

Kaon physics: a lattice perspective

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I critically review recent lattice QCD results relevant for kaon phenomenology, as well as the methods that are used to obtain them. The focus is on calculations with $N_f = 2$ and $N_f = 2 + 1$ flavors of sea quarks. Concerning methodology, the subjects covered include a discussion of how best to extrapolate and/or interpolate results to the physical quark-mass point, a scheme for assessing the extent to which a lattice QCD calculation includes the various effects required to compute a given quantity reliably and a procedure for averaging lattice results. The phenomenological topics that I review comprise leptonic and semileptonic kaon decays, as well as neutral kaon mixing and CP violation in $K \rightarrow \pi\pi$ decays.

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1. Introduction

This talk critically reviews recent lattice QCD results relevant for kaon phenomenology, as well as the methodology that is used to obtain them. The focus is on full QCD calculations, which account for the effects of light sea quarks either partially, as in $N_f = 2$ simulations, where degenerate up and down sea quarks of mass m_{ud} are included, or fully, as in $N_f = 2 + 1$ calculations, where strange sea quarks of mass m_s are also incorporated.

The main motivation for studying kaon physics off and on the lattice is to test the standard model, to determine some of its parameters and to constrain possible new physics scenarios. From a lattice perspective, kaon processes fall into three broad categories. The first are processes, such as leptonic and semileptonic kaon decays, for which lattice QCD methods are already providing high precision results. The second category corresponds to processes for which lattice calculations are delivering results with errors on the level of 10% or less, such as for K^0 - \bar{K}^0 mixing matrix elements. The last category of processes are those for which lattice calculations have failed up until now to provide reliable answers. Amongst them are the $\Delta I = 1/2$ rule and, more critically, direct CP violation in $K \rightarrow \pi\pi$ decays.

Another motivation for studying kaons physics on the lattice is the overlap this physics has with chiral perturbation theory (ChPT). ChPT describes the low-energy dynamics of the pseudo-Nambu-Goldstone bosons of chiral symmetry breaking and has been successful in many phenomenological applications. Moreover, it is a very useful tool for understanding the dependence of lattice results on light quark masses and on volume. Recent $N_f = 2$ and $2 + 1$ calculations, which include pions with masses $M_\pi \lesssim 350$ MeV, are not only using ChPT but are also beginning to provide information about ChPT in return.

The talk begins with a critical discussion of the role that ChPT and other expansions can play in interpolating and extrapolating lattice QCD results to the physical mass point, $(m_{ud}, m_s) = (m_{ud}^{ph}, m_s^{ph})$, in view of the quark mass values currently reached in lattice calculations. In an aside, I present a scheme for visualizing the extent to which a lattice calculation includes the different effects necessary for computing a quantity of interest reliably, and a procedure for averaging lattice results. This is followed by a review of calculations of quantities relevant for leptonic and semileptonic kaon decays, as well as for neutral kaon mixing and CP violation in $K \rightarrow \pi\pi$ decays.

2. Reaching the physical mass point

Using today's algorithms, it is straightforward to perform $N_f = 2 + 1$ calculations with a strange quark whose mass is around its physical value. The physical strange quark mass point is thus recovered simply by interpolation.

Reaching the physical up and down quark mass point is much more difficult. Though the results of PACS-CS [1] announce that calculations will soon be done directly at this point in physically large volumes, for the moment all other simulations are being performed with larger quark masses. Thus, reaching the physical point still requires conducting a number of computationally intensive calculations with $m_{ud} < m_{ud}^{max} \sim m_s^{ph}/2$, extending preferably below $m_s^{ph}/12$, and performing a delicate extrapolation in m_{ud} to $m_{ud}^{ph} \simeq m_s^{ph}/26$.

To guide the interpolation to m_s^{ph} and extrapolation to m_{ud}^{ph} , a natural candidate is $SU(3)$ ChPT, since it provides a concise framework for describing the dependence of hadronic quantities on the masses of the up, down and strange quarks. Moreover, ChPT in its various quenched and partially-quenched guises has served the lattice community well. Nevertheless, lattice calculations are reaching regions of parameter space and precisions never attained before, and it is worth considering the following two questions candidly:

- What is the best way to interpolate to $m_s = m_s^{ph}$?
- What is the best way to extrapolate from $m_s^{ph}/12 \lesssim m_{ud} < m_{ud}^{max} \sim m_s^{ph}/2$ to $m_{ud} = m_{ud}^{ph}$?

There are, I believe, three physically motivated options to choose from:

- (1) As already mentioned, $SU(3)$ ChPT is a natural candidate. It has the advantage of addressing both problems together, within a compact and constrained framework. Its drawback is that it provides similar solutions to two problems which are of a quite different nature: the first concerns a simple interpolation rather far away from the chiral point while the second involves a difficult extrapolation which reaches much deeper into the chiral regime.
- (2) $SU(2)$ ChPT provides a means of distinguishing these two problems. For the extrapolation in m_{ud} , it brings to bear all of the power of chiral expansions. The interpolation in m_s is not directly addressed, but it suffices to supplement the chiral expansion with a regular mass—or what I call “*flavor*”—expansion about m_s^{ph} , and to perform a simple polynomial interpolation.
- (3) The idea of a *flavor* expansion can also be applied to the extrapolation in m_{ud} . To reduce uncertainties, this expansion should be performed about the midpoint of the interval between the physical point and the largest up and down quark mass considered, i.e. $\bar{m}_{ud} = [m_{ud}^{ph} + m_{ud}^{max}]/2$. In this scheme, both the extrapolation in m_{ud} and the interpolation in m_s can be performed with polynomial *flavor* expansions.

Let us now review these three alternatives in more detail.

2.1 $SU(3)$ versus $SU(2)$ ChPT and *flavor* expansions: what’s the difference?

The *flavor* expansions are performed about regular points \bar{m}_{ud} and m_s^{ph} (i.e. they are Taylor expansions). This is not the case for the chiral expansions. $SU(2)$ ChPT is an expansion about the singular point $(m_{ud}, m_s) = (0, m_s^{ph})$. $SU(3)$ ChPT makes the additional assumption that the strange quark is chiral so that the expansion is around $(m_{ud}, m_s) = (0, 0)$.

In *flavor* expansions of quantities which do not vanish in the $SU(2)$ chiral limit, it is the “distance” from the expansion points, \bar{m}_{ud} or m_s^{ph} , in units of the QCD scale, which determines how well the series converges (hence my use of the adjective “*flavor*”). Thus, the expansion parameters are $(m_{ud} - \bar{m}_{ud})/M_{QCD}$ and $(m_s - m_s^{ph})/M_{QCD}$, where $M_{QCD} \sim 1 \text{ GeV}$ is a typical QCD scale. On the other hand, $SU(3)$ ChPT expressions are expansions in $m_{ud,s}/\Lambda_\chi$, with $\Lambda_\chi \sim 4\pi F_\pi = O(M_{QCD})$ the chiral symmetry breaking scale. In $SU(2)$, the expansions are in m_{ud}/m_s and m_{ud}/Λ_χ .

Because m_{ud} and m_s are not measured directly in experiment, it is convenient to replace these masses by observables which are sensitive to them. ChPT suggests that M_π and $M_K^\chi \equiv [M_K^2 - M_\pi^2/2]^{1/2}$, with $M_\pi^{ph} \simeq 135 \text{ MeV}$ and $M_K^{\chi,ph} \simeq 486 \text{ MeV}$, are particularly appropriate. Indeed, LO

ChPT yields $M_\pi^2 = 2Bm_{ud}$ and $(M_K^\chi)^2 = Bm_s$, with $B = O(M_{QCD})$. In terms of these variable, the $SU(2)$ ChPT expansion parameters can be written $(M_\pi/\sqrt{2}M_K^\chi)^2$ and $(M_\pi/\Lambda_\chi)^2$, while $SU(3)$ ChPT is an expansion in $(M_{\pi,K,\eta}/\Lambda_\chi)^2$. Similarly, the *flavor* expansion parameters become $\Delta_\pi \equiv (M_\pi^2 - \bar{M}_\pi^2)/2M_{QCD}^2$ and $\Delta_K \equiv [(M_K^\chi)^2 - (M_K^{\chi,ph})^2]/M_{QCD}^2$. It is worth noting that this definition for Δ_K remains appropriate if one assumes that M_K^χ itself obeys a *flavor* expansion in m_s , i.e. $M_K^\chi = M_K^{\chi,ph}[1 + C_K(m_s - m_s^{ph})/M_{QCD} + \text{h.o.t.}]$, with C_K a constant. Indeed, in that case we also have $\Delta_K = O[(m_s - m_s^{ph})/M_{QCD}]$. On the other hand, M_π 's *flavor* expansion in m_{ud} , $M_\pi = \bar{M}_\pi[1 + 2C_\pi(m_{ud} - \bar{m}_{ud})/M_{QCD} + \text{h.o.t.}]$, is poorly behaved for the range of m_{ud} currently considered in lattice calculations, since the NLO plus higher order terms can be 50% or more of the LO term. However, this fact does not invalidate the use of *flavor* expansions in m_{ud} for quantities which do not vanish in the $SU(2)$ chiral limit. It merely signals that, in current calculations, the relative variation in M_π is large, while the change in M_π with respect to M_{QCD} remains small.

