

Figure 3: Dimensionless condensate versus dimensionless bare mass for $N_f = 2.0$.

Next, we fitted the data for the dimensionless condensate at different values of N_f and m, and lattice sizes 54^3 and 80^3 to a renormalization group inspired equation of state that includes a finite size scaling term [20]:

$$m = A((\beta-\beta_c)+CL^{-\frac{1}{\nu}})(\beta^2\langle\bar{\psi}\psi\rangle)^p + B(\beta^2\langle\bar{\psi}\psi\rangle)^\delta, \tag{3.2}$$

where $p = \delta - 1/\beta_m$. The results extracted from this fit are: $A = 0.0477(38), B = 0.79(2), C = 10.7(8), N_{fc} = 1.52(6), \delta = 1.177(7), p = 0.73(2)$. The data and the fitting functions are shown in Fig. 5. These results are consistent with a relatively smooth second order phase transition.

4. Summary

We studied numerically the equation of state of non-compact QED$_3$. The extrapolations to the chiral limit on lattices with small finite size effects show with high accuracy that the $N_f = 2.0$ theory

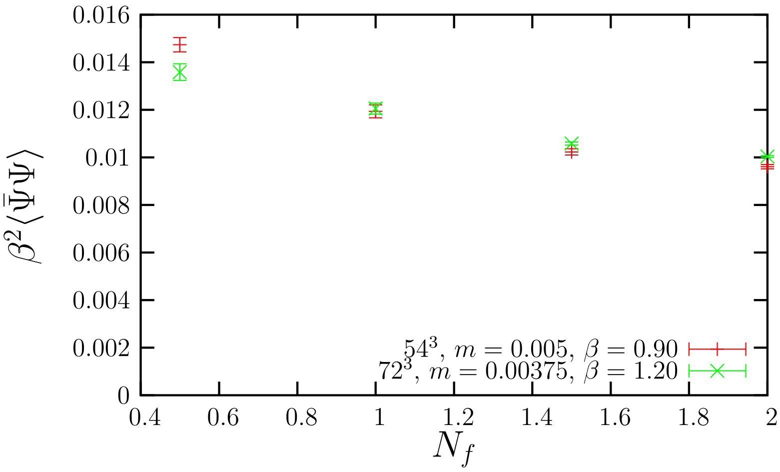

Figure 4: Dimensionless condensate versus N_f at $\beta = 0.90$ and 1.20 with constant L/β and βm.

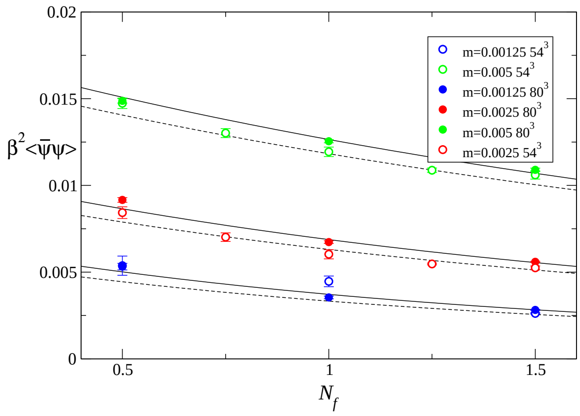

Figure 5: Fits of $\beta^2\langle\bar{\Psi}\Psi\rangle$ vs. N_f to a finite volume scaling form of the Equation of State.

is chirally symmetric with an accuracy of $O(10^{-7})$. The preliminary results extracted from fits to a finite volume Equation of State are consistent with a second order phase transition scenario and $N_{fc} \approx 1.5$. However, in order to reach a decisive conclusion on the value of N_{fc} and the properties of the chiral phase transition we have to extend our simulations to larger lattices, generate better statistics, and include more data points close to the continuum limit.

Acknowledgements

Discussions with Simon Hands and Pavlos Vranas are greatly appreciated.

