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ChPT Progress on Non-Leptonic and Radiative Kaon Decays

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I discuss recent developments on non-leptonic and radiative kaon decays mainly related to direct CP-violation within the combined ChPT and $1/N_c$ expansion approaches. In particular, I review the status of $K \to \pi\pi$, ε'_K, direct CP-violating $K^+ \to 3\pi$ Dalitz plot slope g and decay rate asymmetries, and the Standard Model prediction for $Br(K_L \to \pi^0 e^+ e^-)$.

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1. Motivation

Non-leptonic and radiative kaon decays have attracted a lot of attention in various respects. Testing the Standard Model (SM) and unveiling flavour structure beyond it is one of them. This can be done very effectively using precision tests of the scalar sector where *direct* CP-violating effects involving kaons provides with some of the most promising opportunities. Indeed, *direct* CP violation in kaon decays is experimentally very well known in $K \to \pi\pi$ [1, 2]

$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right) = (1.63 \pm 0.23) \times 10^{-3}. \tag{1.1}$$

I discuss the present theoretical status of the SM prediction for this quantity in Section 3.1 while CP-violating $K^+ \to 3\pi$ Dalitz plot slope g and decay rate asymmetries are in Section 3.2 .

As a typical example of radiative kaon decays, I discuss in Section 4 the theoretical advances predicting the CP-violating decay $K_L \to \pi^0\gamma^* \to \pi^0 e^+e^-$ within the SM.

A deeper understanding of the strong-weak dynamics interplay at low energy is also a very interesting aspect of the study of non-leptonic and radiative kaon decays. Finally, I also report on the recent theoretical advances based in large N_c approaches to low-energy QCD.

2. Theoretical Framework

The SM effective action at energies around or below the charm quark mass is well known. For the $\Delta S = 1$ sector, this has been done to next-to-leading order (NLO) in two renormalization schemes (NDR and HV) by two groups, [3] and [4]. It contains ten four-quark operators, Q_1 to Q_{10}, and two magnetic dipole operators, Q_{11} and Q_{12}, which are chirally suppressed, see e.g. [3] for definitions. In the presence of electroweak interactions, there appear another two operators, Q_{7V} and Q_{7A}, which contribute to radiative kaon decays, see e.g. [3]. Short-distance information enters via Wilson coefficients multiplying the operators of the effective action. This short-distance information is the one we want to extract from measurements of non-leptonic and radiative kaon decays.

For the explicit expression of the $\Delta S = 1$ SM effective action and a very detailed discussion of low-energy SM effective action see [3]. Here I use the same notation as there.

Chiral Perturbation Theory (ChPT) [5, 6] is the effective field theory that describes the SM interactions among the lowest-energy degrees of freedom: pions, kaons, photons, $\cdots$ For reviews with emphasis on kaon physics see [7]. There have been recent advances and a lot of work in understanding the long-distance–short-distance matching between the effective SM action and ChPT, both using analytical large N_c methods and lattice QCD – for lattice, see Chris Sachrajda and Bob Mawhinney's talks. As yet, there remains a lot of work to be done, mainly for rare kaon decays.

Within ChPT, one constructs the most general Lagrangian compatible with all SM symmetries and in particular, with the structure generated by the QCD chiral symmetry breaking $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \to \mathrm{SU}(3)_V$. ChPT provides then with a low-energy Taylor expansion of amplitudes in external momenta and meson masses which in general depends on unknown couplings. This is still very predictive because, at lower orders, there appear only few of them, e.g. just the pion $\pi^+ \to \mu^+\nu$ decay constant in the chiral limit, F_0, and the lowest pseudo-Goldstone boson octet masses in the strong sector at leading-order (LO). This fact allows to relate different decays with the same unknowns. Also in SU(3) but at next-to-leading order (NLO) and without electromagnetism (EM), ten additional physical couplings, L_1 to L_{10}, [6] are needed in the $\Delta S = 0$ sector. One more coupling appears when including EM at LO.

