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Exploring the QCD phase structure with density fluctuations

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We briefly summarize the properties of conserved charge fluctuations as a sensitive probe for the QCD phase transitions. We discuss the denisty fluctuations which play a significant role to search for the critical end point. The importance of spinodal instabilities to distinguish the first-order phase transition is also indicated.

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1. Introduction

The existence of a critical end point (CEP) in QCD phase diagram is one of the striking expectations [1, 2], which has been explored based on calculations in effective models and on universality arguments. The appearance of the CEP in the temperature T and quark chemical potential μ_q plane was also studied in terms of Lattice Gauge Theory (LGT) [3]. The search for the CEP has recently attracted considerable attention in the context of heavy ion phenomenology [2]. Thus, it is of particular interest to identify the position of the CEP in the phase diagram and to study generic properties of thermodynamic quantities in its vicinity. The analysis of fluctuations is a powerful method for characterizing the thermodynamic properties of a system. Modifications in the magnitude of fluctuations or the corresponding susceptibilities have been suggested as a possible signal for deconfinement and chiral symmetry restoration [4, 2, 5, 6]. In this context, fluctuations related to conserved charges are of particular interest [7].

In this contribution we briefly summarize the properties of the conserved charge fluctuations to probe the QCD phase structure. The role of the fluctuations in order to identify the location of the CEP as well as the phase boundaries is discussed. We also show that the enhanced baron or electric charge density fluctuations could signal the first order phase transition in the presence of spinodal decomposition.

2. Fluctuations of conserved charges

In our study of fluctuations we adopt the Nambu–Jona-Lasinio (NJL) model as an effective chiral model under the mean field approximation [8]. The model describes the chiral phase transition where the dynamically generated quark mass M acts as an order parameter. In Fig. 1 we show the phase diagram of the two-flavored NJL model for an isosymmetric system in the chiral limit. The position of the phase boundary and the tricritical point (TCP) depends crucially on the model parameters, like e.g. on the strength of the four-fermion interactions. In the figure we illustrate the dependence on the scalar-isoscalar G_S and vector-isoscalar G_V couplings. With increasing G_V, the phase transition line at fixed T is shifted to larger μ_q due to strong repulsive forces among the constituent quarks. Consequently, for sufficiently large value of the vector coupling the TCP disappears from the phase diagram [9, 10].

The position of the phase boundary and the order of the chiral phase transition can be also identified through thermodynamic observables, like net baryon number fluctuations which are sensitive probes of the phase transition [11, 12, 4, 2, 6]. Furthermore, fluctuations of conserved charges are directly accessible in experiments. Thus, it is of importance to explore the behavior of such fluctuations in the vicinity of the phase boundary.

The quark number and iso-vector susceptibilities, χ_q and χ_I respectively, describe the response of the net quark density n_q and the isovector density n_I to the change of the corresponding chemical potentials. Thus, χ_q and χ_I are defined as derivatives of n_q and n_I with respect to μ_q and μ_I;

$$\chi_q = \frac{\partial n_q}{\partial \mu_q}, \qquad \chi_I = \frac{\partial n_I}{\partial \mu_I}, \tag{2.1}$$

with μ_q and μ_I being the net quark and isovector chemical potential respectively.

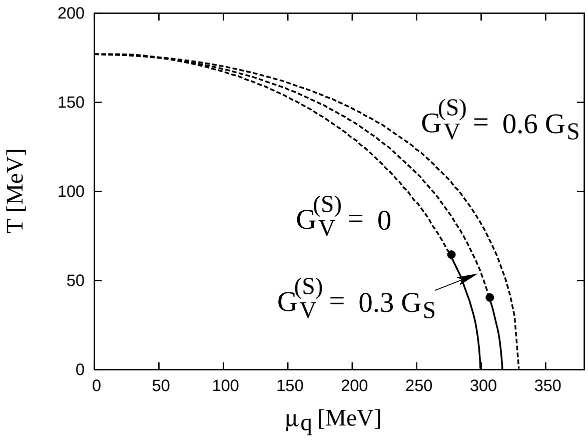

Figure 1: The phase diagram of the NJL model for an isosymmetric system in the chiral limit [10]. The tricritical point is indicated by a dot.

The temperature dependence of χ_q shows characteristic features, which vary rapidly with μ_q. The phase boundary is signaled by a discontinuity in the susceptibility. The size of the discontinuity grows with increasing μ_q up to the TCP, where the susceptibility diverges. Beyond the TCP the discontinuity is again finite. On the other hand, at $\mu_q = 0$ the discontinuity vanishes and the susceptibility shows a weaker non-analytic structure at the transition temperature, resulting in a discontinuity of $\partial\chi_q/\partial T$. These critical properties of χ_q are consistent with that expected for a second order phase transition belonging to the universality class of $O(4)$ spin model in three dimensions [2, 6, 13].

