PROCEEDINGS OF SCIENCE

A determination of the B_s^0 and B_d^0 mixing matrix elements using 2+1 lattice QCD

R. Todd Evans*, **Aida X. El-Khadra, and Elvira Gámiz**
Department of Physics, University of Illinois, Urbana, IL 61801, USA
E-mail: rtevans@illinois.edu, axk@illinois.edu, megamiz@illinois.edu

Fermilab Lattice and MILC Collaborations

We present a calculation of the hadronic matrix elements relevant to $B^0-\bar{B}^0$ mixing and a preliminary result for the $SU(3)$ breaking ratio $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}} = 1.205(52)$. The calculation is done on the MILC lattices with 2+1 flavors of Asqtad-improved sea quarks. We use the Asqtad action for the light valence quarks and the Fermilab action for the b quark.

The XXVI International Symposium on Lattice Field Theory
July 14 - 19, 2008
Williamsburg, Virginia, USA

*Speaker.

PoS(LATTICE 2008)052

1. Introduction

Experimental measurements of the mass differences between the heavy and light mass eigenstates of the B and B_s systems, ΔM_d and ΔM_s respectively, have uncertainties of less than 1% [1][2]. The Standard Model expressions for the mass difference depend on the CKM matrix elements $|V_{td}|$ and $|V_{ts}|$, and hadronic matrix elements which describe the nonperturbative QCD corrections to this process. Hence, the experimental measurements together with theoretical predictions of the matrix elements can help test the unitarity of the CKM matrix and provide constraints on the possible contributions to new physics.

The B_q-$\bar{B}_q$, $q = d, s$, meson system's mass difference, ΔM_q, is parametrized as

$$\Delta M_q = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B(\mu_B) S_0(m_t, m_W) |V_{tq} V_{tb}^*|^2 \langle \bar{B}_q | Q_q^1(\mu_B) | B_q \rangle, \tag{1.1}$$

where η_B is a Wilson coefficient, $S_0(m_t, m_W)$ is known as the Inami-Lim function, and the scale $\mu_B \approx m_B$. The hadronic matrix element is conventionally parametrized as

$$\langle \bar{B}_q | Q_q^1 | B_s \rangle = \langle \bar{B}_q | \bar{b}\gamma_\mu(1-\gamma_5)q\bar{b}\gamma^\mu(1-\gamma_5)q | B_q \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}, \tag{1.2}$$

where f_{B_q} is the decay constant of the B_q meson, and B_{B_q} is the bag parameter. The ratio of CKM matrix elements, $|V_{td}/V_{ts}| = \xi \sqrt{\frac{\Delta M_d}{\Delta M_s} \frac{m_{B_s}}{m_{B_d}}}$, is an important input into unitarity triangle fits, where the dominant source of uncertainty is contained in the hadronic quantity $\xi = f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$. This uncertainty can only be reduced using lattice QCD techniques. This report presents our preliminary calculation of the $SU(3)$ breaking ratio of the hadronic matrix elements, ξ, using 2+1 lattice QCD, including a careful analysis of the systematic errors.

2. Simulation Details

We perform the calculation on the MILC 2+1 flavor lattices. The MILC gauge action used in the generation of these lattices has discretization errors starting at $\mathcal{O}(a^2\alpha_s, a^4)$ [3, 4]. The staggered Asqtad action is used for the sea quarks and has errors starting at $\mathcal{O}(a^2\alpha_s, a^4)$ [5]. The Asqtad action is also used for the light valence quark propagators, where the staggered propagator is converted to a naive propagator before construction of the correlation functions. The Fermilab action is used to construct the b quark propagator, with discretization errors starting at $\mathcal{O}(\alpha_s\Lambda_{\mathrm{QCD}}/M, (\Lambda_{\mathrm{QCD}}/M)^2)$ [6]. To improve Q_q^1 to the same order as the Fermilab action we rotate the b quark.

