PROCEEDINGS OF SCIENCE

# Transverse Momentum of Partons and Single-Spin Asymmetries

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We find the light-cone wavefunction representations of the Sivers and the Boer-Mulders distribution functions. A necessary condition for the existence of these functions is that the light-cone wavefunctions have complex phases. We induce the complex phases by incorporating the final-state interactions into the light-cone wavefunctions in the scalar diquark model, and then we calculate explicitly the Sivers and the Boer-Mulders distribution functions from the obtained light-cone wavefunctions.

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## 1. Introduction

It was found that the final-state interaction of quark and gluon induces the single-spin asymmetry in the semi-inclusive deep inelastic scattering at the twist-two level [1]. Then, this time-odd twist-two effect was interpreted as the Sivers effect by finding that the final-state interaction can be treated as the source of the time-odd Sivers distribution function [2, 3, 4, 5, 6]. It is also often referred to as "naively $T$-odd", because the appearance of this function does not imply a violation of time-reversal invariance, since they can arise through the final-state interactions. With these developments, the existence of the Sivers distribution function has gained a firm theoretical support. The Sivers distribution function $f_{1T}^{\perp}$ describes the difference between the momentum distributions of quarks inside the nucleon transversely polarized in opposite directions. There is another quark distribution function of the nucleon induced by the final-state interaction of quark and gluon, which is called the Boer-Mulders distribution function $h_1^{\perp}$. $h_1^{\perp}$ describes the difference between the momentum distributions of the quarks transversely polarized in opposite directions inside the unpolarized nucleon. The distribution functions $f_{1T}^{\perp}$ and $h_1^{\perp}$ are depicted in Figs. 1 and 2.

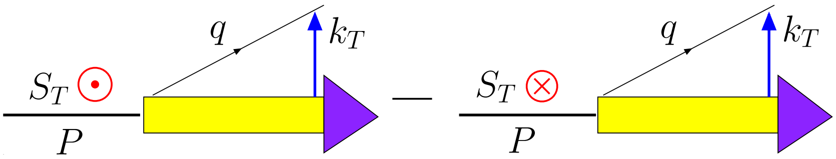


**Figure 1:** *Schematic depiction of the Sivers distribution function $f_{1T}^{\perp}$. The spin vector $S_T$ of the nucleon points out of and into the page, respectively, and $k_T$ is the transverse momentum of the extracted quark.*

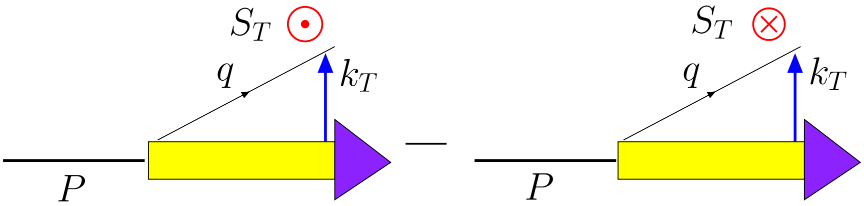


**Figure 2:** *Schematic depiction of the Boer-Mulders distribution function $h_1^{\perp}$. The spin vector $S_T$ of the quark points out of and into the page, respectively, and $k_T$ is the transverse momentum of the extracted quark.*

The light-cone wavefunctions are valuable for studying the hadronic processes by treating the non-perturbative effects in a relativistically covariant way [7, 8]. The formulas which express the electromagnetic form factors and the generalized parton distribution functions in terms of the light-cone wavefunctions were found in Refs. [9, 10, 11] and Refs. [12, 13], respectively. In Ref. [14] the light-cone wavefunction representation of the nucleon electric dipole moment was found by introducing the complex phases of the light-cone wavefunctions, and studied a general relation connecting nucleon electric dipole and anomalous magnetic moments.

In this paper we find the formulas which express the Sivers and the Boer-Mulders distribution functions in terms of the matrix elements of the nucleon spin states. We find the light-cone wavefunction representations of the Sivers and the Boer-Mulders distribution functions, and we calculate these functions for the scalar diquark model by using these light-cone wavefunction representations.