In Fig. 2 we show the net quark and isovector susceptibilities along the phase boundary. The singularity of χ_q indicates the existence of the TCP. In the absence of a TCP, the net quark susceptibility would be a monotonic function of T along the phase boundary, as illustrated in the Fig. 2-left by a dashed-dotted line. We note that on the qualitative level the critical behavior of the net quark number susceptibility can be also obtained in the Landau theory [6, 10]: First, the discontinuity across the phase boundary vanishes at $\mu_q = 0$. Second, the singularity of χ_q shows up only in the chirally broken phase, while the susceptibility in the symmetric phase is monotonous along the phase boundary and shows no singular behavior. The isovector fluctuations χ_I, contrary to χ_q, are neither singular nor discontinuous at the chiral phase transition for finite chemical potential. We find a rather smooth increase of χ_I with increasing μ_q along phase boundary line. At the TCP the χ_I remains finite. The non-singular behavior of χ_I at the TCP is consistent with the observation that there is no mixing between isovector excitations and the isoscalar sigma field due to isospin conservation [14]. Recent LGT results [12] also show a smooth change of the isovector fluctuations around deconfinement transition and a fairly weak dependence of χ_I on the quark chemical potential μ_q.

The net quark number χ_q and the isovector χ_I susceptibilities are related with fluctuations of the electric charge χ_Q as

$$\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I + \frac{1}{6}\frac{\partial^2 P}{\partial\mu_q\partial\mu_I}\,, \tag{2.2}$$

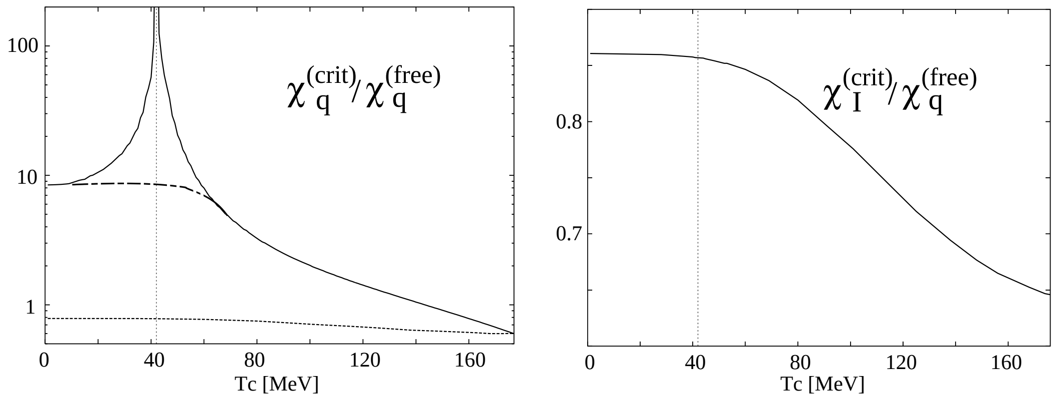

Figure 2: (Left) The net quark number susceptibility χ_q along the phase boundary in the chiral limit. The solid (dashed) line denotes χ_q in the chirally broken (symmetric) phase. The vertical dotted line indicates the position of the TCP. (Right) The isovector susceptibility along the phase boundary [10].

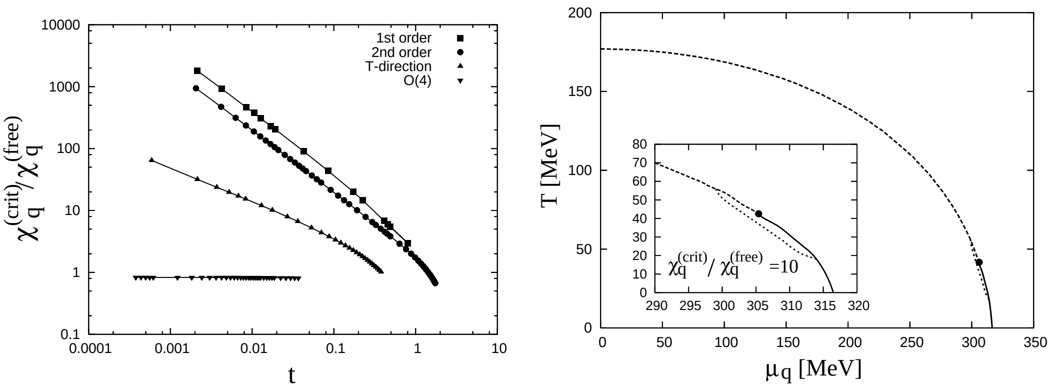

Figure 3: (Left) The quark number susceptibility near the TCP as a function of the reduced temperature. (Right) The critical region where the χ_q is enhanced by an order of magnitude compared to the free one [10].