We calculate the matrix elements at two lattice spacings, $a = 0.12$ and 0.09 fm, and at 6 light sea quark masses and 6 light valence quark masses for a total of 36 mass combinations, summarized in Table 1. The lightest sea pion mass is $m_{\pi,sea} \sim 250$ MeV and the lightest valence pion mass is $m_{\pi,val} \sim 240$ MeV. To improve our statistics we calculate the correlators on each ensemble and mass combination at 4 time sources and average them.

3. Correlators

The operator Q_q^1 is fixed at the origin with the positions of the $\bar{B}_q$ and B_q mesons varied. The matrix elements are extracted from the three-point correlators at spatial momentum $\vec{p} = 0$ by fitting

Table 1: The masses and lattice spacings used in the calculation. The ensembles are labeled by the sea quark mass content in lattice units, where m_L refers to the mass of the 2 degenerate light sea quarks and m_h refers to the mass of the heavier strange sea quark. The mass of the light valence quark is m_q and $N_{configs}$ is the number of configurations on which the calculations are performed.

am_L/am_h	am_q	$N_{configs}$
	a=0.12 fm	
0.005/0.050	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415	529
0.007/0.050	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415	833
0.010/0.050	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415	580
0.020/0.050	0.005, 0.007, 0.01, 0.02, 0.03, 0.0415	460
	a=0.09 fm	
0.0062/0.031	0.0031, 0.0044, 0.0062, 0.0124, 0.0272, 0.031	553
0.0124/0.031	0.0031, 0.0042, 0.0062, 0.0124, 0.0272, 0.031	534

to the expression

$$C_{Q_q^1}(t_1,t_2) = \sum_{\vec{x}_1,\vec{x}_2} \langle \bar{B_q}(t_1,\vec{x}_1)|Q_q^1(0)|B_q(t_2,\vec{x}_2)\rangle = \sum_{i,j}((-1)^{t_1+1})^i((-1)^{t_2+1})^j \frac{Z_i Z_j O_{ij}}{(2E_i)(2E_j)} e^{-E_i t_1 - E_j t_2}, \tag{3.1}$$

where i,j number the states and $O_{00} = \langle \bar{B_q}|Q_q^1|B_q\rangle = \frac{8}{3} M_{B_q}^2 f_{B_q}^2 B_{B_q}$. We smear the interpolating b quark using a 1S wave-function to improve overlap with the ground state. In order to isolate O_{00} and extract the quantity $\beta_q = f_{B_q}\sqrt{M_{B_q} B_{B_q}}$, we simultaneously fit the two-point pseudoscalar B_q correlator to the following function,

$$C_2^q(t) = \sum_{\vec{x}} \langle B_q(t,\vec{x})|\bar{q}(0)\gamma_5 b(0)\rangle = \sum_i ((-1)^{t+1})^i \frac{|Z_i|^2}{2E_i} e^{-E_i t}. \tag{3.2}$$

Bayesian fitting techniques are used [7]. The states oscillating in Euclidean time are due to the temporal doublers from the naive light quark in the operator and are straightforward to remove [8].

The lattice data for β_q are shown in Fig. (1). Some characteristics of the data relevant to the chiral fits are the following: mild sea quark mass dependence, mild lattice spacing dependence, and statistical errors that vary between 2-5%.

4. Chiral Fits and Extrapolations

We fit our numerical lattice data for the parameters β_q and their ratio ξ to expressions derived in heavy meson staggered Chiral Perturbation Theory (HMSχPT) [9]. We work to lowest-order in the heavy quark mass, $\mathcal{O}(1/m_b)$, and next-to-leading order (NLO) in the light quark masses and lattice spacing. We determine the light quark mass dependence and then extrapolate to the physical u- and d-quark masses and interpolate to the physical s-quark mass. Because we have data at two lattice spacings, we can also determine the size of $\mathcal{O}(a^2)$ taste-breaking light quark discretization effects and largely remove them. The NLO HMSχPT expression for the B_q-$\bar{B_q}$ mixing matrix

