PROCEEDINGS OF SCIENCE

K to π semileptonic form factor with 2+1 flavor domain wall Fermions on the lattice

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UKQCD and RBC Collaborations

We present preliminary results from the RBC and UKQCD collaborations for the $Kl3$ form factor $f_0^{K\pi}(0) = f_+^{K\pi}(0)$ with 2+1 flavours of dynamical domain wall quarks. Simulations are performed on $16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$ lattices with three values of the light quark mass, allowing for an extrapolation to the chiral limit

Kaon International Conference
May 21-25, 2007
Laboratori Nazionali di Frascati dell'INFN

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PoS(KAON)010

1. Introduction

The RBC and UKQCD collaborations are currently performing a joint full lattice QCD simulation of the K to π vector form factor $f_+^{K\pi}(0)$ from first principles. Our motivation lies in the fact that the theoretical uncertainty in $f_+^{K\pi}(0)$ is still dominating the extraction of the CKM matrix element $|V_{us}|$ from the experimentally very precisely measured decay rates. Palutan presented a review of the experimental situation at this conference [1] and we quote his estimate $|V_{us}\, f_+^{K\pi}(0)| = 0.21668(45)$ (cf. the PDG 2006 value $|V_{us}\, f_+^{K\pi}(0)| = 0.2169(9)$ [2]).

The current experimental precision is not fully appreciated in a determination of $|V_{us}|$ since it is still common practice to determine it using the phenomenological estimate $f_+^{K\pi}(0) = 0.961(8)$ given by Leutwyler & Roos [3] in 1984 because a first principles calculation of $f_+^{K\pi}(0)$ on the lattice with a reliable control of the systematic uncertainties is lacking.

The calculation on the lattice is straight forward in principle. The technique which allows to achieve sub-per cent level precision for the prediction of $f_+^{K\pi}(0)$ has been set out in [4] and it has subsequently been applied in several computations [5, 6, 7, 8]. However, all these computations were carried out in quenched or partially quenched QCD (i.e. with a quenched strange quark and two dynamical light quarks) or with rather heavy up and down quark masses resulting in unphysical pion masses of 500 MeV or heavier. This made it difficult to reliably extrapolate the data to the physical point using predictions for the quark mass dependence from chiral perturbation theory.

In this talk we present the status of our calculation of $f_+^{K\pi}(0)$ with $N_f = 2+1$ dynamical quark flavors of domain wall quarks with an unprecedented light pion mass of around 300 MeV which will allow to estimate the systematics due to the chiral extrapolation more reliably [9, 10].

2. The simulation

The $K \to \pi$ matrix element

$$\langle \pi(p_\pi)|V_\mu|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K + p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K - p_\pi)_\mu\,, \qquad (q = p_K - p_\pi)\,, \tag{2.1}$$

can be extracted from suitable Euclidean three-point correlation functions at large values of the Euclidean time. We compute these correlation functions directly from the discretized Euclidean QCD path integral by means of a Monte Carlo integration. In particular, we use the Iwasaki gauge action [11] and the domain wall fermion action which for our simulation parameters turns out to exhibit good chiral properties [12]. The ensembles of gauge configurations on which we evaluate the correlation functions have been jointly generated by the RBC and UKQCD collaborations [12]. For our choice of the bare coupling $\beta = 2.13$ the inverse lattice spacing is $a^{-1} \approx 1.62$ GeV.

For precision results one has to control and understand the sources of systematic errors of which we summarise the most significant ones in the following:
quark masses: We simulate QCD with a fixed dynamical strange quark of approximately physical mass and a pair of dynamical degenerate light up and down quarks. Since it is currently not feasible to simulate with the latter at their physical mass in a large lattice volume we generate results for a number of unphysically heavy up and down quark masses. These results are then extrapolated to the physical point ideally using predictions for the quark mass dependence of the form factor from chiral perturbation theory.