In other cases, it can be shown that LO chiral loops are finite and no unknown counterterm at that order appears –these are parameter free predictions at that order. To this class belong the radiative decays $K_S \to \gamma\gamma$ [8] and $K_L \to \pi^0\gamma\gamma$ [9]. For both decays there have been reported new measurements at this Conference. In the case of $K_S \to \gamma\gamma$, the KLOE result [10] nicely confirms the LO ChPT prediction while the KTeV preliminary result [11] agrees with a previous NA48 measurement pointing to the need of large NLO ChPT corrections. For a complete discussion of these two decays and for a comprehensive list of works applying ChPT to non-leptonic and rare kaon decays see [12].

At LO in the $|\Delta S| = 1$ SM sector and within SU(3), there appear three couplings [1] of order p^2 plus one of order $e^2 p^0$, namely, G_8, G'_8, G_{27} and G_E, respectively. The corresponding Lagrangian reads

$$\begin{aligned} \mathscr{L}^{(2)}_{|\Delta S|=1} &= C F_0^6\, e^2 G_E\, \mathrm{tr}\left(\Delta_{32} u^\dagger Q u\right) + C F_0^4 \, \Big[G_8 \mathrm{tr}\left(\Delta_{32} u_\mu u^\mu\right) \\ &+ \; G'_8\, \mathrm{tr}\left(\Delta_{32}\chi_+\right) + G_{27}\, t^{ij,kl}\, \mathrm{tr}\left(\Delta_{ij} u_\mu\right) \mathrm{tr}\left(\Delta_{kl} u^\mu\right)\Big] \end{aligned} \tag{2.1}$$

with $C = -(3/5) G_F V_{ud} V^*_{us}/\sqrt{2} \simeq -1.08\times 10^{-6}\,\mathrm{GeV}^{-2}$, $u_\mu \equiv i u^\dagger (D_\mu U) u^\dagger$, $U \equiv u^2 = \exp(i\sqrt{2}\Phi/F_0)$, $\Delta_{ij} = u \lambda_{ij} u^\dagger$, $(\lambda_{ij})_{ab} = \delta_{ia}\delta_{jb}$, $\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$, $\chi = \mathrm{diag}(m_u, m_d, m_s)$ and the $t^{ij,kl}$ tensor can be found in [14]. The SU(3) × SU(3) matrix Φ collects pion, kaon and eta pseudo-Goldstone boson fields. In this normalization, $G_8 = G_{27} = 1$ at large N_c. At NLO in ChPT, the $|\Delta S| = 1$ SM sector was constructed within SU(3) in [9, 15, 16].

3. Non-Leptonic Kaon Decays

3.1 $K \to \pi\pi$ and ε'_K: Status

The decays $K \to \pi\pi$ are fully known to NLO in ChPT, i.e. including isospin breaking effects from quark masses and EM [14, 17 – 20]. The rôle of final state interactions (FSI) in those decays is also clarified. For a recent summary of the theory status of both the $\Delta I = 1/2$ rule in kaons and ε'_K see [21].

In [22], the authors performed a combined fit to both data on $K \to \pi\pi$ and $K \to 3\pi$ which is also known fully at NLO in ChPT including isospin breaking [22, 23] and obtained

$$\mathrm{Re}\, G_8 = (7.0 \pm 0.6) \left(87\,\mathrm{MeV}/F_0\right)^4 \; ; \; G_{27} = (0.50 \pm 0.06) \left(87\,\mathrm{MeV}/F_0\right)^4 \tag{3.1}$$

which represents the $\Delta I = 1/2$ rule for kaons. Recent analytical advances on the quantitative understanding of this rule can be found in [24, 25] using $1/N_c$ approaches. In particular, the $\Delta I = 1/2$ rule is reproduced within 40 % in [21, 24] at NLO in $1/N_c$ using the ENJL model [26] at low energies and with analytical short-distance independence.