PoS(LATTICE 2008)052

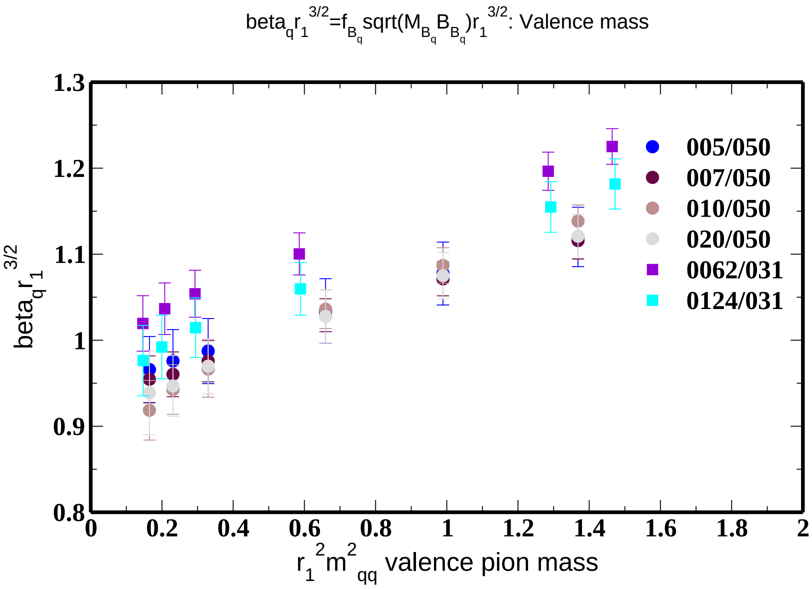

Figure 1: The fit results for β_q. The results are plotted versus the mass of a pion constructed from the light valence quarks. The data from different ensembles are labeled by their sea quark content, m_L/m_h. The circular data points are from ensembles with lattice spacing 0.12 fm and the square data points are from ensembles with lattice spacing 0.09 fm. The results are converted from lattice units to r_1 units [11].

element is [10],

$$\langle \bar{B}_q | Q_q^1 | B_q \rangle_{QCD} = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_q = m_{B_q} \langle \bar{B}_q | Q_1^q | B_q \rangle_{HQET} = \\ m_{B_q} \beta \left[1 + (\mathscr{Q}_q + \mathscr{W}_q + \mathscr{T}_q) + L_v m_q + L_s (2m_L + m_h) + L_a a^2 \right] . \tag{4.1}$$

The low-energy constants β, L_v, L_s, and L_a are determined from the lattice data, and m_q, m_L, and m_h denote the light valence, light sea, and strange sea quarks respectively. The light sea quarks are treated as degenerate, so the isospin average is used, $m_L = \frac{m_u + m_d}{2}$.

The chiral logs, $\mathscr{T}_q$, $\mathscr{W}_q$, $\mathscr{Q}_q$, due to the tadpole, wavefunction renormalization, and sunset diagrams respectively, each have a similar structure,

$$\begin{aligned} \mathscr{W}_q &= -\frac{3g^2_{B*B\pi}}{16\pi^2 f^2} \left\{ \frac{1}{16} \left[\sum_{\Xi = I,P,4V,4A,6T} \bar{h}_\Xi^q \right] + \frac{1}{3} h_I^q + a^2 (\delta'_V h_V^q + \delta'_A h_A^q) \right\}, \\ \mathscr{T}_q &= -\frac{1}{16\pi^2 f^2} \left\{ \frac{1}{16} \left[\sum_{\Xi = I,P,4V,4A,6T} \bar{h}_\Xi^q + \bar{I}_\Xi \right] + \frac{2}{3} h_I^q + a^2 (\delta'_V h_V^q + \delta'_A h_A^q) \right\}, \\ \mathscr{Q}_q &= \frac{3g^2_{B*B\pi}}{16\pi^2 f^2} \left\{ \frac{1}{16} \left[\sum_{\Xi = I,P,4V,4A,6T} \bar{I}_\Xi^q \right] + \frac{1}{3} h_I^q \right\}. \end{aligned} \tag{4.2}$$