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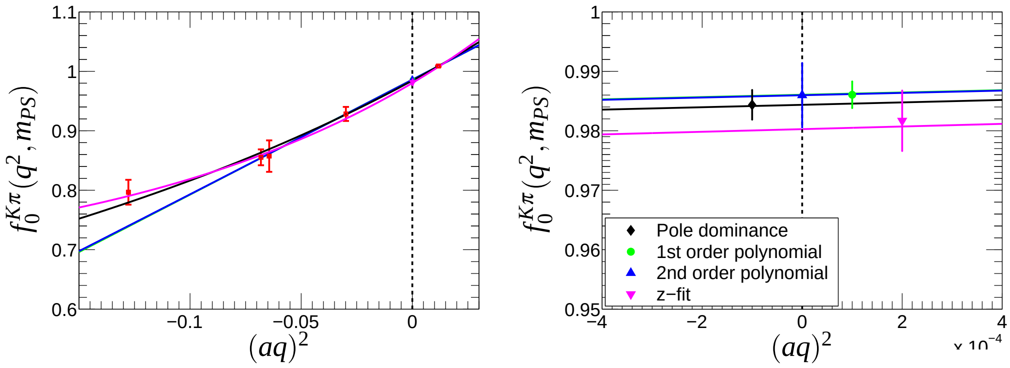

Figure 1: Typical result for $f_0^{K\pi}(q^2)$ ($am_l = 0.01$). Left: Data points for $q^2_{\rm max}$ and $(|\mathbf{p}_K|, |\mathbf{p}'_\pi|) \in 2\pi/L\{(0,1),(1,0),(\sqrt{2},0),(0,\sqrt{2})\}$. Right: Zoom into $q^2 = 0$-region with scattering of interpolation results from using different ansätze.

finite volume: The systematics due to the finite extent of the lattice can be investigated in chiral perturbation theory [4]. Here we directly assess the influence of the spatial boundary by comparing the scaling of observables between two simulations with equivalent physical parameters but two different lattice sizes: $(L/a)^3 \times (T/a) \times L_s = 16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$ (L and T are the spacial and time-extent of the lattice, respectively and L_s the size of the 5th dimension). In physical units the spacial volumes are $(1.9\text{fm})^3$ and $(2.9\text{fm})^3$.
In a finite volume, the momenta of the kaon and pion $|\vec{p}_K|$ and $|\vec{p}_\pi|$ are quantized and take the values 0, $2\pi/L$ and $\sqrt{2}2\pi/L, \ldots$. Thus, the kinematical point $q^2 = 0$ is generally not directly accessible in lattice simulations and one has to interpolate the form factor in q^2.
cut off effects: The domain wall fermions simulated here are chirally symmetric to a good approximation and therefore cut-off effects are naively expected to be of order $(a\Lambda_{\rm QCD})^2 \approx 4\%$ (assuming $\Lambda_{\rm QCD} = 300$MeV). We are currently extending our project by a simulation with a larger cut-off which will allow us to assess cut off effects in a more reliable way.

We carry out the following 3-step procedure [4] to extract the form factor $f_+^{K\pi}(0) = f_0^{K\pi}(0)$:

1) Compute the scalar form factor

$$f_0^{K\pi}(q^2) = f_+^{K\pi}(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-^{K\pi}(q^2) \tag{2.2}$$

at $q^2_{\rm max} = (m_K - m_\pi)^2$ from

$$R(t,t') = \frac{C_4^{K\pi}(t',t;\mathbf{0}',\mathbf{0})C_4^{\pi K}(t',t;\mathbf{0}',\mathbf{0})}{C_4^{KK}(t',t;\mathbf{0}',\mathbf{0})C_4^{\pi\pi}(t',t;\mathbf{0}',\mathbf{0})} \overset{t,(t'-t)\to\infty}{\longrightarrow} \frac{(m_K+m_\pi)^2}{4m_K m_\pi}\left[f_0^{K\pi}(q^2_{\rm max})\right]^2 , \tag{2.3}$$

where

$$C_\mu^{K\pi}(t',t,\vec{p}\,',\vec{p}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\,'(\vec{y}-\vec{x})} e^{-i\vec{p}\vec{x}} \langle 0|\mathcal{O}_\pi|\pi\rangle\langle\pi|V_\mu|K\rangle\langle K|\mathcal{O}_K^\dagger|0\rangle , \tag{2.4}$$

are Euclidean three point correlation functions from which the matrix element (2.1) can be extracted for large values of the Euclidean times t.

