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High precision study of $B^*B\pi$ coupling in unquenched QCD

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The $B^*B\pi$ coupling is a fundamental parameter of chiral effective Lagrangian with heavy-light mesons and can constrain the $B \to \pi l \nu$ form factor in the soft pion limit which will be useful for precise determination of $|V_{ub}|$. We compute the $B^*B\pi$ coupling with the static heavy quark and the $O(a)$-improved Wilson light quark. Simulations are carried out with $n_f = 2$ unquenched $12^3 \times 24$ lattices at $\beta = 1.80$ generated by CP-PACS collaboration. Following the quenched study by Negishi et al., we employ the all-to-all propagator with 200 low eigenmodes as well as HYP smeared link to improve the statistical accuracy.

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1. Introduction

One of the major subjects in particle physics is to determine the CKM matrix elements in order to test the standard model and find a clue to the physics beyond. Among all the components, $|V_{ub}|$ is very attractive, since it is determined from the electro-weak (EW) tree-level processes and hence gives a good reference point for the standard model prediction.

Despite its importance, $|V_{ub}|$ is known only with 10% accuracy from inclusive decay and 20-30% accuracy from exclusive decay. Although the inclusive determination has been developed for the last decades, it suffers from sizable higher order correction in OPE due to the limited kinematic range to avoid the background from $B \to X_c l\nu$. In order to reduce the error, significant theoretical improvement is needed. On the other hand, the determination from the exclusive $B \to \pi l\nu$ decay is more promising, owing to the significant progress in the experiment. Recently BABAR collaboration gave a very precise determination of the form factor up to $|V_{ub}|$ [1]. The only problem is the theoretical uncertainty in the form factor. In view of the present and future experimental situation, in order to determine $|V_{ub}|$ one only needs to know the form factor at any single point in q^2 in principle. In practice, what would be the best choice of q^2? Normally one can choose the q^2 for the smallest nonzero recoil momenta. However, there is an alternative choice of $q^2 \sim m_B^2$, where one can use the symmetry relation. In this choice the $B \to \pi l\nu$ form factor $f^+(q^2)$ can be expressed in terms of the B^* meson decay constant f_{B^*} and the $B^*B\pi$ coupling $\hat{g}_b$ as

$$f^+(q^2) = -\frac{f_{B^*}}{2f_\pi}\left[\hat{g}_b\left(\frac{m_{B^*}}{v\cdot k - \Delta} - \frac{m_{B^*}}{m_B}\right) + \frac{f_B}{f_{B^*}}\right], \tag{1.1}$$

where v is the velocity of the B meson, k is the pion momentum and $\Delta = m_{B^*} - m_B$. Therefore one can reduce problems of computing the form factor to simpler problems of computing the decay constant and effective coupling. Since the B^* state is below the $B\pi$ threshold, it is not possible to measure the $B^* \to B\pi$ decay experimentally and only lattice QCD can provide precise information of the $B^*B\pi$ coupling.

In lattice simulation, the heavy-light decay constant has been studied extensively and is expected to be determined precisely with nonperturbative accuracy in unquenched QCD near future. However, there has not been much progress in the study of the $B^*B\pi$ coupling, in particular for unquenched QCD. Therefore, in this report we present our recent study of high precision determination of the $B^*B\pi$ coupling in unquenched QCD.

At present, one of the promising approaches to computing the $B^*B\pi$ coupling is to use the nonperturbative heavy quark effective theory(HQET) including $1/M$ corrections. However, it is very difficult to calculate the matrix elements for heavy-light systems with HQET. This is because in the heavy-light system the self-energy correction gives a significant contribution to the energy so that the noise to signal ratio of the heavy-light meson correlators grows exponentially as a function of time. In fact, recent results of $\hat{g}_\infty$ are

$$\hat{g}_\infty = 0.51 \pm 0.03_{\text{stat}} \pm 0.11_{\text{sys}} \qquad \text{for} \quad n_f = 0 \text{ [2]}, \tag{1.2}$$

$$\hat{g}_\infty = 0.51 \pm 0.10_{\text{stat}} \qquad \text{for} \quad n_f = 2 \text{ [3]}, \tag{1.3}$$

these have about 5% and 15% statistical errors for quenched and unquenched cases, respectively. But such accuracies will not be sufficient to test new physics. Therefore significant improvements for statistical precision in HQET are needed. Fortunately the two techniques to reduce the statistical error are developed recently, which are the HYP smearing [4] and the all-to-all propagators with low mode averaging [5]. Calculation of $\hat{g}_\infty$ in quenched QCD using these two methods was carried out recently by Negishi et al. [6]. It is found that the statistical accuracy is drastically improved as

$$\hat{g}_\infty = 0.517(16)_{\text{stat.}} \quad \text{for} \quad n_f = 0. \tag{1.4}$$

Our ultimate goal is to extend the above strategies to unquenched simulation and give precise values of the $B^*B\pi$ coupling with $2+1$ flavors in the continuum limit. In this report, we present our high precision study of the static $B^*B\pi$ coupling in $n_f = 2$ unquenched QCD combining two techniques of the HYP smeared link and the all-to-all propagators. Our results also suggest that these techniques can be useful to precisely calculate other physics parameters for heavy-light systems.