The labels I, P, V, A, and T refer to the taste of the meson in the chiral logarithm. For staggered quarks the taste-nonsinglet pseudoscalar meson masses are split

$$M_{ij,\Xi}^2 = \mu(m_i + m_j) + a^2 \Delta_\Xi, \tag{4.3}$$

where m_i, m_j are quark masses and the sixteen meson masses are labeled by their taste representation, $\Xi = P, A, T, V, I$. The parameters μ and Δ_Ξ are determined from lattice calculations for pions and kaons [11]. The B-B^*-π coupling, $g^2_{B*B\pi}$, is constrained by the CLEO measurement, $g^2_{D*D\pi} = 0.35 \pm 0.14$, since heavy-quark symmetry implies $g^2_{B*B\pi} \approx g^2_{D*D\pi}$ [12]. For the parameter f we use the PDG value for the pion decay constant, $f \sim f_\pi \approx 130$ MeV. In the actual fits all dimensionful parameters are made dimensionless by multiplication of appropriate powers of r_1 as explained in Ref. [11]. The prior values and widths for δ'_V and δ'_A are also determined from lattice calculations for pions and kaons [11].

Next-to-next-to-leading order (*NNLO*) analytic terms are also included in our fits, with priors and constraints estimated by power counting in HMSχPT. The *NNLO* analytic terms are included because the s quark mass breaks the chiral symmetry to a large enough degree that *NNLO* terms may be important. The error associated with leaving out the *NNLO* chiral logs and higher order terms is estimated by including and excluding all combinations of the *NNLO* analytic terms, using the extremes of the range in fit results as our light quark discretization and chiral extrapolation Ansatz error.

In the continuum limit the chiral logarithms $\bar{h}_\Xi$, $\bar{I}_\Xi$, and h_I combine to form the continuum chiral expression, and the taste violating chiral logarithms, h_V and h_A, are set to 0. When extrapolating to the physical results we set $\Delta_\Xi = \delta'_{A,V} = a = 0$, $m_L \to (m_u + m_d)/2$, and $m_h \to m_s$. We then obtain $\langle \bar{B}_d^0 | Q_d^1 | B_d^0 \rangle$ and $\langle \bar{B}_s^0 | Q_s^1 | B^0 \rangle$ by setting $m_q \to m_d$ or m_s. We fit the data simultaneously to all 6 sea and 6 valence quark masses (36 lattice data points) to determine the low energy constants.

The fits and extrapolations for $\xi' = f_{B_s}\sqrt{M_{B_s} B_{B_s}} / f_{B_d}\sqrt{M_{B_d} B_{B_d}}$ are shown in Figs. (2)-(3). The perturbative matching corrections have not yet been included in these results. At one-loop the operator $\langle \bar{B}_q^0 | \bar{b}(1-\gamma_5) q \bar{b}(1-\gamma_5) q | B_q^0 \rangle$ mixes with Q_q^1. The one-loop correction to ξ is expected to be at the less than 1% level, which is much smaller than most other uncertainties in our calculation.

5. Results

Our preliminary results for ξ' are shown in Table 2. This table also lists the error budget for β_d and β_s. We find for $\xi = \xi' \sqrt{M_{B_d}/M_{B_s}}$,

$$\xi = 1.205(52). \tag{5.1}$$

Our largest sources of uncertainty in ξ' are from statistical errors in the chiral extrapolation, and the light quark discretization and Ansatz errors in the chiral fit and extrapolation. The smaller higher-order matching and heavy quark discretization errors are estimated using power counting. The uncertainties due to the chiral fits' input parameters, r_1, $g_{BB^*\pi}$, and light quark masses, are estimated by individually varying the input parameters within their range of uncertainty determined in Ref. [11]. The largest sources of uncertainty in β_d and β_s are due to statistical errors in the chiral extrapolations, higher-order matching terms, and the determination of the scale. The uncertainties due to these parameters are estimated in the same way as those of ξ'.