Using the calculations quoted above, one can get the prediction for ε'_K fully at NLO in ChPT

$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right) \simeq -\left[(1.9 \pm 0.5)\ \mathrm{Im} G_8 + (0.34 \pm 0.15)\, \mathrm{Im}(e^2 G_E)\right] \tag{3.2}$$

[1] There appears one more octet singlet coupling within U(3), see [13].

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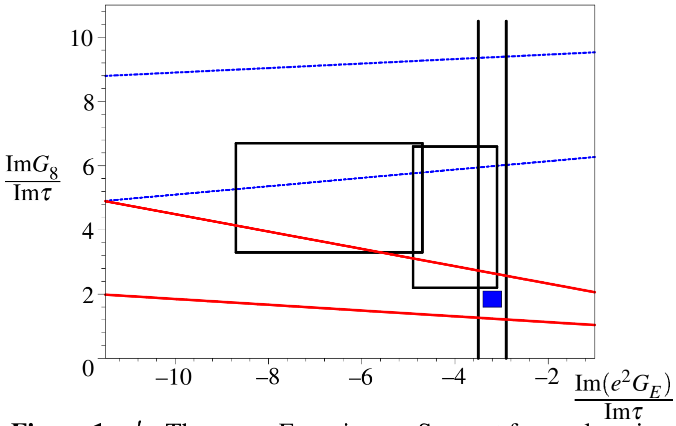

Figure 1: ε'_K: Theory vs Experiment. See text for explanation.

where $\mathrm{Im}\, G_8$ and $\mathrm{Im}\,(e^2 G_E)$ are proportional to the CP-violating phase $\mathrm{Im}\,\tau \equiv -\mathrm{Im}\,(\lambda_t/\lambda_u)$ with $\lambda_i \equiv V_{id} V_{is}^*$ and V_{ij} are Cabibbo–Kobayashi–Maskawa matrix elements. Notice that it does not appear any p^4 counterterm $\mathrm{Im}\,\widetilde{K}_i$ –see [22] for their definition– in the previous NLO in ChPT expression for $\mathrm{Re}\,(\varepsilon'_K/\varepsilon_K)$ because they have been estimated to be negligible within large N_c [18].

Putting together the experimental result in (1.1) and the NLO ChPT formula in (3.2), one obtains that the pair ($\mathrm{Im}\,(e^2 G_E)$, $\mathrm{Im}\, G_8$) has to lie between the two lower horizontal (red) lines in Fig. 1. An immediate consequence of (3.2) is that for typical values of $\mathrm{Im}\,(e^2 G_E)$ and $\mathrm{Im}\, G_8$ –say large N_c values– though there is some cancellation between the two terms there, it is however not as large as it was previously thought and still sometimes argued.

Recently, several analytical works have been devoted to calculating $\mathrm{Im}\,(e^2 G_E)$ [27 – 30] –see [29] for a comparison– and $\mathrm{Im}\, G_8$ [25, 31], both at NLO in the $1/N_c$. The nice feature of $\mathrm{Im}\,(e^2 G_E)$ is that it can be related via dispersion relations to $VV - AA$ spectral two-point function in the chiral limit [27, 28, 30]. The results found for the pair ($\mathrm{Im}\,(e^2 G_E)$, $\mathrm{Im}\, G_8$) in [28, 31] are represented in Fig. 1 by the rectangle on the right while the results in [25, 29] are represented by the rectangle on the left. In these two calculations, part of the large uncertainties come from two input parameters, namely, the quark condensate in the chiral limit which present uncertainty is around 20 % and enters squared and L_5 which uncertainty is around 45 %. The large N_c result is the (blue) filled square to which I have not assigned any uncertainty since it does not include the NLO in $1/N_c$ different planar topology. The lattice result for $\mathrm{Im}\,(e^2 G_E)$ [32] is also shown in Fig. 1, it lies between the two vertical lines. Unfortunately, we still don't have a reliable value for $\mathrm{Im}\, G_8$ from lattice QCD but one can assess from Fig. 1 what that value has to be if compatible with the measurement of ε'_K.