References

[1] R.D. Pisarski, Phys. Rev. **D29** (1984) 2423;
T.W. Appelquist, M. Bowick, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. **D33** (1986) 3704;
T.W. Appelquist, D. Nash and L.C.R. Wijewardhana, Phys. Rev. Lett. **60** (1988) 2575.

[2] V.A. Miransky and K. Yamawaki, Phys. Rev. **D55** (1997) 5051 [hep-th/9611142];
V.P. Gusynin, V.A. Miransky and A.V. Shpagin, Phys. Rev. **D58** (1998) 085023 [hep-th/9802136].

[3] M.R. Pennington and D. Walsh, Phys. Lett. **B253** (1991) 246.

[4] P. Maris, Phys. Rev. **D54** (1996) 4049 [hep-ph/9606214].

[5] D. Nash, Phys. Rev. Lett. **62** (1989) 3024;
K.-I. Kondo, T. Ebihara, T. Iizuka and E. Tanaka, Nucl. Phys. **B434** (1995) 85 [hep-ph/9404361];
I.J.R. Aitchison, N.E. Mavromatos and D.O. McNeill, Phys. Lett. **B402** (1997) 154 [hep-th/9701087].

[6] M. Franz, T. Pereg-Barnea, D. E. Sheehy and Z. Tesanovic, Phys. Rev. **B68** (2003) 024508 [cond-mat/0211119].

[7] C.S. Fischer, R. Alkofer, T. Dahm, and P. Maris, Phys. Rev. D **70** (2005) 073007 [hep-ph/9404361].

[8] N. Dorey and N.E. Mavromatos, Phys. Lett. **B250** (1990) 907; Nucl. Phys. **B386** (1992) 614;
K. Farakos and N.E. Mavromatos, Mod. Phys. Lett. **A13** (1998) 1019 [hep-lat/9707027];
I.J.R Aitchison, N. Dorey, M. Klein-Kreisler and N.E. Mavromatos, Phys. Lett. **B294** (1992) 91 [hep-ph/9207246];
I.J.R Aitchison, Z. Phys. **C67** (1995) 303.

[9] M. Franz, Z. Tesanovic, and O. Vafek, Phys. Rev. **B66** (2002) 054535 [cond-mat/0204536];
I.F. Herbut, Phys. Rev. **B66** (2002) 094504 [cond-mat/0202491].

[10] I.F. Herbut, Phys. Rev. Lett. **94** (2005) 237001 [cond-mat/0411594].

[11] N.E. Mavromatos, and J. Papavassiliou, [cond-mat/0311421].

[12] T.W. Appelquist, A.G. Cohen and M. Schmaltz, Phys. Rev. **D60** (1999) 045003 [hep-th/9901109].

[13] K. Kaveh, and I.G. Herbut, Phys. Rev. **B71** (2005) 184519 [cond-mat/0411594].

[14] E. Dagotto, A. Kocić and J.B. Kogut, Nucl. Phys. **B334** (1990) 279.

[15] E. Dagotto, A. Kocić and J.B. Kogut, Phys. Rev. Lett. **62** (1989) 1083;
J.B. Kogut and J.-F. Lagaë, Nucl. Phys. **B**(Proc. Suppl.)**30** (1993) 737 [hep-lat/9211059].

[16] V. Azcoiti and X.-Q. Luo, Mod. Phys. Lett. **A8** (1993) 3635 [hep-lat/9212011].

[17] S.J. Hands and J.B. Kogut, Nucl. Phys. **B335** (1990) 455.

[18] S.J. Hands, J.B. Kogut and C.G Strouthos, Nucl. Phys. **B645** (2002) 321 [hep-lat/0208030];
S.J Hands, J.B. Kogut, L. Scorzato and C.G Strouthos, Phys. Rev. **B70** (2004) 056006 [hep-lat/0404013].

[19] S. Christofi, S. Hands, and C. Strouthos, Phys. Rev. **D75** (2007) 101701 [hep-lat/0701016].

[20] L. Del Debbio, S.J. Hands and J.C. Mehegan Nucl. Phys. **B502** (1997) 269 [hep-lat/9701016].

PoS(LATTICE 2007)278