## 2. Sivers and Boer-Mulders Distribution Functions

The $k_T$-dependent unpolarized quark distribution function $f_1(x,\vec{k}_\perp)$, the Sivers distribution function $f_{1T}^\perp(x,\vec{k}_\perp)$ and the Boer-Mulders distribution function $h_1^\perp(x,\vec{k}_\perp)$ are parts of the proton correlation function $\Phi(x,\vec{k}_\perp : P,S)$ [15]:

$$\Phi(x,\vec{k}_\perp : P,S) = \frac{M}{2P^+}\Big[f_1(x,\vec{k}_\perp)\frac{\gamma\cdot P}{M} + f_{1T}^\perp(x,\vec{k}_\perp)\varepsilon_{\mu\nu\rho\sigma}\frac{\gamma^\mu P^\nu k_\perp^\rho S_T^\sigma}{M^2} + h_1^\perp(x,\vec{k}_\perp)\frac{\sigma_{\mu\nu}k_\perp^\mu P^\nu}{M^2} + \cdots\Big] , \tag{2.1}$$

from which we find that $f_1(x,\vec{k}_\perp)$ and $f_{1T}^\perp(x,\vec{k}_\perp)$ can be defined through matrix elements of the bilinear vector current:

$$\begin{aligned}&\int \frac{dy^- d^2\vec{y}_\perp}{16\pi^3}\, e^{ixP^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\, \langle P,\vec{S}_\perp|\overline{\psi}(0)\,\gamma^+\,\psi(y)\,|P,\vec{S}_\perp\rangle\Big|_{y^+=0} \\ &= \frac{1}{2P^+}\Big[\, f_1(x,\vec{k}_\perp)\,\overline{U}(P,\vec{S}_\perp)\,\gamma^+\,U(P,\vec{S}_\perp) \;+\; f_{1T}^\perp(x,\vec{k}_\perp)\,\frac{k_\perp^i}{M}\,\overline{U}(P,\vec{S}_\perp)\,\sigma^{i+}\,U(P,\vec{S}_\perp)\Big] ,\end{aligned} \tag{2.2}$$

where

$$\frac{1}{2P^+}\overline{U}(P,\vec{S}_\perp)\sigma^{i+}U(P,\vec{S}_\perp) = \varepsilon^{ji}S_\perp^j \qquad \text{with } \varepsilon^{12} = -\varepsilon^{21} = 1 . \tag{2.3}$$

For an explicit calculation, let us consider the case of $\vec{S}_\perp = (S_\perp^1, S_\perp^2) = (0,1)$ for the transverse spin in (2.2). Then, the proton state is given by $(|P,\uparrow\rangle + i|P,\downarrow\rangle)/\sqrt{2}$ and Eq. (2.2) becomes

$$\mathscr{A}\;\frac{\langle P,\uparrow| - i\langle P,\downarrow|}{\sqrt{2}}\overline{\psi}(0)\,\gamma^+\,\psi(y)\,\frac{|P,\uparrow\rangle + i|P,\downarrow\rangle}{\sqrt{2}}\Big|_{y^+=0} = f_1(x,\vec{k}_\perp) \;-\; S_\perp^2\,\frac{k_\perp^1}{M}\,f_{1T}^\perp(x,\vec{k}_\perp) , \tag{2.4}$$

where

$$\mathscr{A} \;\equiv\; \int \frac{dy^- d^2\vec{y}_\perp}{16\pi^3}\, e^{ixP^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} . \tag{2.5}$$

From (2.4) we have

$$f_1(x,\vec{k}_\perp) \;=\; \mathscr{A}\;\frac{1}{2}\Big[\langle P,\uparrow|J^+(y)|P,\uparrow\rangle + \langle P,\downarrow|J^+(y)|P,\downarrow\rangle\Big]\Big|_{y^+=0} , \tag{2.6}$$

$$-\frac{k_\perp^1}{M}\,f_{1T}^\perp(x,\vec{k}_\perp) \;=\; \mathscr{A}\;\frac{i}{2}\Big[\langle P,\uparrow|J^+(y)|P,\downarrow\rangle - \langle P,\downarrow|J^+(y)|P,\uparrow\rangle\Big]\Big|_{y^+=0} , \tag{2.7}$$

where $J^+(y) = \overline{\psi}(0)\,\gamma^+\,\psi(y)$.