3.2 $K \to 3\pi$ and Direct CP-Violating Dalitz-Plot Slope g Asymmetries

Non-leptonic $K \to 3\pi$ decays have also attracted a lot of work recently. Theses decays were calculated at NLO in ChPT in [17] but unfortunately the complete expressions were not available and as mentioned above, recently they were redone in [22, 23]. Using those calculations, one can predict the Dalitz plot slopes –see e.g. [23] for their definition– at NLO in ChPT for $K^+ \to \pi^+\pi^+\pi^-$ and $K^+ \to \pi^0\pi^0\pi^+$ which are in very good agreement with recent measurements [33].

It is possible to define CP-violating asymmetries using the Dalitz plot slope g [23, 34]. Previous predictions within the SM were done using LO ChPT plus various NLO estimates [34]. There is work looking for large SUSY effects in this asymmetries as well [35]. The first full NLO in ChPT results were presented in [23] where one can also find the $K^+ \to 3\pi$ decay rate CP-violating asymmetries. At NLO in ChPT in the isospin limit, one gets for the $K^+ \to \pi^+\pi^+\pi^-$ slope g_C

$$10^2 \times \Delta g_C \simeq (0.7\pm0.1)\,\mathrm{Im}\,G_8 - (0.07\pm0.02)\,\mathrm{Im}\,(e^2 G_E) + (4.3\pm1.6)\,\mathrm{Im}\,\widetilde{K}_2 - (18.1\pm2.2)\,\mathrm{Im}\,\widetilde{K}_3 \tag{3.3}$$

where $\widetilde{K}_i$ are order p^4 counterterms [22, 23]. More details and similar expressions for Δg_N and the decay rate asymmetries can be found in [23]. It turns out that Δg_C is quite stable against unknown NLO ChPT counterterms while Δg_N is somewhat less stable [23]. The results obtained are [23]

$$\Delta g_C = -(2.4\pm1.2)\times10^{-5};\ \Delta g_N = (1.1\pm0.7)\times10^{-5}. \tag{3.4}$$

Variation of input values and other uncertainties are within the quoted error. Experimentally, the final results of the NA48/2 experiment were presented at this Conference [36],

$$\Delta g_C = -(1.5\pm2.1)\times10^{-4};\ \Delta g_N = (1.8\pm1.8)\times10^{-4}. \tag{3.5}$$

which are compatible with previous measurements [37] but with significantly smaller uncertainty.

In Fig. 1, the region between the two upper horizontal (blue) lines is where the pair ($\mathrm{Im}\,(e^2 G_E)$, $\mathrm{Im}\,G_8$) would have to lie if $\Delta g_C = -(4.0\pm0.5)\times10^{-5}$ was measured. Any measurement of Δg_C between this value and the present experimental limits would lead to an allowed region for the pair ($\mathrm{Im}\,(e^2 G_E)$, $\mathrm{Im}\,G_8$) which moves toward the upper side of that figure when the modulus of Δg_C increases. Therefore, if we require this region to cross with the allowed region for ε'_K then we would need a negative very large value in modulus for $\mathrm{Im}\,(e^2 G_E)/\mathrm{Im}\,\tau$. Using the results from calculations of this coupling both using analytic techniques [27 – 30] and lattice QCD –see Chris Sachrajda and Bob Mawhinney's talks at this Conference, this would clearly call for the presence of new physics independently of the hadronic uncertainties in $\mathrm{Im}\,G_8$. This plot also points to an experimental accuracy of around 0.2×10^{-4} as the goal to be reached.

4. Radiative Kaon Decays

As a typical example of radiative kaon decay, I discuss here the status and make some comments on the $K \to \pi\gamma^* \to \pi\ell^+\ell^-$ decays. ChPT at LO plus NLO dominant effects analysis have been done and unknown couplings appear [16, 38]. The short-distance contribution to the SM effective action description is also known at NLO order in two schemes (HV and NDR) [3, 39, 40]. On the experimental side, the CP-conserving $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$, which are dominated by the long distance process $K \to \pi\gamma^* \to \pi\ell^+\ell^-$, have been measured.