where P is the thermodynamic pressure. For isospin symmetric system the last term vanishes. Hence in this case all relevant susceptibilities are linearly dependent. Clearly, since χ_I is finite at the TCP, the electric charge fluctuations χ_Q diverge with the same critical behavior as χ_q. At finite μ_I the properties of χ_I at the chiral phase transition change. Then, since the isoscalar sigma field mixes with the isospin density [14], the isovector susceptibility exhibits a similar structure as χ_q, with a singularity at the TCP.

The strength of singularities is governed by the critical exponents whose values are different depending on paths approaching the phase transition [15]. The mean-field exponents of the TCP and the CEP can be obtained from Landau theory [10]. In Fig. 3-left we illustrate the critical behavior of χ_q near the O(4) critical line and at the TCP. For paths approaching a TCP asymptotically tangential to the phase boundary, the quark number susceptibility diverges with the critical exponent $\gamma_q = 1$. On the other hand, approaching the TCP along the first-order transition line, the pre-factor of the singular contribution to the quark susceptibility is twice as large as that obtained when approaching TCP along the O(4) critical line. For other paths the critical exponent is $\gamma_q = \frac{1}{2}$. At the O(4) critical line, the susceptibility remains finite. The corresponding critical exponent of the O(4) universality class is $\alpha \simeq -0.2$, while in the NJL model under the mean-field approximation, $\alpha = 0$. For non-zero quark mass, at the critical endpoint, the mean-field critical exponent along a path not tangential to the phase boundary is $2/3$, while along the phase boundary it remains equal to unity [2]. When quantum fluctuations are included, the first exponent is renormalized to that of the 3D Ising model universality class [16], i.e. $\varepsilon = 0.78$.

In Fig. 3-right we show the “critical” region, where the susceptibility exceeds its value in the ideal quark gas by more than an order of magnitude [1]. The differences in values of the critical exponents for different "paths" are reflected in the shape of the critical region around TCP. It is elongated along the phase boundary, where the singularity is strongest.

3. Baryon number susceptibility in the presence of spinodal instabilities

In the previous section we have argued that the enhancement of the baryon number fluctuations could be a clear indication for the existence of the critical end point in the QCD phase diagram. However, the finite density fluctuations along the first order transition appear under the assumption that this transition happen in equilibrium. In non-equilibrium system, a first order phase transition is intimately linked with the existence of a convex anomaly in the thermodynamic pressure [17]. There is an interval of energy density or baryon number density where the derivative of the pressure, $\partial P/\partial V > 0$, is positive. This anomalous behavior characterizes a region of instability in the (T, n_q)-plane which is bounded by the spinodal lines, where the pressure derivative with respect to volume vanishes. The derivative taken at constant temperature and that taken at constant entropy,

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \qquad \text{and} \qquad \left(\frac{\partial P}{\partial V}\right)_S = 0\,, \tag{3.1}$$

define the isothermal and isentropic spinodal lines respectively.

For finite vale of the quark mass and within large range of parameters the NJL model exhibits a critical end point (CEP) that separates the cross over from the first order chiral phase transition. The relevant part of the phase diagram in the (T, n_q)–plane is shown in Fig. 4. If the first order phase transition takes place in equilibrium, there is a coexistence region, which ends at the CEP. However, in a non-equilibrium first order phase transition, the system supercools/superheats and, if driven sufficiently far from equilibrium, it becomes unstable due to the convex anomaly in the thermodynamic pressure. In other words, in the coexistence region there is a range of densities and temperatures, bounded by the spinodal lines, where the spatially uniform system is mechanically unstable.

In Fig. 5-left we show the evolution of the net quark number fluctuations along a path of fixed $T = 50$ MeV in the (T, n_q)–plane. When entering the coexistence region, there is a singularity in χ_q that appears when crossing the isothermal spinodal lines and where the fluctuations changes the sign. In between the spinodal lines, the susceptibility is negative. Consequently, this implies

[1] By “critical” region we mean here the region where the susceptibility is large due to fluctuations and not the region where the critical exponents deviates from their mean-field values.