On the other hand, from (2.1) the Boer-Mulders distribution function $h_1^\perp(x,\vec{k}_\perp)$ can be defined through matrix elements of the bilinear tensor current:

$$\begin{aligned}&\int \frac{dy^- d^2\vec{y}_\perp}{16\pi^3}\, e^{ixP^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp}\, \langle P,\vec{S}_\perp|\,\overline{\psi}(0)\,\sigma^{i+}\,\psi(y)\,|P,\vec{S}_\perp\rangle\Big|_{y^+=0} \\ &= \frac{1}{2P^+}\Big[\, h_1^\perp(x,\vec{k}_\perp)\,\frac{k_\perp^i}{M}\,\overline{U}(P,\vec{S}_\perp)\,\gamma^+\,U(P,\vec{S}_\perp)\Big] ,\end{aligned} \tag{2.8}$$

which gives

$$\frac{k_\perp^i}{M}\,h_1^\perp(x,\vec{k}_\perp) \;=\; \frac{1}{2}\,\mathscr{A}\,\Big(\Big[\langle P,\uparrow|\,\overline{\psi}(0)\,\sigma^{i+}\,\psi(y)\,|P,\uparrow\rangle\Big] + \Big[\langle P,\downarrow|\,\overline{\psi}(0)\,\sigma^{i+}\,\psi(y)\,|P,\downarrow\rangle\Big]\Big)\Big|_{y^+=0} . \tag{2.9}$$

## 3. Light-Cone Wavefunction Representations of Sivers and Boer-Mulders Functions

The expansion of the proton eigensolution $|\,\psi_p\rangle$ on the eigenstates $\{|n\rangle\}$ of the free Hamiltonian $H_{LC}$ gives the light-cone Fock expansion:

$$\left|\psi_p(P^+,\vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\,16\pi^3}\;16\pi^3\delta\left(1-\sum_{i=1}^{n}x_i\right)\delta^{(2)}\left(\sum_{i=1}^{n}\vec{k}_{\perp i}\right) \times \psi_n(x_i,\vec{k}_{\perp i},\lambda_i)\left|n;\,x_iP^+,x_i\vec{P}_\perp+\vec{k}_{\perp i},\lambda_i\right\rangle . \tag{3.1}$$

The plus component momentum fractions $x_i = k_i^+/P^+$ and the transverse momenta $\vec{k}_{\perp i}$ of partons represent the relative momentum coordinates of the light-cone wavefunctions. The physical transverse momenta of partons are $\vec{p}_{\perp i} = x_i\vec{P}_\perp + \vec{k}_{\perp i}$. The $\lambda_i$ label the light-cone spin projections of the partons along the quantization direction $z$. The $n$-particle states are normalized as

$$\left\langle n;\,p_i'^+,\vec{p}\,'_{\perp i},\lambda_i'\,\middle|\,n;\,p_i^+,\vec{p}_{\perp i},\lambda_i\right\rangle = \prod_{i=1}^{n}16\pi^3 p_i^+\,\delta(p_i'^+-p_i^+)\;\delta^{(2)}(\vec{p}\,'_{\perp i}-\vec{p}_{\perp i})\;\delta_{\lambda_i'\lambda_i}\ . \tag{3.2}$$

From (2.6) and (2.7) we get

$$f_1(x,\vec{k}_\perp) = \mathscr{B}\,\frac{1}{2}\left[\psi^{\uparrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\;\psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\;+\;\psi^{\downarrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\;\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\right], \tag{3.3}$$

$$-\frac{k_\perp^1}{M}\,f_{1T}^\perp(x,\vec{k}_\perp) = \mathscr{B}\,\frac{i}{2}\left[\psi^{\uparrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\;\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\;-\;\psi^{\downarrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\;\psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_i)\right], \tag{3.4}$$

where

$$\mathscr{B}\;\equiv\;\sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\;16\pi^3\delta\left(1-\sum_{j=1}^{n}x_j\right)\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)\delta(x-x_1)\;\delta^{(2)}(\vec{k}_\perp-\vec{k}_{\perp 1})\,. \tag{3.5}$$

As we see in (3.4), the Sivers distribution function is given by the product of the light-cone wavefunctions which have opposite proton spin states and same quark spin states.