The expected accuracy at NLO in the $SU(2)$ expansion around the physical mass point is much better than for the $SU(3)$ case. Indeed, in $SU(2)$ this accuracy is given by $(M_\pi^{ph}/\sqrt{2}M_K^{\chi,ph})^4 \sim 0.1\%$ whereas it is expected to be $(M_\eta^{ph}/4\pi F_\pi)^4 \sim 5\%$ in the $SU(3)$ case. However, with pions of about 450 MeV floating around, as in present day simulations, the $SU(2)$ figure becomes $(M_\pi/\sqrt{2}M_K^\chi)^4 \sim 20\%$, which is much less impressive. Nevertheless, this expansion has the advantage that its convergence improves rapidly as M_π is reduced, while the $SU(3)$ expansion parameter $(M_\eta/4\pi F_\pi)^2$ does not decrease significantly with M_π .

The accuracy of the *flavor* interpolation in strange quark mass is generically very high. Suppose that one has performed the calculation for at least two values of the strange quark mass that bracket m_s^{ph} with a total spread of about 10%. The expansion parameter is then $|\Delta_K| \sim 0.01$. Assuming that the error due to the truncation of the interpolating polynomial is on the order of the first omitted term, the systematic error associated with a linear interpolation in $(M_K^\chi)^2$ (i.e. a linear interpolation) will have an accuracy on the order of $\Delta_K^2 \sim 0.01\%$.

In current lattice calculations, the *flavor* expansion in up and down quark mass is not as good. Assuming that we consider only pions with $M_\pi \leq M_\pi^{max} = 450\text{MeV}$, the expansion parameter is $|\Delta_\pi| \lesssim 0.05$. This means that a linear extrapolation will have a truncation uncertainty on the order of $\Delta_\pi^2 \sim 0.3\%$ (with a coefficient that increases with u/d content). Moreover, it is straightforward to show that, with a quadratic *flavor* expansion, one can fit a chiral logarithm which gives a correction of up to 30% as M_π varies in the range from M_π^{ph} to M_π^{max} , with a systematic accuracy better than 0.5%. So, even in the presence of a chiral logarithm, a *flavor* expansion can be used.

Let me now add a few words about the possible outcomes of implementing the different approaches. $SU(3)$ ChPT provides functional forms which are more constrained, i.e. which have less parameters, at a given order, than the $SU(2)$ chiral and *flavor* expansions. That is one reason why $SU(3)$ ChPT might be appealing. So let me assume, for the moment that we are fitting lattice results to $SU(3)$ ChPT expressions. As M_π is lowered below $\sqrt{2}M_K^\chi$ with fixed m_s , $SU(3)$ ChPT turns into $SU(2)$ ChPT, except that the extended symmetry of the $SU(3)$ theory imposes constraints amongst the $SU(2)$ LECs. These constraints can be released by adding NNLO and higher terms to the $SU(3)$ expansion. If the M_K^2/Λ_χ^2 expansion in the $SU(3)$ theory behaves well, then the LECs obtained with the fits may be $SU(3)$ LECs of QCD, as defined in the $SU(3)$ chiral limit. However, if the assumption that the strange quark is chiral is not borne out in practice, a good fit may still be obtained by adding higher order terms, but the fitted LECs will most likely not be QCD's LECs.

In that case, one may still find that the M_π^2 component of the $SU(3)$ chiral expansion is reasonably well behaved. If this is so, an $SU(2)$ chiral fit ought to work and should give the $SU(2)$ LECs of QCD. However, the expansion may still behave poorly for heavier pions because in that case the expansion parameter $(M_\pi/\sqrt{2}M_K^2)^2$ may not be small. Alternatively one may use the *flavor* expansion approach. It deals with the strange quark mass interpolation in the same way as $SU(2)$ ChPT, but differs in the choice of expansion point for the extrapolation in m_{ud} . ChPT expands observables about $M_\pi = 0$, which is further from the lightest simulated M_π than is the physical point. The *flavor* expansion, on the other hand, is performed about a value of $M_\pi = \bar{M}_\pi$ which is between the heaviest simulated M_π and the physical value. Thus, the *flavor* expansion will be better behaved, though generically less constrained.

ChPT is a worthy object of study in its own right, with applications which go beyond present lattice QCD capabilities. Thus, it is important to test its range of validity and its accuracy where it is applicable. It is also important to determine its LECs, since these can be used to make predictions in a variety of processes. However, if the goal is to determine the value of an observable at the physical point, one should remain agnostic in regards to the expansion used and pick the one which gives the lowest combined statistical and systematic error. Moreover, if the goal is to obtain the LECs of QCD, it may be necessary to perform calculations closer to the chiral limit, especially in the case of $SU(3)$ ChPT.

2.2 $SU(3)$ versus $SU(2)$ ChPT and *flavor* expansions: examples

To further clarify the difference between the different expansions and their applicability to lattice calculations, it is useful to turn to a concrete example. We consider here the expansions of the pion and kaon decay constants, F_π and F_K , at NLO. In the $SU(3)$ theory, we have [2]:

$$F_\pi = F_3 \left\{ 1 - \frac{1}{(4\pi F_3)^2} \left[\chi_1(M_\pi^2) + \frac{1}{2} \chi_1(M_K^2) \right] + 4(L_5 + L_4)(\mu) \frac{M_\pi^2}{F_3^2} + 8L_4(\mu) \frac{M_K^2}{F_3^2} \right\} \quad (2.1)$$

$$F_K = F_3 \left\{ 1 - \frac{1}{(4\pi F_3)^2} \left[\frac{3}{8} \chi_1(M_\pi^2) + \frac{3}{4} \chi_1(M_K^2) + \frac{3}{8} \chi_1(M_\eta^2) \right] + 4(L_5 + 2L_4)(\mu) \frac{M_K^2}{F_3^2} + 4L_4(\mu) \frac{M_\pi^2}{F_3^2} \right\}, \quad (2.2)$$

where $\chi_n(M^2) = M^{2n} \ln(M^2/\mu^2)$ and where F_3 is the pion decay constant in the $N_f = 3$ chiral limit. The up-down and strange quark mass-dependence of these two quantities are obtained here in terms of only *three* parameters: F_3 , L_4 and L_5 .

The $SU(2)$ theory is much less frugal with parameters. At NLO it predicts [3–5]:

$$F_\pi = F_2(1 + \alpha_F \Delta_K) \left\{ 1 - \frac{1}{(4\pi F_2)^2} [\chi_1(M_\pi^2) - \ell_4(\mu) M_\pi^2] \right\} + O(M_\pi^2 \Delta_K) \quad (2.3)$$

$$F_K = F_2^K(1 + \alpha_F^K \Delta_K) \left\{ 1 - \frac{1}{(4\pi F_2)^2} \left[\frac{3}{8} \chi_1(M_\pi^2) - \ell_4^K(\mu) M_\pi^2 \right] \right\} + O(M_\pi^2 \Delta_K), \quad (2.4)$$

where F_2 and F_2^K are the pion and kaon decay constants, respectively, in the $N_f = 2$ chiral limit and where I have included a strange quark mass dependence. Thus, the $SU(2)$ description of the mass-dependence of the two decay constants requires at least *six* parameters (F_2 , ℓ_4 , α_F , F_2^K , ℓ_4^K , α_F^K), *eight* if $O(M_\pi^2 \Delta_K)$ terms are required.

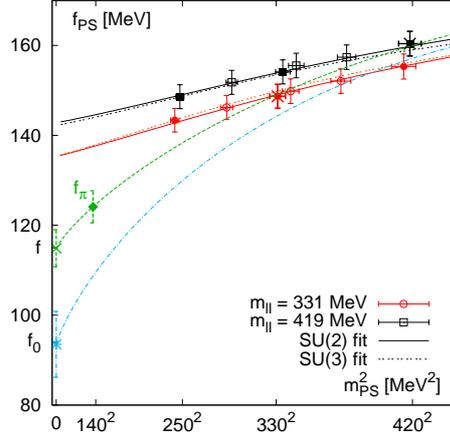


Figure 1: RBC/UKQCD's partially quenched pion decay constants versus pion mass squared, for two values of the sea pion mass, $M_\pi = 331$ and 419 MeV. The only points where sea and valence quarks are degenerate are the square and circle with crosses. Also shown are the unitary $SU(3)$ and $SU(2)$ fits. Conventions are such that $f_\pi = \sqrt{2}F_\pi = 131$ MeV.