2) Compute $f_0^{K\pi}(q^2)$ from ratios similar to (2.3) [1] with the combinations of Fourier momenta of the kaon and pion $(|\mathbf{p}_K|, |\mathbf{p}'_\pi|) \in 2\pi/L\{(0,1),(1,0),(\sqrt{2},0),(0,\sqrt{2})\}$. A typical example of the resulting data is shown in the left plot in figure 1. We use this data to constrain an interpolation to $q^2 = 0$. The corresponding ansatz is a priori not known and we estimate the resulting systematic uncertainty from the scattering of the results at $q^2 = 0$ as obtained from different fit ansätze:

$$\begin{aligned} f_0^{\text{pole}}(q^2) &= f_0(0)/(1-q^2/M^2)\,, \\ f_0^{\text{lin}}(q^2) &= f_0(0)(1+q^2/M_0^2)\,, \\ f_0^{\text{quad}}(q^2) &= f_0(0)(1+q^2/M_0^2+q^4/M_1^4)\,, \\ f_0^{\text{z-fit}}(q^2) &= \frac{1}{\phi(q^2,q_0^2,Q^2)}\sum_{k=0}^{2} a_k(q_0^2,Q^2)z(q^2,q_0^2)^k\,. \end{aligned} \tag{2.5}$$

For the z-fit [13] in the last line we use the same parameters as already described in [9]. An example of the performance of the different ansätze is given in the right plot in figure 1. We mention that a new approach for the computation of $f_0^{K\pi}(0)$ on the lattice has been developed and tested in [14] in which the interpolation is completely avoided, thus eliminating one source of systematic uncertainty.

3) At this point the results for $f_0^{K\pi}(0)$ are still for unphysically heavy up- and down quark masses. Contact with the physical point is made using predictions from chiral perturbation theory. Here it is crucial to note that firstly $f_0^{K\pi}(0) = 1$ in the $SU(3)$-symmetric limit and that thanks to the Ademollo-Gatto theorem [15] $f_0^{K\pi}(0)$ is analytic up to including p^4 contributions and can be expressed purely in terms meson masses [3],

$$f_0^{K\pi}(0,m_K,m_\pi) = 1 + f_2(m_K,m_\pi) + O(p^6)\,, \tag{2.6}$$

where we take $f_2(m_K,m_\pi)$ from [3]. In our simulation we therefore only compute the corrections beyond p^4. To this purpose we determine the quantity

$$R_{\Delta f}(m_\pi,m_K) = \frac{f_0^{K\pi}(0,m_\pi,m_K) - (1+f_2(m_\pi,m_K))}{(m_K^2-m_\pi^2)^2}\,, \tag{2.7}$$

and extrapolate it linearly in $(m_K^2+m_\pi^2)$ to the physical point. The linear fit-ansatz is supported by visual inspection of the data in the l.h.s. plot in figure 2. We also extrapolate linearly the quantity

$$\Delta f(m_K,m_\pi) = f_0^{K\pi}(0,m_K,m_\pi) - (1+f_2(m_K,m_\pi))\,, \tag{2.8}$$

and take the discrepancy between the result at the physical point obtained in this way and the result at the physical point for $R_{\Delta f}$ as an estimate of the systematic error.

We repeat the result at the physical point which we already quoted at the CKM 2006 workshop [9]:

$$\Delta_f = R_{\Delta_f}(m_K^2-m_\pi^2)^2 = -0.0161(46)(15)(16)(7)\,. \tag{2.9}$$

[1] For more details please refer to [4].