At LO in ChPT a single coupling governs the $K \to \pi\gamma^*$ form factor [16]. In the case of $K^+ \to \pi^+\ell^+\ell^-$ this coupling is of order N_c and was called ω_+. The authors of [38] pointed out that adding a NLO momenta dependent term to the form factor improves considerably the fit. Including this NLO term and using the measurement at BNL [41], one gets $\mathrm{Re}\,\omega_{+,e} = 1.49\pm0.02$ or equivalently, $\mathrm{sign}(G_8)\,a_{+,e} = -(0.59\pm0.01)$. Subscript e refers to the electron mode, see [38] for the definition of a_+. The corresponding decay into muons has also been measured giving compatible results [42]. Notice that both ω_+ and $\mathrm{sign}(G_8)\,a_+$ are global sign convention independent.

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Analogously, one can obtain the coupling that governs the $K^0 \to \pi^0\gamma^*$ form factor at LO from the measurement of $K_S \to \pi^0 e^+e^-$, in this case this coupling is of order one in $1/N_c$ and was called ω_S. The different N_c counting of ω_+ and ω_S already tells us that they are unrelated as noticed in [43, 44]. In this case, NLO momenta dependent terms in the form factor cannot be determined from a fit to data due to the smallness of the non-analytic contributions [45]. The result one gets using the NA48/1 results [46] has a twofold ambiguity $\mathrm{Re}\,\omega_{S,e} = [2.53^{+0.56}_{-0.45}, -(1.87^{+0.56}_{-0.45})]$ which does not fix the sign of the coupling. Equivalently, using the notation of [38] one gets $|a_{S,e}| = 1.12^{+0.29}_{-0.23}$. The corresponding decay into muons has also been measured giving compatible results [47].

The closely related CP-violating $K_L \to \pi^0\ell^+\ell^-$ decay has received a great deal of attention both within the SM [16, 38 – 40, 44, 45, 48 – 51] and as tool of unveiling beyond the SM flavour structure [52]. A pretty precise prediction for this decay within the SM can be made. In particular, it was shown in [45, 48] that the CP-conserving $K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+e^-$ decay contribution is negligible. Updating [45, 50] and using [51], one gets

$$Br(K_L \to \pi^0 e^+e^-) = \Bigg[(3.41\pm0.03)\,W_{S,e}^2 + (3.91\pm0.05)\,W_{S,e}\,(\widehat{y}_{7V}+M_{6V})\left(\frac{\mathrm{Im}\lambda_t}{10^{-4}}\right) + (2.36\pm0.06)\,\left[\widehat{y}_{7A}^2+(\widehat{y}_{7V}+M_{6V})^2\right]\left(\frac{\mathrm{Im}\lambda_t}{10^{-4}}\right)^2\Bigg]\times10^{-12} \quad (4.1)$$

with $W_{S,e}^2 \equiv 10^9\times Br(K_S\to\pi^0e^+e^-)/1.20$ and, to a very good approximation, $W_{S,e} = \mathrm{Re}\,\omega_{S,e} - 1/3$, and $\widehat{y}_{7V(A)} \equiv y_{7V(A)}/\alpha$ [3]. The term M_{6V} is the hadronic penguin operator Q_6 contribution to the direct CP-violating term. The Q_{7V} relevant matrix element is $3/4\pi\alpha$ and the Q_6 one is, at large N_c,

$$\langle Q_6\rangle\big|_{N_c}(\nu) = 32\frac{\langle\overline{q}q\rangle^2(\nu)}{F_0^6}\,[2C_{63}^r - C_{65}^r]\,(M_\rho)\,. \quad (4.2)$$