This number of parameters is comparable to that required in the *flavor* expansion of F_K and F_π . Six parameters are needed if the M_π^2 dependence turns out to be linear and *eight* if curvature is observed, corresponding to an expansion to $O(\Delta_\pi^2, \Delta_K)$.

Let us now investigate how these considerations play out with real lattice results. I begin with a partially quenched, $N_f = 2 + 1$ study of F_π and F_K performed by RBC/UKQCD [5], whose results were presented at this conference by E. Scholz [6]. These results are shown in Fig. 1, where the pion decay constant is plotted against the valence pion mass squared for two values of the sea pion mass (331 MeV and 419 MeV). Details of the simulation are given below in Table 1.

In their calculation, the $SU(3)$ ChPT expansion parameters are, at $M_\pi^{max} = 419$ MeV: $(M_\pi^{max}/4\pi F_\pi^{ph})^2 \simeq 0.1$ and $(M_\eta/4\pi F_\pi^{ph})^2 \simeq 0.3$. The $SU(2)$ expansion at M_π^{max} is not any better: $(M_\pi^{max}/\sqrt{2}M_K^{\chi,ph})^2 \simeq 0.4$. Thus it is not clear, a priori, which of the two expansions is better at the top of the M_π range. Of course, as already mentioned, as M_π decreases the $SU(2)$ expansion improves rapidly whereas the $SU(3)$ expansion parameter, $(M_{K,\eta}/4\pi F_\pi^{ph})^2$ stays roughly constant. Assuming that $SU(3)$ ChPT is applicable, they find very large NLO corrections to the pion decay constant, even at their lightest unitary point, $M_\pi = 311$ MeV, where they are of order 70%. They also find that the NLO forms do not describe their kaon results, where the down quark is replaced by a strange. This is perhaps not too surprising since their kaons have masses of up to approximately 570 MeV.

With $SU(2)$ ChPT, on the other hand, they obtain good fits and find much more reasonable NLO corrections, that are on the order of 30% at $M_\pi = 311$ MeV. They use this information, together with that obtained from fits with partial NNLO terms and more massive pions, to conclude that $SU(3)$ ChPT fails in the range of masses explored, while $SU(2)$ ChPT is reliable.

A few comments are in order. The first is that the fits do not take into account correlations which are obviously strong at fixed sea quark mass. This makes getting a meaningful figure of merit for the fits difficult. The second is that the results display none of the logarithmic behavior which becomes relevant in the extrapolation to physical M_π : at NLO in partially quenched ChPT,

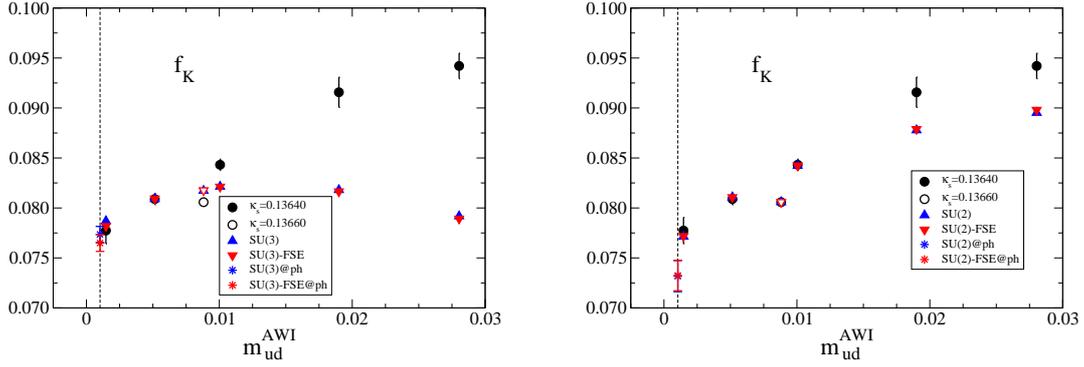


Figure 2: From PACS-CS, F_K as a function of the isospin averaged up and down quark masses, both in lattice units, at a single value of the lattice spacing, $a \simeq 0.09$ fm [1]. The black circles are the decay constants obtained from the different simulations, corresponding to $M_\pi \simeq 156, 296, 385, 411, 570$ and 702 MeV. The left panel shows these decay constants together with results from a variety of NLO $SU(3)$ ChPT fits (triangles) while the right panel displays the same results with NLO $SU(2)$ ChPT fits (triangles). If all fits were good, triangles at each m_{ud}^{AWI} would sit atop the corresponding circle. In their conventions, $f_K = \sqrt{2}F_K = 159$ MeV.

the dependence on valence quark mass is linear and with only two values of the sea quark mass, one cannot distinguish between a straight line and a chiral logarithm. Thus, the lattice results are not inconsistent with $SU(2)$ ChPT, but they cannot be claimed, either, to confirm the relevance of this expansion in the quark mass range considered. Moreover, the value of F_π obtained by linear fit would be significantly larger than the one found in the plot, though consistent within the final systematic error quoted by the authors. Finally, it should be remembered that the analysis is performed at a single, rather large value of the lattice spacing ($a \simeq 0.11$ fm), and mass dependent discretization errors could distort the physical chiral behavior.

PACS-CS has also investigated the applicability of the two variants of ChPT to their results for the decay constants [1], as shown at this conference by Y. Kuramashi [7]. Their calculations are performed for six different values of the pion mass, ranging from ~ 700 MeV all the way down to 156 MeV. Moreover, they consider only unitary points, for which valence and sea quarks of the same flavor have identical masses (i.e. no partial quenching). The parameters of their simulations are given below in Table 1. Their studies of the dependence of F_K on the isospin averaged up and down quark mass, m_{ud} , are shown in Fig. 2. The left hand panel displays the decay constants obtained directly from the simulations together with the values of these constants which result from fitting the simulation data to various $SU(3)$ ChPT forms. The fits are restricted to points with $M_\pi \lesssim 410$ MeV. The fit results above this point are extrapolations. They find that NLO $SU(3)$ ChPT fails to reproduce the M_π^2 dependence of F_K above $M_\pi \sim 400$ MeV. Moreover, they find that it fails to predict the strange quark mass dependence of F_K around m_s^{ph} and for $M_\pi \simeq 400$ MeV.

Again, the situation is quite different for $SU(2)$ ChPT fits. There they find that the m_{ud} dependence is well reproduced up to $M_\pi \simeq 410$ MeV and only deviates from the simulation result by 5% at $M_\pi \simeq 570$ MeV. Moreover the m_s dependence is correctly reproduced, as it should since there are two m_s values and this dependence is fitted by a line.

These calculations, performed almost all the way down to the physical point, are a real prowess. For the moment, however, the volume considered for their lightest pion ($M_\pi \simeq 156$ MeV) is small,

corresponding to $LM_\pi \sim 2.3$. This may make it difficult to control finite-volume effects at low M_π . Moreover, the calculations have only been performed at a single lattice spacing for now, so that alterations of the mass dependence by discretization errors have not yet been investigated.

Combining the experiences of RBC/UKQCD and PACS-CS, the following conclusion seems to emerge: $SU(3)$ ChPT appears to break down at the physical strange quark mass, at least in the presence of heavier up and down quarks, with masses larger than $m_{ud} \simeq 9m_{ud}^{ph} \simeq m_s^{ph}/3$, corresponding to $M_\pi \gtrsim 400$ MeV.

3. Aside on a classification scheme for lattice simulations and on the averaging of lattice results

Before turning to quantities of phenomenological interest, I wish to say a few words about the methodology that I will follow in reviewing lattice results.

3.1 Of stars and lattice calculations

The FLAVIANet Lattice Averaging Group (FLAG) is in the process of putting together a classification scheme for lattice calculations. The goal is to provide tables which, at a glance, give the reader a sense of how thoroughly a given calculation includes all of the necessary ingredients, based on a list of pre-defined, objective criteria. Since this collective work has not yet been finalized, I propose a personal version of the scheme here. It is based on a starring system, reminiscent of the one used in a famous, red restaurant guide:

- *** indicates that this aspect of the calculation is fully satisfactory;
- ** indicates that the corresponding ingredient has not been fully included, but that the investigations performed allow for a reasonable estimate of the ensuing systematic error;
- * indicates that the calculations performed are not sufficient to provide a reliable estimate of what is missing.