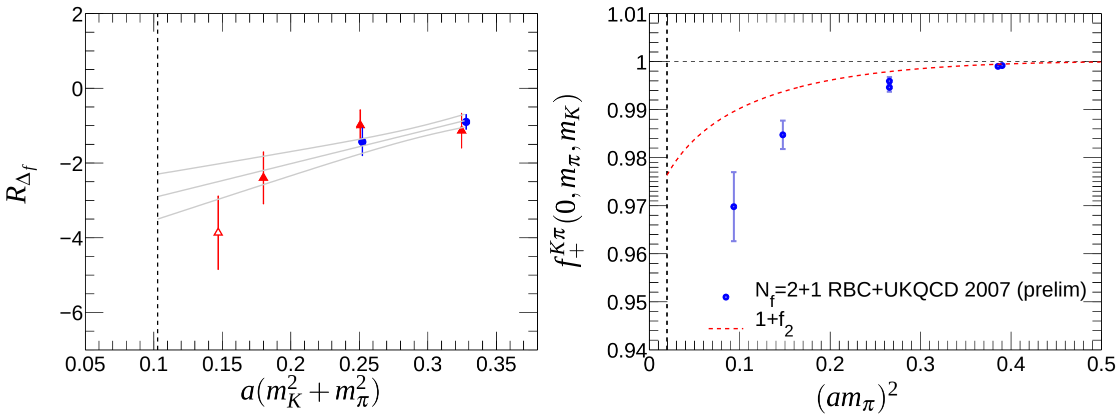

Figure 2: Left: Linear chiral extrapolation of the form factor data in terms of the ratio R_{Δ_f}. Circles represent data points from the 16^3 data set and triangles the ones from the 24^3 data set. Empty symbols have not yet been included into the extrapolation. Right: Results for $f_+^{K\pi}(0,m_\pi,m_K)$ in comparison to the form factor with only leading $SU(3)$-breaking contribution $1+f_2(m_\pi,m_K)$.

The errors are statistical, systematic due to the extrapolation to the chiral limit, systematic due to the interpolation in q^2 and cut-off effects. Since this quantity is only the $O(p^6)$-correction to $f_0^{K\pi}(0)$ the current precision is sufficient to yield a sub-per cent error for the form factor itself. Our final result is then readily obtained by using

$$f_0^{K\pi}(0) = 1 + f_2^{\text{phys}} + R_{\Delta_f}^{\text{phys}}(m_K^2 - m_\pi^2)^2\,, \tag{2.10}$$

evaluated at the physical point.

We here quote our preliminary result [9]: $f_0^{K\pi}(0) = 0.9609(51)$. If we combine this result with Palutan's estimate for $|V_{us} f_+^{K\pi}|$ from his review at this conference we determine $|V_{us}| = 0.2255(4)_{\text{exp}}(12)_{f_+^{K\pi}(0)}$.
In the r.h.s. plot of figure 2 we also plotted our preliminary results for $f_+^{K\pi}(m_\pi,m_K)$ as a function of the pion mass squared, $(am_\pi)^2$. Our results suggest that $f_+^{K\pi}(0)$ receives $SU(3)$-breaking contributions beyond $f_2(m_\pi,m_K)$ which are of the same sign and of about the same magnitude as $f_2(m_\pi,m_K)$ (dashed red line), itself.

3. Outlook and conclusions

We presented our most recent data for the precision determination of the $Kl3$ form factor from lattice simulations of full ($N_f = 2+1$) QCD with good chiral properties. We will shortly finalize our study with a single lattice spacing. A more reliable estimate of cut-off effects will be possible in the near future after analysing the relevant correlation functions on a gauge field ensemble with a larger cut-off which is currently being generated by the RBC and UKQCD collaborations.

Since the CKM 2006 workshop [9] we extended our simulation by the data point at $am_l = 0.005$ which yields pions of mass $am_\pi \approx 300$MeV. We are currently increasing statistics on it. Once this data point is finalised it will help to further constrain the chiral extrapolation of our data.

We hope to be able to present final numbers including this data point at this year's Lattice conference.

Acknowledgements: We thank D.Chen, N.Christ, S.Cohen, C.Cristian, Z.Dong, A.Gara, A. Jackson, C. Jung, R.Kenway, C.Kim, L. Levkova, X. Liao, G. Liu, R.Mawhinney, S.Ohta, K.Petrov, T.Wettig and A.Yamaguchi for developing with us the QCDOC machine and its software. This development and the resulting computer equipment used in this calculation were funded by the U.S. DOE grant DE-FG02- 92ER40699, PPARC JIF grant PPA/J/S/1998/00756 and by RIKEN. This work was supported by DOE grants DE-FG02-92ER40699 and DE-AC02- 98CH10886 and PPARC grants PPA/G/O/2002/00465, PP/D000238/1, PP/C504386/1, PPA/G/S/2002/00467 and PP/D000211/1 and JSPS Grant-In-Aid for Scientific Research (C) (No.19540265). We thank BNL, EPCC, RIKEN, and the U.S. DOE for supporting the computing facilities essential for the completion of this work.

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