2. Lattice observables

The $B^*B\pi$ coupling can be obtained from the form factor at zero recoil which corresponds to the matrix element

$$\langle B^*(p_{B^*},\lambda)|A_i|B(p_B)\rangle|_{\overrightarrow{p_{B^*}}=\overrightarrow{p_B}=0} = (m_B + m_{B^*})A_1(q^2=0)\varepsilon_i^{(\lambda)}, \tag{2.1}$$

where $A_1(q^2=0)$ is the matrix element of B to B^* at zero recoil with axial current $A_i \equiv \bar{\psi}\gamma_5\gamma_i\psi$ and λ stands for polarization [7]. In the static limit,

$$\hat{g}_\infty = A_1(q^2=0) \tag{2.2}$$

holds, so that $\hat{g}_\infty$ can be evaluated by lattice calculations from the ratio of the 3-point and 2-point correlation functions as

$$\hat{g}_\infty = R(t,t_A) \equiv \frac{C_3(t,t_A)}{C_2(t+t_A)}, \tag{2.3}$$

where $C_2(t) \equiv \langle 0|\mathscr{O}_B(t)\mathscr{O}_B^\dagger(0)|0\rangle$, $C_3(t,t_A) \equiv \langle 0|\mathscr{O}_{B_i^*}(t+t_A)A_i(t_A)\mathscr{O}_B^\dagger(0)|0\rangle$ are 2-point and 3-point functions and $\mathscr{O}_B$, $\mathscr{O}_{B_i^*}$ are some interpolation operators for the B meson and B^* meson with polarization in the i-th direction. The lattice HQET action in the static limit is defined as

$$S = \sum_x \bar{h}(x)\frac{1+\gamma_0}{2}\left[h(x) - U_4^\dagger(x-\hat{4})h(x-\hat{4})\right], \tag{2.4}$$

where $h(x)$ is the heavy quark field. The static quark propagator is obtained by solving the time evolution equation. As is well known, HQET propagator is very noisy, and it becomes increasingly serious as the continuum limit is approached. In order to reduce the noise, the Alpha collaboration studied HQET action in which the link variables are smeared to suppress the power divergence [4]. They found that the HYP smearing significantly suppresses the noise of the static heavy-light meson. The error is further suppressed by applying the all-to-all propagator technique developed by the TrinLat collaboration [5]. We divide the light quark propagator into two parts: the low mode part and the high mode part. The low mode part can be obtained using low eigenmodes of Hermitian Dirac operator. The high mode part can be obtained by the standard random noise methods with time, color, and spin dilutions. Combining these propagators, we can obtain the 2-point functions for the heavy-light meson which are averaged all over the spacetime. Similarly, the 3-point functions can be divided into four parts : low-low, low-high, high-low and high-high parts.

3. Simulation details

Numerical simulations are carried out on $12^3 \times 24$ lattices with two flavors of $O(a)$-improved Wilson quarks and the Iwasaki gauge action at $\beta = 1.80$ corresponding to $a^{-1} = 0.9177(92)$GeV. We make use of 100 gauge configurations provided from CP-PACS collaboration through JLDG [8]. We use the $O(a)$-improved Wilson fermion for light valence quark with clover coefficient $c_{sw} = 1.60$. In the HQET, we use static action with HYP smeared links with smearing parameter values $(\alpha_1, \alpha_2, \alpha_3) = (0.75, 0.6, 0.3)$. The B and B^* meson operators are smeared with a function $\phi(r) = \exp(-0.9r)$. We obtain the low-lying eigenmodes of the Hermitian Dirac operator using implicitly restarted Lanczos algorithm. The low mode correlation functions are computed with N_{ev} = 200 eigenvectors of the low-lying eigenmodes of the lattice Hermitian Dirac operator. The high mode correlation functions are obtained by using the quark propagator with the source vector generated by the complex random Z_2 noise. The number of the random noise and the number of time dilution for each configuration are set to $N_r = 1$ and $N_{t_0} = 24$, respectively. This setup is based on the experience from the work by Negishi et al. [6].