More specifically, here are the criteria which I use for starring the calculations reviewed below:

- **publication status**
 - *** published
 - ** preprint
 - * proceedings, talk
- **action, unitarity**
 - *** local action, unitary calculation
 - ** non-local action and/or discretization induced unitarity violations
- **flavors**
 - *** all dynamical flavors required for the process under study are included
 - ** some dynamical flavors missing, but at least $N_f \geq 2$
 - * $N_f = 0$ (i.e. quenched calculation)
- **renormalization**
 - *** nonperturbative with nonperturbative running

- ★★ nonperturbative with perturbative running at GeV energies, or perturbative at two-loops or more
- ★ one-loop perturbative and/or discretization which leads to poorly controlled operator mixing

- **extrapolation/interpolation to physical mass point**

Let $M_{\pi,typ}^{min}$ be a mass that is representative (“typical”) of the masses of the lightest pion variants that contribute to the m_{ud} dependence of the quantities studied.¹

- ★★★ $M_{\pi,typ}^{min} \leq 200\text{MeV}$ with NLO or better ChPT or any other demonstrably controlled functional mass dependence
- ★★ $M_{\pi,typ}^{min} \leq 350\text{MeV}$ and reliable estimate of extrapolation error
- ★ $M_{\pi,typ}^{min} > 350\text{MeV}$

- **continuum extrapolation**

- ★★★ 3 or more lattice spacings with at least one $a < 0.08\text{fm}$ and controlled scaling
- ★★ 2 or more lattice spacings with one $a \lesssim 0.1\text{fm}$
- ★ a single lattice spacing or all $a > 0.1\text{fm}$

- **finite volume**

- ★★★ $LM_{\pi}^{min} \geq 4$ (and numerical volume scaling study)
- ★★ $3 < LM_{\pi}^{min} \leq 4$ and well motivated analytical corrections
- ★ $LM_{\pi}^{min} \leq 3$ or $3 < LM_{\pi}^{min} \leq 4$ and no quantification of finite-volume effects

where M_{π}^{min} is the mass of the lightest pion contributing to finite-volume effects.

3.2 Averaging of lattice results

Now that results for various quantities of phenomenological interest are emerging from lattice calculations in which most effects are realistically taken into account, it is important to set forth objective, quantity independent averaging procedures. In particular, that means taking literally the statistical and systematic error estimates provided by the authors in a refereed publication. It also means only considering calculations in which all relevant sources of systematic uncertainty have been accounted for. Since we are still in the early days of realistic lattice calculations, this rule might have to be bent slightly at first to include results which are close to reaching this goal.

The averaging procedure which I adopt is the following. Given a list of results which satisfy the basic criteria described above, I perform their weighted average, with an inverse weight obtained by adding the statistical and systematic covariance matrices in quadrature. To determine the statistical error on the average, I construct a χ^2 with only the statistical correlation matrix and perform a standard $\Delta\chi^2$ analysis. For the systematic error, since one does not generically expect them to compensate from one calculation to the next, I take the smallest total systematic uncertainty amongst those obtained in the most complete calculations. In cases where either statistical or systematic errors are not symmetric, I symmetrize them.

¹ This “typical” mass depends on the fermion formulation used, on the quantities studied, etc. Since it is meant to be indicative, I have kept its determination simple. For staggered calculations I take the RMS of the masses of the different tastes; for non-staggered on staggered, the RMS of the valence and the sea taste-singlet pion masses; for Wilson, overlap, domain-wall, the RMS of the valence and sea pion masses (i.e. simply the lightest pion mass for unitary calculations); and for tmQCD, I have taken the charged pion mass, though some sort of isospin averaging should probably be performed. I thank C. Aubin, J. Laiho, S. Sharpe and R. Van de Water for enlightening correspondence.

There will be some statistical correlations between results obtained from the same set or from subsets of a given ensemble of gauge configurations. There will also be some correlations in the systematic errors of calculations which make use of similar methods. However, such correlations have not yet been analyzed in any detail and I choose to neglect them here when computing world averages. For computing an average's statistical error, though, I keep only the statistical error of the calculation, amongst those performed on a same set of configurations, that makes use of the largest fraction of these configurations. Correlations may be added more systematically later, once they are better understood.

In situations where some results have significantly smaller systematic uncertainties, for reasons which are not fully understood, one can provide an average with and without those results.

4. $|V_{us}|$ from experiment and the lattice

A precise determination of the magnitude of the CKM matrix element V_{us} allows for a precision test of CKM unitarity as well as of quark-lepton universality and provides constraints on new physics, through:

$$\frac{G_q^2}{G_\mu^2} \left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \right] = 1 + O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right), \quad (4.1)$$

where (V_{ud}, V_{us}, V_{ub}) forms the first row of the CKM matrix and where G_q is the Fermi constant as measured in quark decays, whereas G_μ is the same constant as determined in muon decays. Eq. (4.1) accounts for the fact that what is actually measured are not the CKM matrix elements, $V_{qq'}$, but $(G_q^2/G_\mu^2) \times |V_{qq'}|$. The large amounts of new experimental results from BNL-E685, KLOE, KTEV, ISTRA+ and NA48 provide the opportunity for testing this aspect of the standard model with unprecedented accuracy.

The current situation on the measurement of the relevant CKM matrix elements is:

- $|V_{ud}| = 0.97425(22)$ [0.02%] from nuclear β decays [8]
- $|V_{us}| = 0.2246(12)$ [0.5%] from $K_{\ell 3}$ decays [9]
- $|V_{us}/V_{ud}| = 0.2321(15)$ [0.6%] from $K_{\ell 2}$ decays [9]
- $|V_{ub}| = 3.87(47) \cdot 10^{-3}$ [12%] from exclusive and inclusive $b \rightarrow u\ell\nu$ decays [10]

where a factor of (G_q/G_μ) is implicit, as per Eq. (4.1), and where the percentages in square brackets indicate, for convenience, the relative error of the measurement.

The Flavianet Kaon Working Group combined the first three measurement to squeeze out a little additional precision on $|V_{us}|$ [9]. I have updated their analysis here to take into account the new result for $|V_{ub}|$ [8]:

- $|V_{ud}| = 0.97425(22)$ [0.02%], which implies the following contribution to the uncertainty in Eq. (4.1): $\delta|V_{ud}|^2 = 4.3 \cdot 10^{-4}$,
- $|V_{us}| = 0.2252(9)$ [0.4%], which implies the following contribution to the uncertainty in Eq. (4.1): $\delta|V_{us}|^2 = 4.2 \cdot 10^{-4}$,
- and the contribution from V_{ub} to Eq. (4.1), $|V_{ub}|^2 \simeq 1.5 \cdot 10^{-5}$, is so small that its error bar is irrelevant.

Ref.	N_f action	a [fm]	LM_π^{min} typ/val	M_π^{min} [MeV] typ/val	F_K/F_π
PDG'08 [11]					1.193(6)
ETM'08 [12]	2 tmQCD	0.07, 0.09, 0.10[F_π]	3.6/3.6	260/260	<i>1.196(13)(7)(8)</i>
NPLQCD'06 [13]	2+1 DWF/ KS _{MILC}	0.13[r_0]	5.1/3.5	420/290	1.218(2) $^{+11}_{-24}$
MILC'04-07 [14, 15]	2+1 KS _{MILC} ^{AsqTad}	0.06, 0.09, 0.12, 0.15[F_π]	5.3/4.2	300/240	1.197(3) $^{+6}_{-13}$
HPQCD/'07 UKQCD [16]	2+1 KS _{MILC} ^{HISQ}	0.09, 0.12, 0.15[Υ]	4.8/4.1	360/310	1.189(2)(7)
RBC'08 UKQCD [5]	2+1 DWF	0.11[Ω]	4.1/3.4	290/240	1.205(18)(62)
ALV'08	2+1 DWF/ KS _{MILC}	0.09, 0.12[Υ/F_π]	5.3/4.2	300/240	<i>1.191(16)(17)</i>
PACS-CS'08[1]	2+1 NP-SW	0.09[Ω]	2.3/2.3	160/160	1.189(20)
BMW'08	2+1 SW	0.065, 0.085, 0.125[Ξ]	4/4	190/190	<i>1.19(1)(1)</i>

Table 1: Parameters of the simulations used by various collaborations for calculating F_K/F_π , together with their result for that quantity (results in italic were presented at this conference). The column N_f indicates the number of sea quark flavors considered. The symbols in brackets in the a [fm] column indicate the quantity used to set the scale. Also given are the masses of the lightest valence pion simulated, as well as the “typical” lightest pion mass defined in footnote 1.