From (2.9) we have

$$\begin{aligned}\frac{k_\perp^1}{M}\,h_1^\perp(x,\vec{k}_\perp) \;=\; \frac{\mathscr{B}}{2}\,(-i)\,\Big(\Big[\;&\psi^{\uparrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1'=\downarrow,\lambda_{i\neq 1})\;\psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1=\uparrow,\lambda_{i\neq 1})\\ -\;&\psi^{\uparrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1'=\uparrow,\lambda_{i\neq 1})\;\psi^{\uparrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1=\downarrow,\lambda_{i\neq 1})\;\Big]\\ +\Big[\;&\psi^{\downarrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1'=\downarrow,\lambda_{i\neq 1})\;\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1=\uparrow,\lambda_{i\neq 1})\\ -\;&\psi^{\downarrow\,*}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1'=\uparrow,\lambda_{i\neq 1})\;\psi^{\downarrow}_{(n)}(x_i,\vec{k}_{\perp i},\lambda_1=\downarrow,\lambda_{i\neq 1})\;\Big]\Big)\,.\end{aligned} \tag{3.6}$$

As we see in (3.6), the Boer-Mulders distribution function is given by the product of the light-cone wavefunctions which have same proton spin states and opposite quark spin states, whereas we found in (3.4) that the Sivers distribution function is given by the product of the light-cone wavefunctions which have opposite proton spin states and same quark spin states.

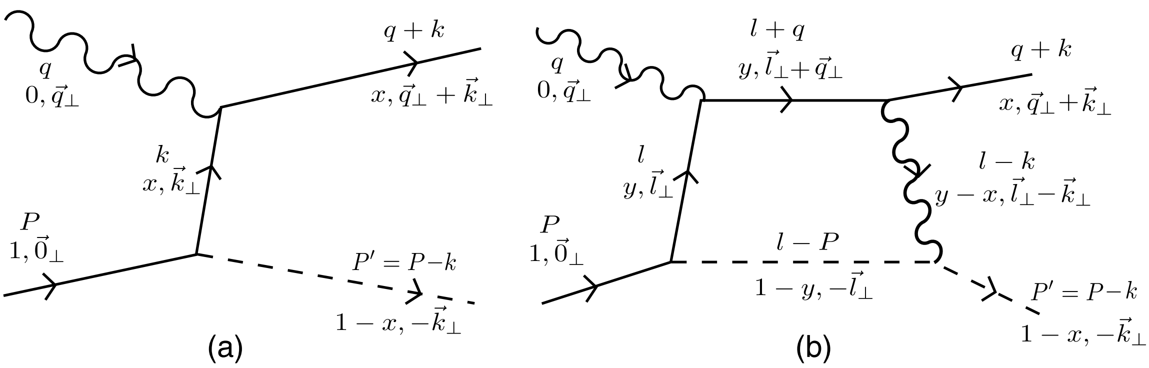


**Figure 3:** (a) Tree level diagram and (b) diagram with final-state interaction.

## 4. Explicit Calculations with Scalar Diquark Model

The final-state interactions in semi-inclusive deep inelastic scattering are commonly treated as a part of the proton distribution function [2, 5]. If we adopt the same treatment for the wavefunctions, we can consider that the final-state interactions for the scalar diquark model depicted in Fig. 3 induce the spin-dependent complex phases to the wavefunctions:

$$\begin{cases} \psi^{\uparrow}_{+\frac{1}{2}}(x,\vec{k}_\perp) = \frac{(m+xM)}{x}\Big(1+ia_1\Big)\,\varphi\ , \\ \psi^{\uparrow}_{-\frac{1}{2}}(x,\vec{k}_\perp) = -\frac{(+k^1+ik^2)}{x}\Big(1+ia_2\Big)\,\varphi\ , \end{cases} \tag{4.1}$$