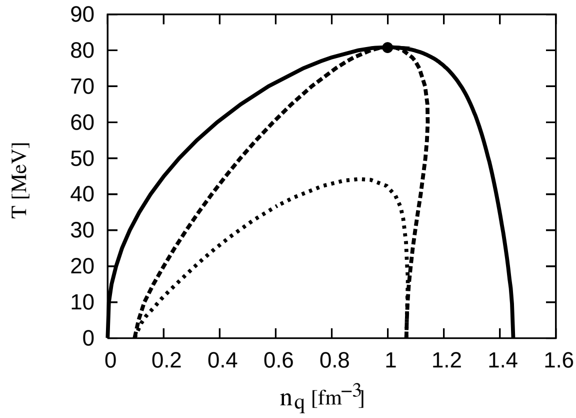

Figure 4: The phase diagram of the NJL model [18]. The filled point indicates the CEP. The full lines starting at the CEP represent boundary of the coexistence region in equilibrium. The dashed curves are the isothermal whereas the dotted ones are the isentropic spinodal lines.

instabilities in the baryon number fluctuations when crossing from meta-stable to unstable mixed phase. The above behavior of χ_q is a direct consequence of the thermodynamics relation

$$\left(\frac{\partial P}{\partial V}\right)_T = -\frac{n_q^2}{V}\frac{1}{\chi_q}\,. \tag{3.2}$$

Along the isothermal spinodals the pressure derivative in Eq. (3.2) vanishes. Thus, for non-vanishing density n_q, χ_q must diverge to satisfy (3.2). Furthermore, since the pressure derivative $\partial P/\partial V|_T$ changes sign when crossing the spinodal line, there must be a corresponding sign change in χ_q, as seen in Fig. 5-left. Due to the linear relation between χ_q, the isovector susceptibility χ_I and the charge susceptibility χ_Q (2.2), the charge fluctuations are also divergent at the isothermal spinodals. Thus, in heavy-ion collisions, fluctuations of the baryon number and electric charge could show enhanced fluctuations across the 1st order transition if the spinodal decomposition appears in a system.

In Fig. 5-right we show the evolution of the singularities from the spinodal lines when approaching the CEP. The critical exponent at the isothermal spinodal line is found to be $\gamma = 1/2$, with $\chi_q \sim (\mu - \mu_c)^{-\gamma}$, while $\gamma = 2/3$ at the CEP [18]. Thus, the singularities at the two spinodal lines conspire to yield a somewhat stronger divergence as they join at the CEP. The critical region of enhanced susceptibility around the TCP/CEP is fairly small [16, 10], while in the more realistic non-equilibrium system one expects fluctuations in a larger region of the phase diagram, i.e., over a broader range of beam energies, due to the spinodal instabilities.

The rate of change in entropy with respect to temperature at constant pressure gives the specific heat expressed as

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \left(\frac{s}{n_q}\right)^2\chi_q\right]\,. \tag{3.3}$$

The entropy χ_{TT} and mixed $\chi_{\mu T}$ susceptibilities exhibit the same behaviors as that of χ_q shown in Fig. 5-left. Thus C_P also divergences on the isothermal spinodal lines and becomes negative in the

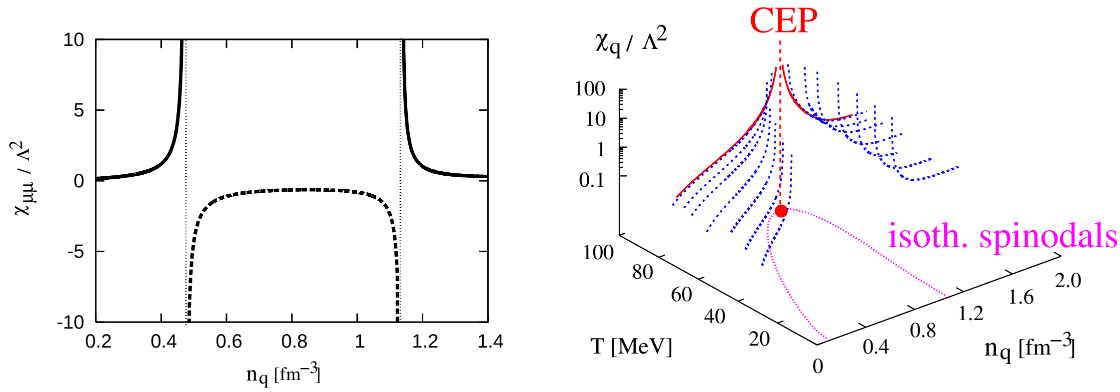

Figure 5: (Left) The net quark number susceptibility at $T = 50$ MeV as a function of the quark number density across the first order phase transition. (Right) The net quark number susceptibility in the stable and meta-stable regions [18].

mixed phase [2]. It was argued that in low energy nuclear collisions the negative specific heat could be a signal of the liquid-gas phase transition [19]. Its occurrence has recently been reported as the first experimental evidence for such an anomalous behavior [20].

