

## Moving NRQCD and $B \rightarrow K^* \gamma$

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**Stefan Meinel, Ronald Horgan, Lew Khomskii, Laurent C. Storonni, and Matthew Wingate\***

*DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK*

*E-mail: M.Wingate@damtp.cam.ac.uk*

The formulation of NRQCD discretized in a reference frame boosted relative to the  $B$  rest frame will enable calculation of  $B$  form factors over a larger range of momentum transfer. We have initiated a program to calculate form factors describing the rare decay  $B \rightarrow K^* \gamma$ . We discuss the strategy and challenges of the project. As a first step in the numerical calculations, we present first results for bottomonium quantities using the  $O(\Lambda_{\text{QCD}}^2/m^2, v_{\text{NR}}^4)$  moving NRQCD action.

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\*Speaker.

Matrix element	Form factor	Relevant decay(s)
$\langle P   \bar{q} \gamma^\mu b   B \rangle$	$f_+, f_0$	$\begin{cases} B \rightarrow \pi \ell \nu \\ B \rightarrow K \ell^+ \ell^- \end{cases}$
$\langle P   \bar{q} \sigma^{\mu\nu} q_\nu b   B \rangle$	$f_T$	$B \rightarrow K \ell^+ \ell^-$
$\langle V   \bar{q} \gamma^\mu b   B \rangle$	$V$	$\begin{cases} B \rightarrow (\rho/\omega) \ell \nu \\ B \rightarrow K^* \ell^+ \ell^- \end{cases}$
$\langle V   \bar{q} \gamma^\mu \gamma^5 b   B \rangle$	$A_0, A_1, A_2$	$\begin{cases} B \rightarrow (\rho/\omega) \ell \nu \\ B \rightarrow K^* \ell^+ \ell^- \end{cases}$
$\langle V   \bar{q} \sigma^{\mu\nu} q_\nu b   B \rangle$	$T_1$	$\begin{cases} B \rightarrow K^* \gamma \\ B \rightarrow K^* \ell^+ \ell^- \end{cases}$
$\langle V   \bar{q} \sigma^{\mu\nu} \gamma^5 q_\nu b   B \rangle$	$T_2, T_3$	$\begin{cases} B \rightarrow K^* \gamma \\ B \rightarrow K^* \ell^+ \ell^- \end{cases}$

**Table 1:** Full list of  $B$  semileptonic form factors.

## 1. Motivation

We at this conference know very well the importance lattice QCD calculations have in the global flavor physics program. Calculations of the  $B$  meson decay constant,  $B \rightarrow \pi$  form factors, and  $B - \bar{B}$  mixing matrix elements have been pursued and refined for over a decade, and they are important ingredients in constraining parameters governing quark flavor-changing interactions.

It is now clear that the CKM mechanism of the Standard Model accurately describes flavor physics up to present precision. In order to probe the couplings to the non-Standard Model physics we expect, we must further refine experimental measurements and theoretical calculations.

In the latter pursuit, lattice QCD must extend its focus. Rare  $B$  decays offer a promising avenue for improvement from the status quo. One difference between the rare  $B$  decays and the processes on which lattice QCD usually focuses is that the former require more assumptions, *e.g.* neglect of long distance contributions and hard spectator effects. Nevertheless, lattice calculations can still play an important role in the phenomenology of exclusive  $b \rightarrow s$  decays by reducing uncertainties in hadronic matrix elements.

## 2. Plan for calculation

In this section we outline our strategy for computing  $B \rightarrow K^* \gamma$  form factors. Ultimately we would like to calculate all of the semileptonic  $B$  decay form factors (Table 1). Presently we concentrate on the radiative decay because it stands to be the most greatly improved.

The main new component to be used is moving NRQCD (mNRQCD). As with conventional NRQCD, this is an effective field theory which permits lattice calculations with the physical bottom quark mass. The formulation in a frame where the lattice is boosted relative to the  $B$  rest frame will permit calculations over a larger range of momentum transfer  $q^2$  than non-moving NRQCD. We discuss mNRQCD in Section 3.