At the time of the conference, $|V_{us}|$ was no longer the dominant source of uncertainty in Eq. (4.1). However, the new result for $|V_{ud}|$ [8] makes it a dead heat. Combining these results yields:

$$\frac{G_q^2}{G_\mu^2} \left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \right] = 0.9999(6) [0.06\%]. \quad (4.2)$$

This result is fully consistent with the standard model. However, within one standard deviation, new physics at a scale $\Lambda_{NP} \gtrsim 3 \text{ TeV}$ cannot be excluded and within three standard deviations, this scale drops down to $\Lambda_{NP} \gtrsim 2 \text{ TeV}$.

4.1 $|V_{us}/V_{ud}|$ from $K, \pi \rightarrow \mu \bar{\nu}$

In 2004, Marciano pointed out a window of opportunity for determining $|V_{us}/V_{ud}|$ from the ratio of leptonic decay rates $\Gamma(K \rightarrow \mu \bar{\nu}(\gamma))/\Gamma(\pi \rightarrow \mu \bar{\nu}(\gamma))$ [17]. Calculating $O(\alpha)$ radiative corrections to this ratio, he obtained (see update in [11]):

$$\frac{|V_{us}|}{|V_{ud}|} \frac{F_K}{F_\pi} = 0.2757(7) [0.25\%]. \quad (4.3)$$

Thus, a precise lattice calculation of F_K/F_π will allow a high precision determination of $|V_{us}/V_{ud}|$. One needs to determine F_K/F_π to:

- 0.5% to match the uncertainty on $|V_{us}|$ obtained in $K \rightarrow \pi \ell \nu$ decays,
- 0.25% to match the experimental uncertainty in $\Gamma(K \rightarrow \mu \bar{\nu}(\gamma))/\Gamma(\pi \rightarrow \mu \bar{\nu}(\gamma))$.

Ref.	publication	action, unit.	N_f	mass extrap	$a \rightarrow 0$	finite volume
ETM'08 [12]	*	***	**	**	***	**
NPLQCD'06 [13]	***	**	***	*	*	***
MILC'04-07 [14, 15]	***	**	***	**	***	***
HP/UKQCD'07 [16]	***	**	***	*	***	***
RBC/UKQCD'08 [5]	***	***	***	**	*	***
ALV'08 [18]	*	**	***	**	**	***
PACS-CS'08 [1]	**	***	***	***	*	*
BMW'08	*	***	***	***	***	***

Table 2: Starring of the simulations used to obtain F_K/F_π , according to the criteria put forth in Sec. 3.1.

F_K/F_π is an $SU(3)$ -flavor breaking effect, i.e.

$$F_K/F_\pi = 1 + O\left(\frac{M_K^2 - M_\pi^2}{\Lambda_\chi^2}\right) \quad (4.4)$$

and it is the deviation from unity that we are actually calculating, which makes the target accuracies a little less forbidding.

In Table 1 I summarize the parameters and results of all unquenched lattice calculation of F_K/F_π . The corresponding consumer report is given in Table 2.

Of all these calculations, the most advanced is that of MILC [14, 15], but the calculation of the BMW collaboration, presented at this conference by S. Dürr, should rival it once completed. The calculation of PACS-CS [1], performed very close to the physical up and down quark mass holds great promise. However, as it stands, it is missing a continuum extrapolation and may also suffer from significant finite-volume errors.

To illustrate lattice results for F_K/F_π , let me briefly present those of BMW. The calculations are performed in volumes up to 4 fm, with pions as light as 190 MeV and lattice spacings down to 0.065 fm. The parameters of the calculation are summarized in Table 1, and details of the ensembles can be found in [19]. The results are plotted in Fig. 3, as a function of M_π^2 in physical units, with the scale set by the Ξ mass as in [19]. The plot shows the extrapolation of the results for F_K/F_π in M_π^2 to the physical point. A large variety of functional forms have been tried, ranging from NLO $SU(2)$ ChPT to polynomial expansions. Three different cuts on pion mass have been made: $M_\pi < 420$ MeV, 470 MeV and 600 MeV. The continuum and mass extrapolations are combined, by allowing for the parameters of the functional mass dependence to acquire a^2 or a corrections. Finite-volume effects are subtracted at two-loops in ChPT, using the results of [20]. The procedure for estimating statistical and systematic uncertainties is very similar to that in [19]. It should be noted that the shift in F_K/F_π from the lightest pion mass to the physical point is less than 2%. The preliminary result is given in Table 1.

Unquenched, lattice results for F_K/F_π are summarized in Fig. 4, where my average for this quantity, obtained as explained in Sec. 3.2, is also given. This average includes only the published

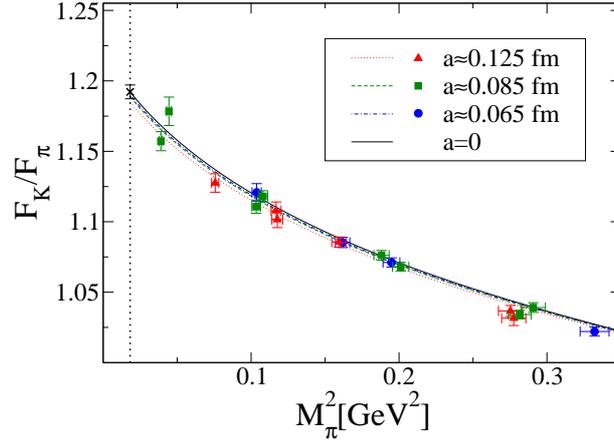


Figure 3: F_K/F_π versus M_π^2 in physical units, as presented by BMW at this conference. The different symbols correspond to different lattice spacings, as indicated in the plot legend. The curves represent the result of a combined chiral and continuum extrapolation fit for each lattice spacing, as well as in the continuum limit. The results have already been interpolated in strange quark mass to the physical value. The particular fit shown corresponds to a NLO, $SU(3)$ ChPT fit with $M_\pi < 470\text{MeV}$ and a^2 discretization errors.

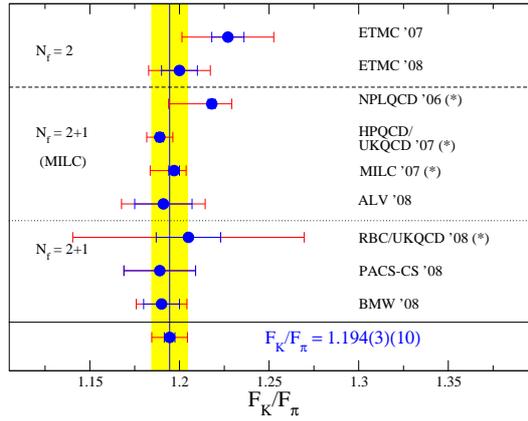


Figure 4: Summary of unquenched lattice results for F_K/F_π , together with my average. The latter is obtained as described in Sec. 3.2 and in the text. The results marked with a “(*)” are those included in the average. The smallest error bar on each point is the statistical error and the larger one, the statistical and systematic errors combined in quadrature. The references are as in Table 1.

$N_f = 2 + 1$ results [13–16, 5] in which many systematic uncertainties have been estimated. The systematic error is taken from [15]. The total uncertainty on this quantity is $\delta(F_K/F_\pi)^{lat} = 0.8\%$. This corresponds to an uncertainty of $\delta(F_K/F_\pi - 1)^{lat} \simeq 5\%$ on the calculated $SU(3)$ -flavor breaking effect, which is much better than the accuracy obtained on the $SU(3)$ -flavor breaking in the form factor for $K \rightarrow \pi \ell \nu$, $\delta f_+(0) \simeq 15\%$. Nevertheless, this uncertainty still leads to a larger theory error in the determination of $|V_{us}|$, i.e. 0.8% vs 0.5%. Since F_K/F_π is a straightforward quantity to calculate, one may expect steady improvements in its lattice determination, especially in light of the recent progress by PACS-CS [1].

4.2 $|V_{us}|$ from $K \rightarrow \pi \ell \nu$

$K \rightarrow \pi \ell \nu$ decays provide an alternative way to determine $|V_{us}|$. This measurement requires the theoretical calculation of the vector form factor, $f_+(q^2)$, defined through:

$$\langle \pi^+(p') | \bar{u} \gamma_\mu s | \bar{K}^0(p) \rangle = (p + p' - q) \frac{M_K^2 - M_\pi^2}{q^2} f_+(q^2) + q_\mu \frac{M_K^2 - M_\pi^2}{q^2} f_0(q^2), \quad (4.5)$$

with $q = p - p'$. The best precision is currently obtained by measuring the form factor shape in experiment and extracting, from the total rate [9],

$$|V_{us}| \times f_+(0) = 0.21664(48) [0.22\%]. \quad (4.6)$$

The experimental error is of similar size as in the ratio of leptonic kaon to pion decay rates. To fully exploit the experimental results requires a determination of $f_+(0)$ to 0.22%.