We find no significant uncertainty due to the determination of κ_b in our preliminary estimates. As a conservative estimate we quote the same uncertainty as in the Fermilab Lattice and MILC collaborations' calculation of f_B and f_{B_s} [13]. The negligible finite volume errors were estimated using the procedure described in Ref. [9].

PoS(LATTICE 2008)052

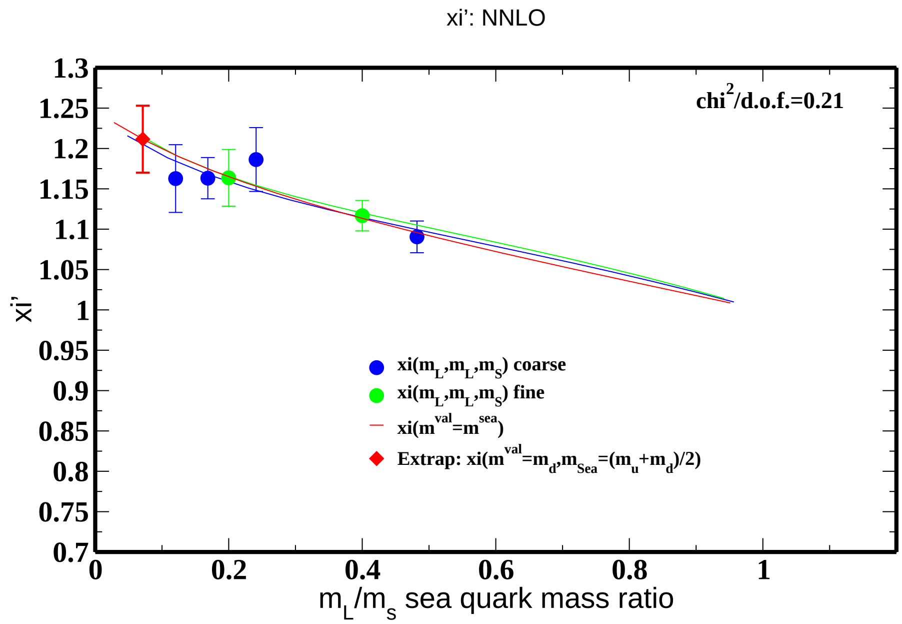

Figure 2: The fit and extrapolation is presented in the sea mass plane, where data is plotted versus the sea quark mass ratio m_L/m_s. The blue circles correspond to the coarse lattice data, and the green circles to the fine lattice data. The same colored lines are the fits to the data. The red line is the continuum extrapolation with the red diamond and error bars corresponding to the physical mass point. The errors are 68% confidence intervals and statistical only.