where C_{63}^r and C_{65}^r are two $\Delta S=0$ $\mathscr{O}(p^6)$ couplings [53]. This same combination of counterterms appears in the EM K^0 charge radius NLO ChPT calculation [54]. Using the PDG [55, 56] experimental value, one gets

$$[2C_{63}^r - C_{65}^r]\,[M_\rho] = (1.8\pm0.7)\,(F_0/87\,\mathrm{MeV})^2\times10^{-5} \quad (4.3)$$

which together with (4.2) yields

$$\frac{M_{6V}}{\widehat{y}_{7V}} \equiv \frac{y_6(\nu)\,\langle Q_6\rangle(\nu)}{y_{7V}(\nu)\,\langle Q_{7V}\rangle} = -(0.2\pm0.1)\,B_{6V} \quad (4.4)$$

where B_{6V} parameterizes non-factorisable corrections. This contribution, which has been argued before to be negligible [3, 40, 45], adds to the direct CP-violating vector part and could be as large as $-(30\sim50)$ % of the Q_{7V} contribution depending of the unknown B_{6V} factor.

The interference term in (4.1) is constructive (destructive) if $\mathrm{Re}\,\omega_{S,e}$ is larger (smaller) than 1/3. Or equivalently, if $\mathrm{sign}(G_8)\,a_{S,e}$ is positive (negative). The Q_{7V} contribution is model independent and gives $\mathrm{sign}(G_8)\,a_S^{Q_{7V}} > 0$, i.e. constructive interference [39]. Assuming VMD for the $K_S\to\pi^0\gamma^*$ form factor and a large non-VMD contribution for the $K^+\to\pi^+\gamma^*$ plus the Q_{7V} relation $a_S^{Q_{7V},VMD} = -a_+^{Q_{7V},VMD}$ produces $\mathrm{sign}(G_8)\,a_S > 0$ and therefore constructive interference if one furthermore identifies $a_S^{Q_{7V},VMD}$ and $a_+^{Q_{7V},VMD}$ with the experimental values for a_S and a_+,

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respectively [45]. This identification is not trivial as these couplings receive sizable contributions from the hadronic operators Q_2 and Q_6 which do not fulfill the above Q_{7V} relation between a_S and a_+ [43, 57].

In [49], the authors saturated $K \to \pi\gamma^*$ form factor by K^* and ρ meson single poles within a large N_c inspired minimal hadronic approximation which used to make a fit to data. They got $\mathrm{Re}\,\omega_+ = 1.4 \pm 0.6 > 1/3$ [i.e. $\mathrm{sign}(G_8)\, a_+ = -(0.5 \pm 0.3) < 0$] and $\mathrm{Re}\,\omega_S = -(2.1 \pm 0.2) < 1/3$ [i.e. $\mathrm{sign}(G_8)\, a_S = -(1.2 \pm 0.1) < 0$] which implies destructive interference [2].

In [43], in addition to the contribution of Q_{7V}, a four-quark effective action model was used to calculate the contributions from $Q_{i=1,\cdots,6}$. These authors predicted $\mathrm{Re}\,\omega_+ = 1.5^{+1.2}_{-0.6} > 1/3$ [i.e. $\mathrm{sign}(G_8)\, a_+ = -(0.6^{+0.6}_{-0.3}) < 0$] and $\mathrm{Re}\,\omega_S = 1.6^{+1.4}_{-0.6} > 1/3$ [i.e. $\mathrm{sign}(G_8)\, a_S = 0.6^{+0.7}_{-0.3} > 0$] which implies constructive interference. In particular, the large N_c result for $\langle Q_6 \rangle$ in [43] is equivalent to

$$[2C_{63}^r - C_{65}^r]\,[1\mathrm{GeV}] = (2.2 \pm 1.1)\,(F_0/87\,\mathrm{MeV})^2 \times 10^{-5} \tag{4.5}$$

which compares well with (4.3). In fact, it is easy to see that Q_6 with the large N_c result in (4.2) together with (4.3) contributes to $\mathrm{Re}\,\omega_S$ with the same sign as Q_{7V} and comparable magnitude. An analysis of the rest of contributions to ω_S from four-quark operators can be performed at NLO in $1/N_c$ [57] using the approaches developed in [28, 31, 58].