To the extent that it is applicable here, the theoretical framework for attacking this problem is $SU(3)$ ChPT [2, 21]. The chiral expansion for $f_+(0)$ is given by:

$$f_+(0) = 1 + f_2 + f_4 + \dots, \quad (4.7)$$

where the Ademollo-Gatto theorem [22] and ChPT yield:

$$f_2 = O\left(\frac{(M_K^2 - M_\pi^2)^2}{M_K^2 \Lambda_\chi^2}\right) = -0.023. \quad (4.8)$$

Thus, there are no contributions for the $O(p^4)$ LECs and this NLO contribution is fully determined by M_K , M_π and F_π .

This means that a sub-percent level determination of $f_+(0)$ requires a calculation of NNLO and higher corrections, since

$$\Delta f \equiv f_+(0) - 1 - f_2 = O\left(\frac{(M_K^2 - M_\pi^2)^2}{\Lambda_\chi^4}\right) \sim 3\% \quad (4.9)$$

is comparable in size to f_2 . To fully exploit the experimental accuracy “only” requires an accuracy of 7% in the calculation of Δf .

What is known about f_4 and more generally Δf ? The NNLO chiral logs have been computed [23, 24], and they require $O(p^6)$ LECs for renormalization. Estimates have been made of these LECs [24–27] and in [24] it is shown that they can be determined from the slope and curvature of $f_+(q^2)$. The reference value for Δf is still taken to be the quark model result, $\Delta f = -0.016(8)$ [21].

In Table 3, I summarize the parameters and results of all unquenched lattice calculations of $f_+(0)$. The corresponding consumer report is given in Table 4.

The lattice methodology for the calculation of $f_+(0) - 1$ was set forth in [33]. It consists of three main steps:

1. Use a double ratio of three-point functions to obtain:

$$f_0(q_{max}^2) = \frac{2\sqrt{M_K M_\pi} \langle \pi | V_0 | K \rangle \langle K | V_0 | \pi \rangle}{M_K + M_\pi \langle \pi | V_0 | \pi \rangle \langle K | V_0 | K \rangle}. \quad (4.10)$$

This yields a determination of $f_0(q_{max}^2)$ with a statistical error less than about 0.1%!

Ref.	N_f	action	a [fm]	L [fm]	M_π^{min} [MeV] typ/val	$f_+(0)$
JLQCD'05 [28]	2	NP SW	0.09	1.8	550/550	0.967(6)
RBC'06 [29]	2	DWF	0.12	2.5	490/490	0.968(9)(6)
ETM'08 [30]	2	tmQCD	0.11	2.7	260/260	0.957(5)
FNAL/MILC'04 [31]	2+1	KS+Wil				0.962(6)(9)
RBC/UKQCD'07[32]	2+1	DWF	0.11	1.8, 2.8	290/240	0.9644(33)(34)(14)

Table 3: Parameters of the simulations used by various collaborations for calculating $f_+(0)$, together with their result for that quantity. The description of the columns can be inferred from the one given in Table 1.

Ref.	publication	action, unit.	N_f	mass extrap	$a \rightarrow 0$	finite volume
JLQCD'05 [28]	*	***	**	*	*	***
RBC'06 [29]	***	***	**	*	*	***
ETM'08 [30]	*	***	**	**	*	**
FNAL/MILC'04 [31]	*	**	***			
RBC/UKQCD'07[32]	***	***	***	**	*	***

Table 4: Starring of the simulations used to obtain $f_+(0)$, according to the criteria put forth in Sec. 3.1.

2. Compute $f_0(q^2)$ at various q^2 and use an ansatz to interpolate and get $f_+(0) = f_0(0)$.
3. Interpolate/extrapolate in light quark mass to the physical mass point.

RBC/UKQCD [32] actually combine steps 2 and 3, using the functional form:

$$f_0(q^2; M_K, M_\pi) = \frac{1 + f_2(M_K, M_\pi) + (M_K^2 - M_\pi^2)^2 (A_0 + A_1(M_K^2 + M_\pi^2))}{1 - q^2 / (M_0 + M_1(M_K^2 + M_\pi^2))}, \quad (4.11)$$

where A_0, A_1, M_0 and M_1 are parameters and where a polynomial ansatz was made for NNLO terms. This combined fit is shown in the two panels of Fig. 5. Their results fit $1 + f_2(M_K, M_\pi) + \text{NNLO}$ well, though their fits do not take correlations into account. The claim that they are sensitive to NNLO effects seems to be justified. Moreover, the extrapolated result is only two standard deviations below the result obtained at their lightest pion mass and the claimed error on $f_+(0) - 1$ is a rather conservative 14%. The caveats are that m_s is approximately 15% too high and the calculations were performed at a single, rather coarse lattice spacing of $a = 0.114(2)$ fm, meaning that discretizations errors can only be guessed. Nevertheless, this is the first convincing lattice calculation of $f_+(0) - 1$.

Lattice and non-lattice results for $f_+(0)$ are summarized in Fig. 6, together with the ‘‘average’’ which I obtain by copying the result of [32]. The total uncertainty on $f_+(0)$ is $\delta f_+(0)^{lat} = 0.5\%$. This means that $K \rightarrow \pi \ell \nu$ decays still give the best accuracy for $|V_{us}|$. Moreover, one can anticipate that the current error, $\delta(f_+(0) - 1)^{lat} = 14\%$ will be reduced thanks to the use of: stochastic

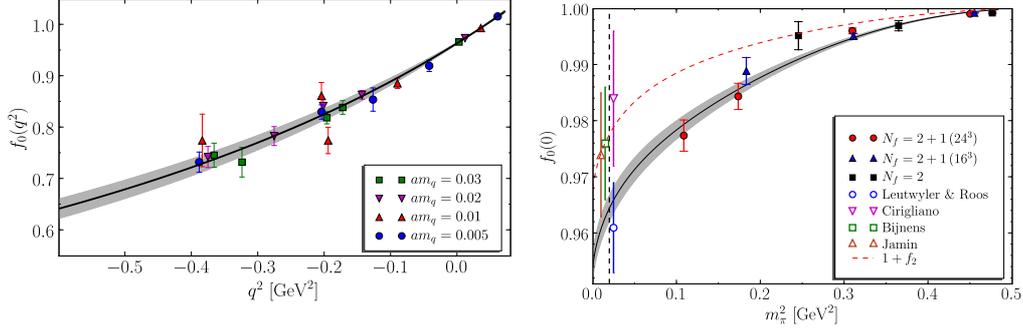


Figure 5: Combined q^2 and chiral fit of $f_0(q^2)$ to Eq. (4.11) by RBC/UKQCD '08 [32]. The left panel displays the lattice values of $f_0(q^2)$ vs q^2 , together with the fit curve obtained at the physical values of M_π and M_K . The lattice points were shifted in pion and kaon mass at fixed q^2 using the fit result. The right panel displays the extrapolation of $f_+(0) = f_0(0)$ in M_π^2 to physical pion mass.

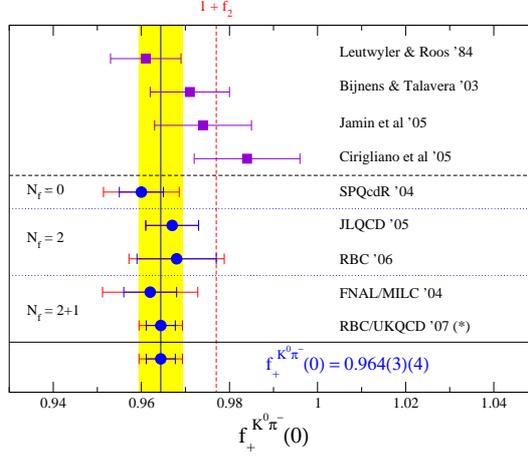


Figure 6: Summary of lattice results for $f_+(0)$, together with the results obtained in various models. Also given is my average of the unquenched lattice results. The latter is obtained as described in Sec. 3.2 and in the text. The results marked with a “(*)” are those included in the average. The smallest error bar on each point is the statistical error and the larger one, the statistical and systematic errors combined in quadrature. The references for the lattice works are as in Table 3 with, in addition, SPQcdR '04 [33]. The others are Leutwyler and Roos '84 [21], Bijens and Talavera '03 [24], Jamin et al '05 [25], Cirigliano et al '05 [26].

sources, as used in [34, 35]; partially twisted boundary conditions [36, 37], applied to form factors in [34, 35, 38–40], which enable to determine $f_+(q^2)$ directly at $q^2 = 0$ [39, 35].