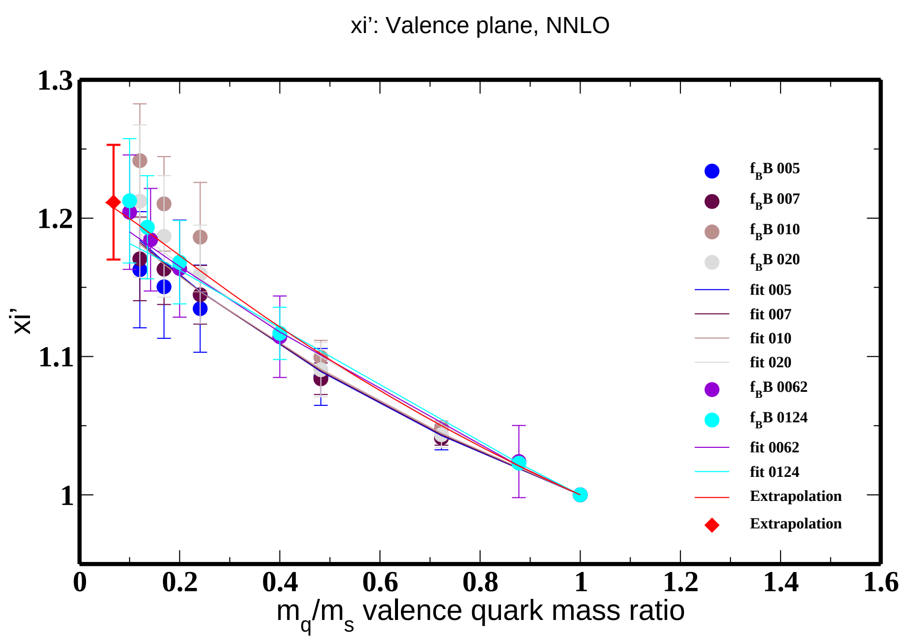

Figure 3: This is the same fit as above shown in the valence quark mass ratio plane. The different colors and fit lines correspond to the different ensembles and fits to them. The red line and diamond correspond to the continuum and mass extrapolations.

Table 2: The total systematic error budget for the B mixing matrix elements calculation. We do not include central values for β_d and β_s. The uncertainty due to κ_b is negligible in our preliminary estimates, and the same uncertainty as in the Fermilab Lattice and MILC collaborations' calculation of f_B and f_{B_s} is used.

Parameter	ξ'	$\beta_d r_1^{3/2}$	$\beta_s r_1^{3/2}$
Central Value	1.215	N/A	N/A
Source of Uncertainty		% Error	
Statistical	3.1	4.0	2.7
Higher Order Matching	~ 0.5	~ 4.0	~ 4.0
Heavy Quark Discretization	0.2	1.8	1.8
Light Quark Discretization + Ansatz	2.8	2.5	0.4
scale uncertainty (r_1)	0.2	3.1	3.0
$g_{BB^*\pi}$	0.3	0.6	0.3
input parameters: m_L,m_d,m_s	0.7	0.5	0.3
finite volume	< 0.1	< 0.1	< 0.1
κ_b	< 0.1	1.1	1.1
Total Systematic	3.0	6.1	5.4
Total % Error	4.3	7.3	6.1

References

[1] A. Abulencia *et al.* [CDF - Run II Collaboration], Phys. Rev. Lett. **97** (2006) 062003 [arXiv:hep-ex/0606027].

[2] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.

[3] C. W. Bernard *et al.*, Phys. Rev. D **64** (2001) 054506 [arXiv:hep-lat/0104002].

[4] M. G. Alford, W. Dimm, G. P. Lepage, G. Hockney and P. B. Mackenzie, Phys. Lett. B **361** (1995) 87 [arXiv:hep-lat/9507010].

[5] G. P. Lepage, Phys. Rev. D **59** (1999) 074502 [arXiv:hep-lat/9809157].

[6] A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D **55** (1997) 3933 [arXiv:hep-lat/9604004].

[7] G. P. Lepage, B. Clark, C. T. H. Davies, K. Hornbostel, P. B. Mackenzie, C. Morningstar and H. Trottier, Nucl. Phys. Proc. Suppl. **106** (2002) 12 [arXiv:hep-lat/0110175].

[8] M. Wingate, J. Shigemitsu, C. T. H. Davies, G. P. Lepage and H. D. Trottier, Phys. Rev. D **67** (2003) 054505 [arXiv:hep-lat/0211014].

[9] C. Aubin and C. Bernard, Phys. Rev. D **73** (2006) 014515 [arXiv:hep-lat/0510088].

[10] Claude Bernard, Jack Laiho, and Ruth Van de Water, Private communication.

[11] C. Aubin *et al.* [MILC Collaboration], Phys. Rev. D **70** (2004) 114501 [arXiv:hep-lat/0407028].

[12] A. Anastassov *et al.* (CLEO) Phys. Rev. D **65** (2002) 032003 [arXiv:hep-ex/0108043].

[13] C. Bernard *et al.* [Fermilab Lattice, MILC and HPQCD Collaborations], PoS **LAT2007** (2007) 370.

[14] M. Di Pierro *et al.* [FermiQCD Collaboration], Nucl. Phys. Proc. Suppl. **129** (2004) 832 [arXiv:hep-lat/0311027].