4. Results

We show the result of the 2-point correlator with $\kappa = 0.1430$ in Fig. 1, which displays whether the improved technique works successfully. Indeed, both the low mode part and the high mode part have small statistical errors. The effective mass of the 2-point function is shown in Fig. 2. We observe a nice plateau at $t \geq 4$. From this result we take t_A=5 as a reasonable choice of the time difference between the current A_i and the B meson source.

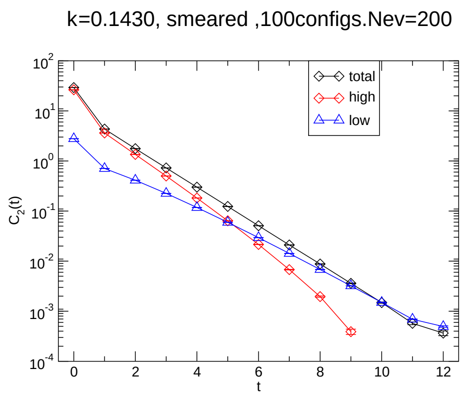

Figure 1: Low mode and high mode contributions to the 2-point functions for $\kappa = 0.1430$. Blue and red symbols represent low and high mode contributions, respectively. We can see low mode becomes dominant for t larger than five.

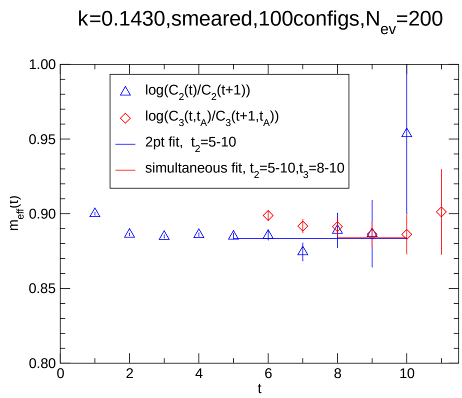

Figure 2: The effective masses of the 2-point and the 3-point functions with $\kappa = 0.1430$ and $t_A = 5$. Blue line represents the fit for the 2-point functions only. Red line corresponds to simultaneous fit for the 2-point and 3-point functions.

Fig. 3 shows the time dependence of the 3-point functions for $\kappa = 0.1430$. Fluctuations of high-modes for 3-point functions are indeed suppressed, so we can get a good plateau for the

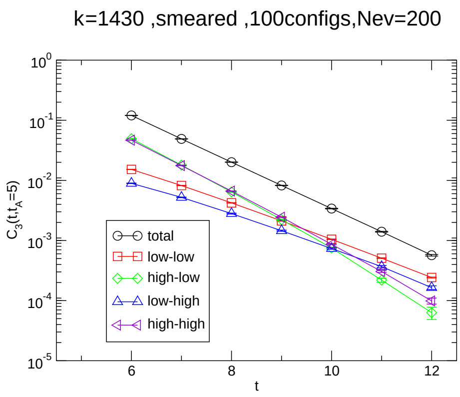

Figure 3: Contributions of low-low, high-low, low-high and high-high parts to the 3-point functions for $\kappa = 0.1430$.

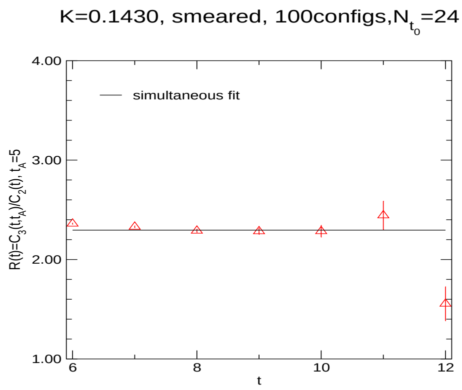

Figure 4: The ratio of the 3-point and 2-point functions at each t for $\kappa = 0.1430$. The solid line represents the ratio of fit parameters Z_3/Z_2.

effective mass of 3-point functions as in Fig 2. We use the following fit functions

$$\begin{aligned} C_2(t) &= Z_2 \exp(-mt), \\ C_3(t) &= Z_3 \exp(-mt), \end{aligned} \tag{4.1}$$

where Z_2 and Z_3 are constant parameters and m corresponds to the heavy-light meson mass. The bare $B^*B\pi$ coupling can be obtained by the ratio of the fit parameters as $\hat{g}_\infty^{\rm bare} = Z_3/Z_2$. Fit ranges for the 2-point and the 3-point functions are $5 \sim 10$ and $8 \sim 10$, respectively. Fig. 2 shows that the effective masses and the fit results are all consistent as we expected.