5. $K \rightarrow \pi\pi$ decays on the lattice

The phenomenology of $K \rightarrow \pi\pi$ decays is extremely rich, and has been highly instrumental in developing the standard model. In the isospin limit, the amplitudes for these decays can be decomposed in terms of amplitudes $A_I e^{i\delta_I}$, $I = 0, 2$, where I is the isospin of the final two-pion state and δ_I is the strong scattering phase in that channel. CP violation implies that $A_I^* \neq A_I$. CP

violation occurs in two ways in K_L decays. K_L is mostly CP odd, and decays predominantly into three pions. But it has a small CP even component, through which it can also decay into two pions. This process is known as indirect CP violation, and is parametrized by [41–43]:

$$\varepsilon = \frac{T[K_L \rightarrow (\pi\pi)_{I=0}]}{T[K_S \rightarrow (\pi\pi)_{I=0}]} \simeq e^{i\phi_\varepsilon} \sin \phi_\varepsilon \left(\frac{\text{Im}M_{12}}{\Delta M_K} + \xi \right), \quad (5.1)$$

with $\xi = \text{Im}A_0/\text{Re}A_0$, $\Delta M_K \equiv M_{K_L} - M_{K_S}$ and M_{12} to be defined below. K_L decays can violate CP through another channel, by having its CP odd component decay directly into two pions. This process is known as direct CP violation, and is parametrized by [41]:

$$\begin{aligned} \varepsilon' &= \frac{1}{\sqrt{2}} \frac{T[K_S \rightarrow (\pi\pi)_{I=2}]}{T[K_S \rightarrow (\pi\pi)_{I=0}]} \left[\frac{T[K_L \rightarrow (\pi\pi)_{I=2}]}{T[K_S \rightarrow (\pi\pi)_{I=2}]} - \frac{T[K_L \rightarrow (\pi\pi)_{I=0}]}{T[K_S \rightarrow (\pi\pi)_{I=0}]} \right] \\ &\simeq \frac{1}{\sqrt{2}} e^{i(\pi/2 + \delta_2 - \delta_0)} \frac{\text{Re}A_2}{\text{Re}A_0} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right]. \end{aligned} \quad (5.2)$$

Experimentally a lot is known about these different processes [11]. The K_L - K_S mass difference is measured to high precision, i.e. $\Delta M_K = (3.483 \pm 0.006) \times 10^{-12} \text{ MeV}$ [0.2%]. $K \rightarrow \pi\pi$ decays exhibit a strong enhancement of the $I = 0$ channel over the $I = 2$ channel, $|A_0/A_2| \simeq 22.2$, known as the $\Delta I = 1/2$ rule, which is still in need of an explanation after over forty years. The parameter for indirect CP violation has also been measured to high accuracy, $|\varepsilon| = (2.229 \pm 0.012) \cdot 10^{-3}$ [0.5%], with a phase $\phi_\varepsilon = (43.5 \pm 0.7)^\circ$ [1.6%]. And after an experimental effort of nearly thirty years, direct CP violation was also measured, yielding $\text{Re}(\varepsilon'/\varepsilon) = (1.65 \pm 0.26) \cdot 10^{-3}$ [16%].

5.1 K^0 - \bar{K}^0 mixing in the standard model and B_K

K^0 - \bar{K}^0 mixing is responsible for the K_L - K_S mass difference as well as for indirect CP violation in $K \rightarrow \pi\pi$. In the standard model, the CP violating contribution occurs through a local $\Delta S = 2$, four-quark operator, once the heavy, standard model degrees of freedom are integrated out. The corresponding amplitude is

$$2M_K M_{12}^* = \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 \rangle = C_1^{\text{SM}}(\mu) \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle,$$

where C_1^{SM} is a short-distance Wilson coefficient and where

$$O_1 = (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} \quad \text{and} \quad \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle = \frac{16}{3} M_K^2 F_K^2 B_K(\mu). \quad (5.3)$$

In terms of these quantities, a revised [42, 43] standard model analysis [44] yields:

$$|\varepsilon| \simeq \kappa_\varepsilon C_\varepsilon \hat{B}_K \left[\text{Im}(\lambda_t^{*2}) \eta_{tt} S_0(x_t) + 2 \text{Im}(\lambda_t^* \lambda_c^*) \eta_{ct} S_0(x_c, x_t) + \text{Im}(\lambda_c^{*2}) \eta_{cc} S_0(x_c) \right], \quad (5.4)$$

where C_ε is determined by well measured quantities, $\hat{B}_K = C_1^{\text{SM}}(\mu) B_K(\mu)$ is the renormalization-group invariant B -parameter, $\lambda_q \equiv V_{qd} V_{qs}^*$ and $\eta_{qq'}$, S_0 are short-distance quantities. κ_ε parametrizes the corrections to the standard analysis [44], which arise from $\sqrt{2} \sin \phi_\varepsilon$ and ξ in Eq. (5.1) at leading order. This approximation is not necessary and should not be used for precision tests of the SM once ξ is known better. A rough estimate yields $\kappa_\varepsilon = 0.92(2)$ [43], which implies an $8 \pm 2\%$ downward shift in the SM prediction for ε . From Eq. (5.4), it is clear that a measurement of $|\varepsilon|$ and a determination of B_K imposes constraints on $\text{Im}\lambda_t^{*2}$, $\text{Im}\lambda_c^{*2}$ and $\text{Im}\lambda_t^* \lambda_c^*$, as shown in Fig. 7.

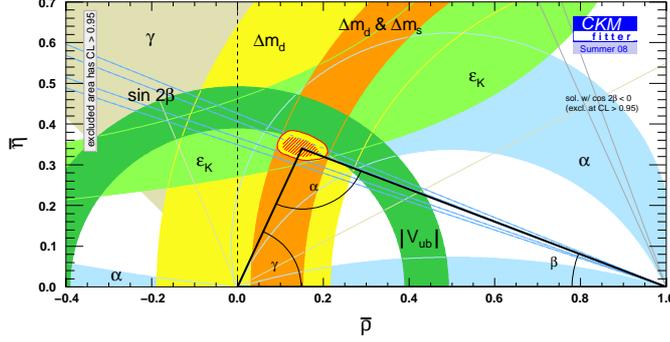


Figure 7: Constraints on the summit $(\bar{\rho}, \bar{\eta})$ of the unitarity triangle from a global CKM fit [10].

Given how accurately $|\varepsilon|$ is measured, one may wonder why the constraint that it gives on the summit of the triangle is not any better. To help answer this question, in Fig. 8 I display SM predictions for $|\varepsilon|$ obtained in different ways. The starting point is a global CKM fit using Eq. (5.4), in which the experimental measurement for $|\varepsilon|$ is not included and where $\hat{B}_K = 0.723(11)(35)$ [5%] (from Fig. 9) and $|V_{cb}| = 0.04059(38)(58)$ [1.7%] [10]. The topmost theoretical prediction for $|\varepsilon|$ is obtained from this global fit, allowing all quantities to fluctuate within their error bars.² The next result is obtained by freezing $|V_{cb}|$ to its central value. The third value results from fixing B_K to its central value. The fourth is obtained by freezing both $|V_{cb}|$ and B_K , and the fifth by fixing B_K to its central value and the four CKM parameters to their best global fit values. The last is the experimental measurement quoted above.

As the second point indicates, a determination of $|V_{cb}|$ to infinite accuracy only reduces the uncertainty on the prediction for $|\varepsilon|$ from 10% to 9%. Significantly improving the accuracy on B_K has a similar effect, since the uncertainty on $|\varepsilon|$ is also 9% in that case. The fourth point indicates that the uncertainty coming from sources other than B_K and $|V_{cb}|$ is a little less than 8%. It is only when the uncertainties on B_K and CKM parameters are assumed to be zero that the error on the SM prediction for $|\varepsilon|$ falls to 5%. The latter is due to perturbative uncertainties and to the error on κ_ε .

It is interesting to note that the SM prediction for $|\varepsilon|$ is now about 18% below the experimental value, down from what it was until recently. The decrease is mainly due to the fact that the central value for \hat{B}_K has dropped by more than 15% over the last decade (see discussion below and caveats) and to the presence of the correction related to κ_ε in Eq. (5.4). This potential tension between theory and observation has led the authors of [45, 43, 46] to investigate new CP violating contributions to $\Delta F = 2$ observables. For the moment, as the top point in Fig. 8 indicates, the discrepancy is about two standard deviations. However, if uncertainties on B_K and on CKM parameters improve substantially, this discrepancy could become significant, as the fifth point of Fig. 8 indicates.