We will use an improved staggered quark action for the light valence and sea quarks. The first calculations will make use of the ensemble of MILC configurations generated with the AsqTad action; later we will use configurations generated with the HISQ action. The virtues and risks of

using rooted, improved staggered quarks have been discussed extensively [1, 2]. A few remarks regarding the  $K^*$  are made in Section 4.

The matching between the continuum and lattice current and penguin operators will be carried out to 1-loop order in perturbation theory. The matching of the vector and axial vector currents for mNRQCD is being finalized presently [3], and the matching for the penguin operator is underway.

A recent lattice calculation used a very different lattice strategy to calculate the  $B \rightarrow K^*$  form factors [4] (see within for earlier lattice calculations). The use of many approaches, sum rules in addition to lattice QCD, is especially desirable given the theoretical uncertainties.

### 3. Moving NRQCD

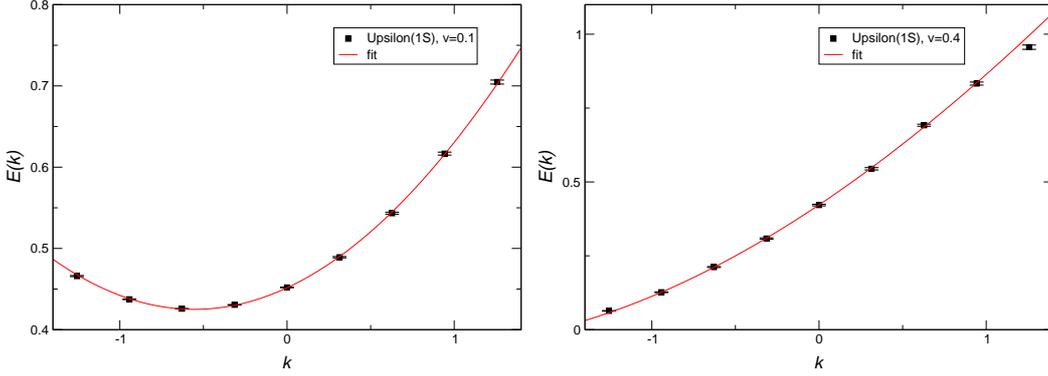
Moving HQET/NRQCD has been a recurring topic for over a decade [5, 6, 7, 8, 9, 10]. Initially it was envisioned for use calculating Isgur-Wise functions at nonzero recoil. Since the  $B \rightarrow D$  form factor shapes are constrained by dispersion relations accurately, only the zero recoil normalization is now necessary from lattice QCD (LQCD). Later, mNRQCD was explored with the idea of extending the reach of LQCD calculations of  $B \rightarrow \pi$  form factors toward large recoil. This is still desirable, but the shape is now being measured competitively by experiment. In the previous 2 cases the LQCD determination of the shape is not imperative, but the LQCD determination of the normalization is still needed. On the contrary, in order to reach the physical point for  $B \rightarrow K^* \gamma$  ( $q^2 = 0$ ) where LQCD can provide the normalization, a lattice calculation of the shape is a necessary step. Moving NRQCD is an important tool to develop and apply.

As with NRQCD, we work with an effective field theory which requires  $m_b > 1/a$ . This condition is satisfied on all present and near-future unquenched lattices. Although one cannot take a continuum limit in the formal sense, we can study and remove discretization errors at least as well as with other heavy quark formulations. There is no *theoretical* problem with working with a finite lattice spacing either. There are no discretization errors on the renormalized trajectory. Of course one can question how close to the renormalized trajectory we can get using the Symanzik improvement program. However, this is a *practical* question, the type of which can be asked of any lattice formulation and can only be answered empirically. Experience has shown NRQCD to be a successful approach.