Fig. 4 compares the results of the $B^*B\pi$ coupling for $\kappa = 0.1430$ determined from two different methods. It is found that the ratio $C_3(t)/C_2(t)$ at each t and the ratio of the fit parameters Z_3/Z_2 give consistent value of the $B^*B\pi$ coupling. We will use the latter result Z_3/Z_2 to determine $\hat{g}_\infty$ in the following analyses. The physical value of the $B^*B\pi$ coupling is obtained by multiplying the renormalization constant. We use the one-loop result of renormalization factor for the axial vector current

$$A_i = 2\kappa u_0 Z_A \left(1 + b_A \frac{m}{u_0}\right) A_i^{lat}, \tag{4.2}$$

$$u_0 = \left(1 - \frac{0.8412}{\beta}\right)^{\frac{1}{4}}, \quad b_A = 1 + 0.0378 g_{\overline{MS}}^2(\mu),$$

where the gauge coupling $g_{\overline{MS}}^2(\mu) = 3.155$ and $Z_A = 0.932$ for $\beta = 1.80$ as given in Ref. [8]. We arrive at our preliminary results of $\hat{g}_\infty$ for our κ values in Table 1.

κ	0.1409	0.1430	0.1445	0.1464
$\hat{g}_\infty$	$0.612(5)_{\rm stat}$	$0.598(5)_{\rm stat}$	$0.591(4)_{\rm stat}$	$0.578(5)_{\rm stat}$

Table 1: Preliminary results of $\hat{g}_\infty$.

We take the chiral extrapolation of the $B^*B\pi$ coupling using the data with four quark masses. Employing the following fit functions (a), (b), (c),

$$
\begin{aligned}
&(\mathrm{a})\ \ g(m_\pi^2)_1 = g(0) + A_1 m_\pi^2, \\
&(\mathrm{b})\ \ g(m_\pi^2)_2 = g(0) + A_1 m_\pi^2 + A_2 (m_\pi^2)^2, \\
&(c)\ \ g(m_\pi^2)_3 = g(0)\left(1 - g(0)^2 \frac{7}{64\pi^2}\frac{m_\pi^2}{f_\pi^2}\log(m_\pi^2)\right) + A_1 m_\pi^2 + A_2 (m_\pi^2)^2,
\end{aligned}
$$

we carry out the linear extrapolation, the quadratic extrapolation, and the quadratic plus chiral log extrapolation where the log coefficient is determined from ChPT[9]. We use three, four and four data point for the fits, respectively. We obtain physical values of the $B^*B\pi$ coupling in the chiral limit as $\hat{g}_\infty = 0.57(1), 0.57(2), 0.52(1)$ from the linear fit, the quadratic fit and the quadratic plus chiral log fit, respectively. We take the average of the results from the linear fit and the quadratic plus chiral log fit as our best value and take half the difference as the systematic error from the chiral extrapolation. Other systematic errors are the perturbative error of $O(\alpha^2)$, and the discretization error of $O((a\Lambda)^2)$. Including these errors estimated by order counting, our preliminary result of $\hat{g}_\infty$ is

$$\hat{g}_\infty^{n_f=2} = 0.55(1)_{\mathrm{stat.}}(3)_{\mathrm{chiral.}}(3)_{\mathrm{pert.}}(6)_{\mathrm{disc.}} \ \text{ at } \ \beta = 1.80. \tag{4.3}$$

We find that the discretization error is dominant in our simulation on this coarse lattice.

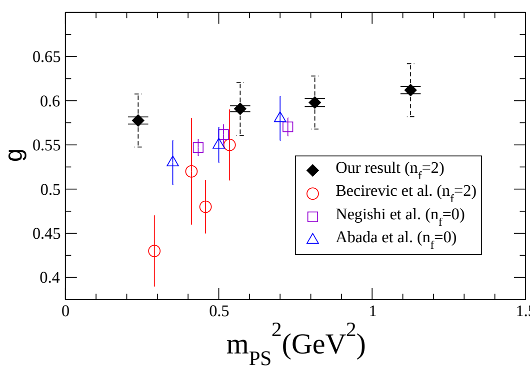

Figure 5: Comparison with other calculations [2, 3, 6]. In our results, small and large errors represents statistics error and perturbative error respectively.

5. Conclusion

In this report, we computed the $B^*B\pi$ coupling on unquenched lattices using the HYP smearing and the all-to-all propagators. Using the low mode averaging with 200 eigenmodes, the statistical error becomes tiny for all the quark masses, giving $\sim 2\%$ in the chiral limit. However, since the dominant error is from the discretization for our simulation on this coarse lattice, we need to simulate on finer lattices. In Fig. 5, we compare the recent results of the $B^*B\pi$ coupling [2, 3, 6]. The improvement in statistical precision is drastic, which proves the power of the improvement techniques examined in this report. This result implies that one can also precisely calculate other quantities such as the B meson decay constant or $\hat{g}$ with $1/M$ corrections.

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