4. Conclusions

We presented a brief discussion of probing the QCD phase structure. We have especially discussed the importance of conserved charge fluctuations. It was shown that the net baryon number susceptibility must yield large contribution around the critical end point (CEP). Consequently, a non monotonic behavior of these fluctuations as functions of the collision energy in heavy ion collisions could be considered as an indication of the CEP in the QCD phase diagram.

We have also shown that in the presence of spinodal instabilities the above picture is modified: The net quark number fluctuations diverge at the isothermal spinodal lines of the first order chiral phase transition. As the system crosses this line, it becomes unstable with respect to spinodal decomposition. The unstable region is in principle reachable in non-equilibrium systems that is most likely created in heavy ion collisions. Consequently, large fluctuations of baryon and electric charge densities are expected not only at the CEP but also when system crosses a non-equilibrium first order transition.

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[2] The specific heat with constant volume, on the other hand, continuously changes with n_q and has no singularities on the mean-field level.

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References

[1] J. Berges and K. Rajagopal, Nucl. Phys. B **538**, 215 (1999).

[2] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998); M. Stephanov, Acta Phys.Polon. B **35**, 2939 (2004); Prog. Theor. Phys. Suppl. **153**, 139 (2004); Int. J. Mod. Phys. A **20**, 4387 (2005).

[3] C. R. Allton *et al.*, Phys. Rev. D **66**, 074507 (2002); R.V. Gavai and S. Gupta, Phys. Rev. D **71**, 114014 (2005); Z. Fodor and S. D. Katz, JHEP **0203**, 014 (2002), JHEP **0404**, 050 (2004).

[4] T. Kunihiro, Phys. Lett. B **271** 395 (1991).

[5] G. E. Brown and M. Rho, Phys. Rept. **363**, 85 (2002).

[6] Y. Hatta and T. Ikeda, Phys. Rev. D **67**, 014028 (2003); H. Fujii, Phys. Rev. D **67**, 094018 (2003); H. Fujii and M. Ohtani, Phys. Rev. D **70**, 014016 (2004).

[7] S. Jeon and V. Koch, *Quark Gluon Plasma 3*, Eds. R.C. Hwa and X. N. Wang, World Scientific Publishing, 2004.

[8] For reviews and applications of the NJL model to hadron physics, see e.g., U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991); S. P. Klevansky, Rev. Mod. Phys. **64** (1992) 649; T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994); M. Buballa, Phys. Rept. **407**, 205 (2005).

[9] M. Asakawa and K. Yazaki, Nucl. Phys. A **504**, 668 (1989); M. Lutz, S. Klimt and W. Weise, Nucl. Phys. A **542**, 521 (1992); M. Kitazawa *et al.*, Prog. Theor. Phys. **108**, 929 (2002).

[10] C. Sasaki, B. Friman and K. Redlich, Phys. Rev. D **75**, 054026 (2007).

[11] S. A. Gottlieb *et al.*, Phys. Rev. Lett. **59**, 2247 (1987), Phys. Rev. D **55** 6852 (1997); R. V. Gavai and S. Gupta, Phys. Rev. D **65** 094515 (2002); C. R. Allton *et al.*, Phys. Rev. D **68**, 014507 (2003); M. D'Elia and M. P. Lombardo, Phys. Rev. D **67**, 014505 (2003); R.V. Gavai and S. Gupta, Phys. Rev. D **72** 054006 (2005), Eur. Phys. J. C **43** 31 (2005), Phys. Rev. D **73** 014004 (2006).

[12] C. R. Allton *et al.*, Phys. Rev. D **71**, 054508 (2005);

[13] F. Karsch, S. Ejiri and K. Redlich, Nucl. Phys. A **774**, 619 (2006).

[14] Y. Hatta and M.A. Stephanov, Phys. Rev. Lett. **91** 102003 (2003).

[15] R. B. Griffiths and J. C. Wheeler, Phys. Rev. A **2**, 1047 (1970).

[16] B. J. Schaefer and J. Wambach, Phys. Rev. D **75**, 085015 (2007).

[17] P. Chomaz, M. Colonna and J. Randrup, Phys. Rept. **389**, 263 (2004).

[18] C. Sasaki, B. Friman and K. Redlich, arXiv:hep-ph/0702254.

[19] P. Chomaz and F. Gulminelli, Nucl. Phys. A **647**, 153 (1999).

[20] M. Schmidt *et al.*, Phys. Rev. Lett. **86** (2001) 1191.