The lattice (m)NRQCD action can be used for both  $Y$  and  $B$  physics. In the latter case, we use standard HQET power counting to order and match operators. The leading uncertainty in some cases is the matching, done with 1-loop perturbation theory so far. The convergence of HQET worsens as the recoil momentum becomes much larger than  $\Lambda_{\text{QCD}}$ ; however, we expect the change to be mild over the range of  $q^2 > 0$  we plan to study directly.

Working with a lattice boosted with respect to the  $B$  meson has the potential to blur the separation between physical and lattice length scales. At rest, hadronic momenta are of order  $\Lambda_{\text{QCD}}$ . In a frame where the  $B$  is boosted with velocity  $v$ , the boosted momentum distribution is of order  $\Lambda_{\text{QCD}} \sqrt{(1+v)/(1-v)}$  in the direction parallel to  $\mathbf{v}$ . That is, discretization errors will be twice as large at  $v = 0.6$  than for non-moving NRQCD. We anticipate that other sources of error will still dominate.

We have independently derived, coded, and tested the moving NRQCD action accurate through  $O(\Lambda_{\text{QCD}}^2/m^2)$  for  $B$  physics (HQET counting) and  $O(v_{\text{NR}}^4)$  (NRQCD counting) for  $Y$  physics. The



**Figure 1:** Preliminary dispersion relation  $E(k)$  as a function of residual momentum  $k$ , both in lattice units. The bare boost velocity is  $v = 0.1$  (left) and  $v = 0.4$  (right).

primary goal of our  $Y$  calculations with mNRQCD is to test the code, checking that we obtain sensible results with reasonable statistical errors as the boost velocity  $v$  increases. As far as we are aware, these are the first mNRQCD calculations with a Lagrangian of this accuracy. These tests were performed on a subset of  $2 + 1$  flavor AsqTad-fermion lattices provided by the MILC Collaboration, with  $\beta = 6.76$ , bare quark masses 0.007 and 0.05,  $V = 20^3 \times 64$  [11]. We used the bare heavy quark mass,  $am = 2.8$ , which gave the correct  $B_s$  and  $Y$  kinetic masses using non-moving NRQCD [12, 13].

First we studied how spectral quantities behaved as the boost velocity  $v$  varied. On Coulomb gauge-fixed lattices, we used smeared interpolating operators of the form

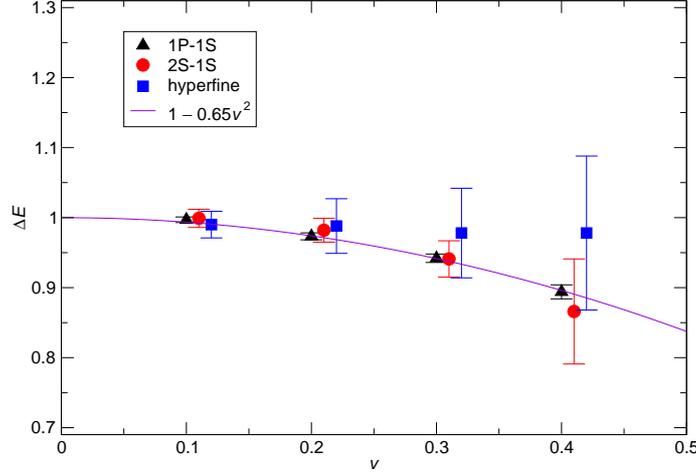
$$O_v(\mathbf{x}, \tau) = \sum_{\mathbf{r}} \bar{\Psi}_v(\mathbf{x}, \tau) f(\mathbf{r}) \Gamma \Psi_v(\mathbf{x} + \mathbf{r}, \tau), \quad (3.1)$$

where  $f(\mathbf{r})$  is a radial smearing function and  $\Gamma$  is a Dirac  $\gamma$  matrix. As in non-moving NRQCD, we decouple the quark and antiquark fields  $\Psi_v = (\psi_v, \chi_v)^T$  and evolve the propagators from the source timeslice to the sink timeslice. At the sink we project onto residual meson momentum  $\mathbf{k}$ . The energies can then be fit to