$$\begin{cases} \psi^{\downarrow}_{+\frac{1}{2}}(x,\vec{k}_\perp) = -\frac{(-k^1+ik^2)}{x}\Big(1+ia_2\Big)\,\varphi\ , \\ \psi^{\downarrow}_{-\frac{1}{2}}(x,\vec{k}_\perp) = \frac{(m+xM)}{x}\Big(1+ia_1\Big)\,\varphi\ , \end{cases} \tag{4.2}$$

where $\varphi = \varphi(x,\vec{k}_\perp) = -gx\sqrt{1-x}/(\vec{k}_\perp^2+B)$ with the nucleon-quark-diquark coupling constant $g$ and $B = -x(1-x)M^2+(1-x)m^2+x\lambda^2$, and $a_1$ and $a_2$ are given by

$$a_{1,2} = \frac{e_1 e_2}{8\pi}\,(\vec{k}_\perp^2+B)\,g_{1,2} \tag{4.3}$$

with [1]

$$g_1 = \int_0^1 d\alpha\,\frac{-1}{\alpha(1-\alpha)\vec{k}_\perp^2+\alpha\lambda_g^2+(1-\alpha)B}\ ,\qquad g_2 = \int_0^1 d\alpha\,\frac{-\alpha}{\alpha(1-\alpha)\vec{k}_\perp^2+\alpha\lambda_g^2+(1-\alpha)B}\ . \tag{4.4}$$

In the above, $e_1$ and $e_2$ are the quark and diquark charge, and $M$, $m$, $\lambda$ and $\lambda_g$ are the nucleon, quark, diquark and gluon mass, respectively. We take $\lambda_g = 0$ at the end of the calculation.

Using the wavefunctions (4.1) and (4.2) in the formulas (3.3), (3.4) and (3.6), we obtain

$$f_1(x,\vec{k}_\perp) = \frac{1}{16\pi^3}\Big[(M+\frac{m}{x})^2+\frac{\vec{k}_\perp^2}{x^2}\Big]\varphi^2\ , \tag{4.5}$$

$$f_{1T}^{\perp}(x,\vec{k}_\perp) = \frac{1}{16\pi^3}\,2\,\frac{M}{x}\,(M+\frac{m}{x})\,\varphi^2\,\frac{e_1e_2}{8\pi}\,(\vec{k}_\perp^2+B)\,\frac{1}{\vec{k}_\perp^2}\,\ln\frac{(\vec{k}_\perp^2+B)}{B}\ , \tag{4.6}$$

$$h_1^{\perp}(x,\vec{k}_\perp) = \frac{1}{16\pi^3}\,2\,\frac{M}{x}\,(M+\frac{m}{x})\,\varphi^2\,\frac{e_1e_2}{8\pi}\,(\vec{k}_\perp^2+B)\,\frac{1}{\vec{k}_\perp^2}\,\ln\frac{(\vec{k}_\perp^2+B)}{B}\ . \tag{4.7}$$

The results in (4.6) and (4.7) agree with those in Refs. [1, 5] with an additional overall minus sign which should be corrected there [16].

## 5. Conclusion

In this paper we find the light-cone wavefunction representations of the Sivers and the Boer-Mulders distribution functions. A necessary condition for the existence of these functions is that the light-cone wavefunctions have complex phases. We induce the complex phases by incorporating the final-state interactions into the light-cone wavefunctions in the scalar diquark model, and then we calculate explicitly the Sivers and the Boer-Mulders distribution functions from the light-cone wavefunctions of the model. The results are the same as those obtained from the direct calculation of the hadronic tensor without employing the concept of the light-cone wavefunction, since the essential interpretation of the final-state interaction is identical in both calculations. However, the analysis in this paper by using the light-cone wavefunction representations is useful to grasp the natures of the Sivers and the Boer-Mulders distribution functions. For example, it shows how the signs of these functions are determined in each model and it helps us understand why the signs of the Sivers and the Boer-Mulders distribution functions are same or different depending on the model used.

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