As pointed out in [50], one can determine experimentally the sign of the interference term in (4.1) using the $K_L \to \pi^0\mu^+\mu^-$ forward-backward asymmetry. Actually, this study can also serve to fix the long-distance contribution to the direct CP-violating term M_{6V} which has to be treated as a further unknown at present. Both, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0\mu^+\mu^-$ modes become then necessary to disentangle new physics [50] and long-distance from short-distance direct CP-violating terms. Using $\mathrm{Im}\,\lambda_t = (1.4 \pm 0.2) \times 10^{-4}$ [55], $\hat{y}_{7A} = -(0.68 \pm 0.03)$, $\hat{y}_{7V} = 0.73 \pm 0.04$ [3, 40] in (4.1) with constructive [destructive] interference, one gets predictions for $Br(K_L \to \pi^0 e^+ e^-)$ between $(2.7^{+0.9}_{-0.7}) \times 10^{-11}$ and $(2.5^{+0.9}_{-0.7}) \times 10^{-11}$ [$(1.3^{+0.9}_{-0.7}) \times 10^{-11}$ and $(1.4^{+0.9}_{-0.7}) \times 10^{-11}$], if one varies B_{6V} between one and two. The present experimental limit is $Br(K_L \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10}$ [59].

4.1 Some Selected Topics

Here, I would like to comment very briefly on two selected topics. First, the NA48/2 very recent first measurement of a *destructive* direct electric emission interference in $K^+ \to \pi^+\pi^0\gamma$ [60]. This interference depends on the sign of one unknown ChPT coupling [61] and naive theoretical predictions tend to tell that it is *constructive* [43, 61]. Clearly, more theory work is needed here.

Secondly, interesting recent work on $\mathrm{U}_A(1)$ anomaly effects in radiative kaon decays using U(3) ChPT was done in [13] reaching a better understanding of $K_L \to \gamma\gamma$ and $K_L \to \gamma\ell^+\ell^-$. One of the conclusions reached there is that $\mathrm{U}_A(1)$ anomaly effects could be sizable in $K_S \to \pi^0\gamma\gamma$ and $K^+ \to \pi^+\gamma\gamma$ and more experimental input on these modes is very welcome.

5. Conclusions

To reach the goals of non-leptonic and radiative kaon decays studies, i.e. to obtain new flavour structure (CP-violating phases) information and/or understand the strong-weak dynamics interplay,

[2]Notice that the sign of the interference term in (4.1) agrees with [38, 39, 45] but is opposite to that used in [49].

one needs in general to combine different modes to disentangle SM from new physics and/or long-distance from short-distance effects. This strategy is both complementary and necessary, see for instance [23, 50, 52] where the cases ε'_K vs $K^+ \to 3\pi$ CP-violating Dalitz plot slopes asymmetries, $K_L \to \pi^0 e^+ e^-$ vs $K_L \to \pi^0 \mu^+ \mu^-$ and ε'_K vs $K_L \to \pi^0 e^+ e^-$ have been studied, respectively.

At the same time, it is obvious the need of theoretical effort predicting unknown ChPT couplings in order to take profit of high precision measurements such as ε'_K, and eventual measurements of the CP-violating Dalitz plot slopes asymmetries, $K_L \to \pi^0 \ell^+ \ell^-$, $\cdots$ as unique probes unveiling physics beyond the SM. For recent efforts in that direction using large N_c hadronic approaches see [58, 62, 63] and references therein. Lattice proposals to study radiative kaon decays also appeared [64] while I refer to Chris Sachrajda and Bob Mawhinney's talks at this Conference for non-leptonic kaon decays lattice efforts.

As a final remark, I believe that with the expected theory and experimental efforts, non-leptonic and radiative kaon decays will continue provide with very nice and interesting physics.

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