$$E(\mathbf{k}) = \sqrt{(2\gamma m v Z_p + \mathbf{k})^2 + M_{\text{kin}}^2} + \Delta_v \quad (3.2)$$

where  $M_{\text{kin}}$  is the kinetic meson mass, and  $\Delta_v$  is an additive energy shift which is a function of  $v$  and is the same for all mesons. Note the physical meson momentum is split into a residual momentum  $\mathbf{k}$ , present explicitly in the calculation of the correlation function, and an external momentum  $2\gamma m v Z_p$ , with  $\gamma = (1 - v^2)^{-1/2}$ .  $Z_p$  accounts for renormalization of the external momentum; we always find it to be consistent with 1 within fitting uncertainties. Dispersion relations for  $\eta_b(1S)$  and  $Y(1S)$  for different boost velocities are plotted in Figure 1.

In Figure 2 we show several energy splittings as a function of  $v$ , computed using correlation functions which project onto residual momentum  $\mathbf{k} = 0$ . We note the statistical errors grow as  $v$  increases from 0 to 0.4, an effect more pronounced for the hyperfine and  $2S - 1S$  splittings than the  $1P - 1S$  splitting. Splittings with non-moving NRQCD were computed in [13].



**Figure 2:** Preliminary bottomonium energy splittings  $\Delta E$  as a function of boost velocity  $\nu$ , plotted relative to  $\Delta E$  computed with  $\nu = 0$ . (Points are offset horizontally for legibility.) The 1P–1S and 2S–1S splittings show a  $1 - c\nu^2$  decrease as expected from the dispersion relation.

Finally, to go beyond energies to matrix elements, we computed the  $\eta_b$  to vacuum matrix element of a fictitious axial vector current  $A^\mu(x) = \bar{\Psi}(x)\gamma_5\gamma^\mu\Psi(x)$ , which we parametrize with a decay constant  $f$  as

$$\langle 0|A^\mu(0)|\eta_b(1S), \mathbf{p}\rangle = if p^\mu \quad (3.3)$$

(in Minkowski spacetime). The appropriate correlation function is constructed by writing this operator in terms of the mNRQCD fields (in the lattice rest frame) using the following transformation:

$$\Psi(x) = S_\Lambda T_{\text{FWT}} e^{-im u \cdot x \gamma^0} T_{\text{TD}} \frac{1}{\sqrt{\gamma}} \Psi_\nu(x) \quad (3.4)$$

where

$$T_{\text{FWT}} = \exp\left(\frac{i}{2m} \gamma^j \Lambda_j^\mu D_\mu\right) \dots \quad (3.5)$$

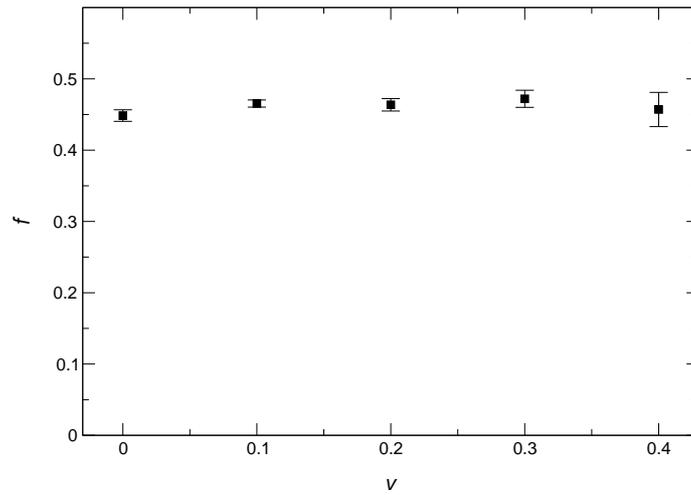
is the Foldy-Wouthuysen-Tani transformation in the boosted frame,

$$T_{\text{TD}} = \exp\left(\frac{i}{4\gamma m} \gamma^0 [(\gamma^2 - 1)D_0 + (\gamma^2 + 1)\mathbf{v} \cdot \mathbf{D}]\right) \dots \quad (3.6)$$