B_K is a quantity which has a long history on and off the lattice. However, because of space constraints, in Table 5 I only summarize the parameters and results of unquenched lattice calculations. The corresponding consumer report is given in Table 6.

²Systematic errors due to theory are assumed to have a *gaussian* distribution, so that the deviation of the predicted from the measured $|\varepsilon|$ can be counted in standard deviations.

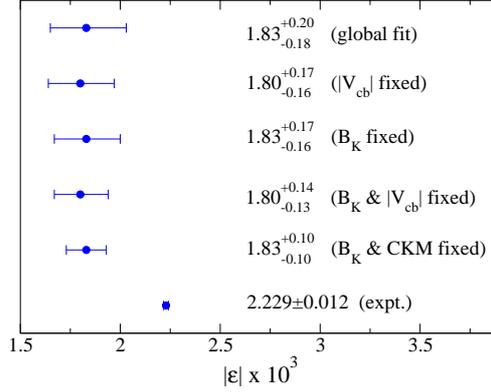


Figure 8: Standard model predictions for $|\epsilon|$, using results from global CKM fits by CKMfitter [10, 47], compared to experiment. The various predictions are obtained by fixing different inputs to their central values, as described in the plot legend and in the text.

Ref.	N_f	action	a [fm]	L [fm]	M_π^{min} [MeV] typ/val	\hat{B}_K
JLQCD'08 [48]	2	Overlap	0.12	1.9	290/290	0.734(5)(55)
ETM'08 [49]	2	OS/tmQCD	0.07,0.09	2.1,2.7	300/300	0.78(3)
HPQCD/ UKQCD'06 [50]	2+1	KS _{MILC} ^{HYP}	0.125	2.5	460/360	0.85(2)(18)
RBC/UKQCD '07-08 [51, 5]	2+1	DWF	0.11	1.8,2.8	290/240	0.717(14)(35)
Bae et al '08 [52]	2+1	KS _{MILC} ^{HYP}	$\gtrsim 0.06$	4	300/240	$\delta\hat{B}_K \rightarrow 3\%$

Table 5: Parameters of the simulations used by various collaborations for calculating the renormalization group invariant parameter \hat{B}_K , together with their result for that quantity. The values of \hat{B}_K in the table have been obtained at NLO from the RI/MOM or \overline{MS} -NDR values given in the papers. The description of the columns can be inferred from the one given in Table 1.

At this conference, A. Vladikas presented new, preliminary results for B_K from ETM [49]. Their calculation makes use of Osterwalder-Seiler valence quarks on the ETM, $N_f = 2$, tmQCD seas to ensure automatic $O(a)$ -improvement as well as multiplicative renormalization of the $\Delta S = 2$, four-quark operator. This implies that their calculation suffers from $O(a^2)$ unitarity violations, which must be controlled. Their plan is to use ETM's three lattice spacings, $a \simeq 0.07, 0.09, 0.010$ fm to extrapolate to the continuum limit. For the moment, though, all results are obtained from simulations performed at $a \simeq 0.09$ fm. They extrapolate in up and down quark mass using NLO, partially-quenched $SU(2)$ ChPT [4, 5] and interpolate in valence strange quark mass linearly. Note that the extrapolated value of B_K is only $\sim 3\%$ below its value at the lightest up and down quark mass, suggesting that this extrapolation is well controlled. The renormalization is performed nonperturbatively in the RI/MOM scheme [53]. The continuum limit and finite-volume corrections are still missing. Discretization induced unitarity violations will have to be investigated.

Unquenched, lattice results for \hat{B}_K are summarized in Fig. 9, where my average for this quantity, obtained as explained in Sec. 3.2, is also given. Only the two $N_f = 2 + 1$ results [50, 51, 5] are taken into account in this average. The systematic error is taken from [51, 5]. However, each

Ref.	publication	action. unit.	N_f	mass extrap	$a \rightarrow 0$	finite volume	renorm
JLQCD'08 [48]	***	***	**	**	*	*	**
ETM'08 [49]	*	**	**	**	**	**	**
HPQCD/ UKQCD'06 [50]	***	**	***	*	*	***	*
RBC/UKQCD '07-08 [51, 5]	***	***	***	**	*	***	**

Table 6: Starring of the simulations used to obtain B_K , according to the criteria put forth in Sec. 3.1.

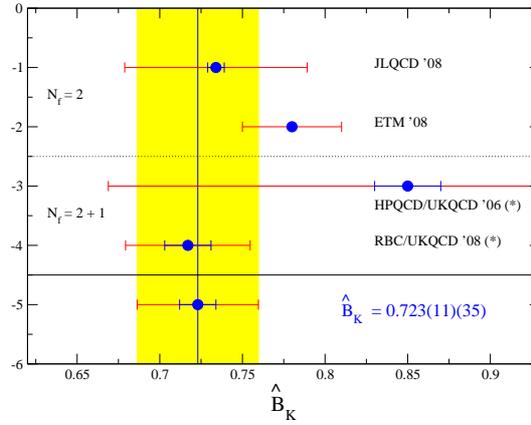


Figure 9: Summary of unquenched lattice results for the renormalization group invariant \hat{B}_K , together with my average. The latter is obtained as described in Sec. 3.2 and in the text. The smallest error bar on each point is the statistical error and the larger one, the statistical and systematic errors combined in quadrature. The results marked with a “(*)” are those included in the average. The references are as in Table 5.

calculation was performed at a single, rather coarse value of the lattice spacing. This means that these results, and thus the average, suffer from a poorly controlled discretization errors.

As noted above, the value of B_K has come down quite significantly compared to JLQCD’s standard quenched value of a decade ago [54]. In particular, $(B_K)_{RBC}^{N_f=2+1} / (B_K)_{JLQCD}^{N_f=0} = 0.83(8)$. This drop cannot really be ascribed to the inclusion of sea quark effects, since comparably low results were obtained in the quenched approximation at comparable lattice spacings [55]. However there, a continuum extrapolation based on two calculations performed at $a \simeq 0.10$ fm and 0.067 fm increased the result to $(B_K)_{RBC}^{N_f=0} / (B_K)_{JLQCD}^{N_f=0} = 0.90(9)$. Thus, it is very important to clarify this situation by investigating the continuum limit of B_K in 2+1 flavor simulations.

The total lattice error on B_K is $\delta B_K^{lat} = 5\%$, which is comparable to the other uncertainties in the standard model prediction for ε . As discussed above, to improve this prediction and possibly reveal new physics, we must not only reduce the error on B_K , but also improve the determination of CKM parameters, of the correction related to κ_ε and eventually of the short distance QCD coefficients.

6. Conclusion

Lattice QCD simulations have made tremendous progress in the last few years. $2 + 1$ flavor lattice calculations with pion masses as low as $M_\pi \sim 190 \text{ MeV}$ in $(4 \text{ fm})^3$ volumes, and lattice spacings down to $\sim 0.065 \text{ fm}$ have already been performed [19]. Moreover, as PACS-CS has shown [1], simulations at physical M_π are around the corner. Thus, it has now become possible to reach the physical QCD point ($M_\pi \simeq 135 \text{ MeV}$, $a \rightarrow 0$, $L \rightarrow \infty$) in a controlled fashion.

Quantities such as F_K/F_π and $f_+^{K^0\pi^-}(0)$ are already being computed with percent or better accuracy and are having an important impact on SM and BSM tests. Quantities such as B_K are reaching the sub 10% accuracy level and have errors which match those from other sources. Calculations of ε'/ε and the $\Delta I = 1/2$ enhancement still have 100% uncertainties despite the impressive $N_f = 2 + 1$ RBC/UKQCD effort [56], but perhaps not for long [56]. Many quantities are still missing continuum extrapolations.

NLO $SU(3)$ ChPT appear to be having trouble at the physical strange quark mass, at least in the presence of heavier up and down quarks, whereas $SU(2)$ ChPT performs better. However, these inferences require further investigation, in particular once continuum limits have been investigated.

Concerning the extrapolations and interpolations required to reach the physical mass point $(m_{ud}, m_s) = (m_{ud}^{ph}, m_s^{ph})$, my advice is to keep an open mind and to pick the approach which gives the lowest combined statistical and systematic error.

To conclude, the age of precision, nonperturbative QCD calculations is dawning, and the next few years should bring many exciting results.

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