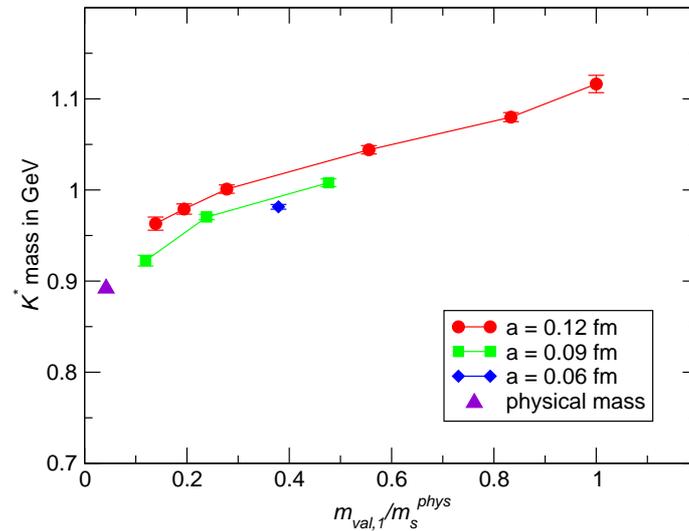
removes unwanted time derivatives, and  $S_\Lambda$  is the Dirac spinor representation of the Lorentz boost.

Figure 3 shows this decay constant computed for several boost velocities. We might expect some dependence on  $\nu$  due to  $\nu$ -dependent operator renormalization and the fact that constant bare heavy quark mass might not correspond to constant  $M_{\eta_b}$ . Nevertheless,  $f$  appears independent of  $\nu$  within the statistical errors.

We note the statistical error increases by a factor of 3. Increasing the signal-to-noise ratio for correlators computed with  $\nu > 0$  will be an important challenge for our planned matrix element calculations. Progress has already been achieved for  $B \rightarrow \pi$  form factors (in the  $\nu = 0$  frame) by using random wall sources [14].



**Figure 3:** Preliminary results for the  $\eta_b$  decay constant  $f$ , in lattice units, as a function of bare boost velocity.



**Figure 4:** Unquenched  $K^*$  mass as a function of light quark mass on MILC lattices (3 spacings), after simple interpolation to physical strange quark mass [11]. (Raw data communicated by D. Toussaint.) Although statistically significant, scaling violations are small compared to other errors anticipated for  $B \rightarrow K^*$  form factors.

#### 4. Vector meson final state

Figure 4 shows the  $K^*$  mass computed by the MILC Collaboration [11]. Discretization errors are visible within the small statistical errors, but are only a few percent, much smaller than the other systematic errors we anticipate for the form factors. Taste splitting effects are negligible between the vector meson masses computed with local and 1-link operators.

There are interesting issues to study regarding threshold effects as the quark mass decreases. Our initial calculations will be done with parameters for which the  $K^*$  is a stable state. (Note that

experimentalists quote branching ratios which treat the vector resonance as a final state.) Given that we do not have a low energy effective theory for the vector mesons, as we do for the pseudoscalar mesons and baryons, the best we can do is empirically extrapolate from our input quark masses to the physical point. The  $B \rightarrow \pi$  form factors have a very mild quark mass dependence, so it is reasonable to expect the same of the  $B \rightarrow K^*$  form factors, up to threshold effects.

## 5. Conclusions

Although more complicated than the standard  $B$  meson matrix elements calculated on the lattice, matrix elements relevant for rare  $B$  meson decays are increasingly important to the flavor physics program. The difficulties involved call for investigation with new tools such as moving NRQCD. We have implemented and tested the mNRQCD action through  $O(\Lambda_{\text{QCD}}^2/m^2, v_{\text{NR}}^4)$ . We present here preliminary results with this action, concentrating on the bottomonium dispersion relation, level splittings, and the  $\eta_b$  decay constant. We are now working on calculations for  $B$  mesons.

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