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Finite Chemical Potential in $N_t = 6$ QCD

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We report results of the investigation of physics at finite chemical potential using the method of Taylor expansion of the free energy for staggered quarks with $m_\pi \simeq 210$ MeV on $N_t = 6$ lattices. When the spatial volume is $(4/T^E)^3$ we estimate that the location of the critical end point is at $T^E/T_c \simeq 0.94(1)$ and $\mu_B^E/T^E \simeq 1.8(1)$. This is to be compared to an earlier estimate of $T^E/T_c \simeq 0.95(1)$ and $\mu_B^E/T^E \simeq 1.3(3)$ made on similar spatial volumes and quark masses but with $N_t = 4$.

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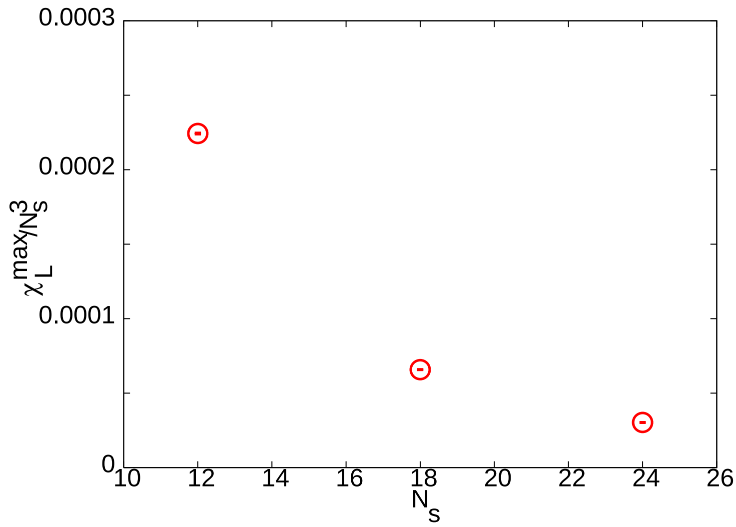

Figure 1: The peak value of χ_L grows slower than N_s^3, hence standard finite size scaling arguments indicate that this is not a first order phase transition.

1. Technical preliminaries

We report results from a simulation of two flavours of staggered quarks with mass $m/T_c = 0.1$ (corresponding to $m_\pi/m_\rho = 0.3$) on lattices of sizes 6×12^3, 6×18^3 and 6×24^3 [1]. The simulations were performed using the R-algorithm. Most of trajectories were of length 1 MD time unit and used time steps of 0.01 units. In tests with time steps of 0.001 units, we found that bulk quantities such as plaquettes, Polyakov loops and the quark condensate remained unchanged. Increasing the trajectory length to 3 MD time units also made no changes to these quantities. However, the longer trajectories gave shorter autocorrelation times.

The finite temperature cross over coupling was monitored using the Polyakov loop susceptibility, χ_L and two operators which enter the quark number susceptibilities (QNS), namely $\langle O_{22} \rangle_c$ and $\langle O_{44} \rangle_c$. All these measures were compatible within the precision of our measurements. We found $\beta_c = 5.425(5)$. Previous results bracket this value: for $m/T_c = 0.15$ it was reported that $\beta_c = 5.438(40)$ [2] and for $m/T_c = 0.075$ it was found that β_c was in the range 5.41–5.43 [3]. We found that 3-loop scaling worked reasonably well between $N_t = 4$ and 6. Standard finite size scaling analysis indicated that the transition is not of first order (see Figure 1). Distinctions between a second order transition and a cross over requires larger spatial volumes.

At each temperature we generated at least 100 statistically independent gauge configurations, and in the region near T_c we around 200 such statistically independent configurations. For each configuration we measured all the QNS upto the eighth order using a noisy determination with 500 random vectors. Twenty matrix inversions are required to do this for each random vector. Nevertheless, the CPU time spent in these measurements were an order of magnitude smaller than that spent in generating the configurations.

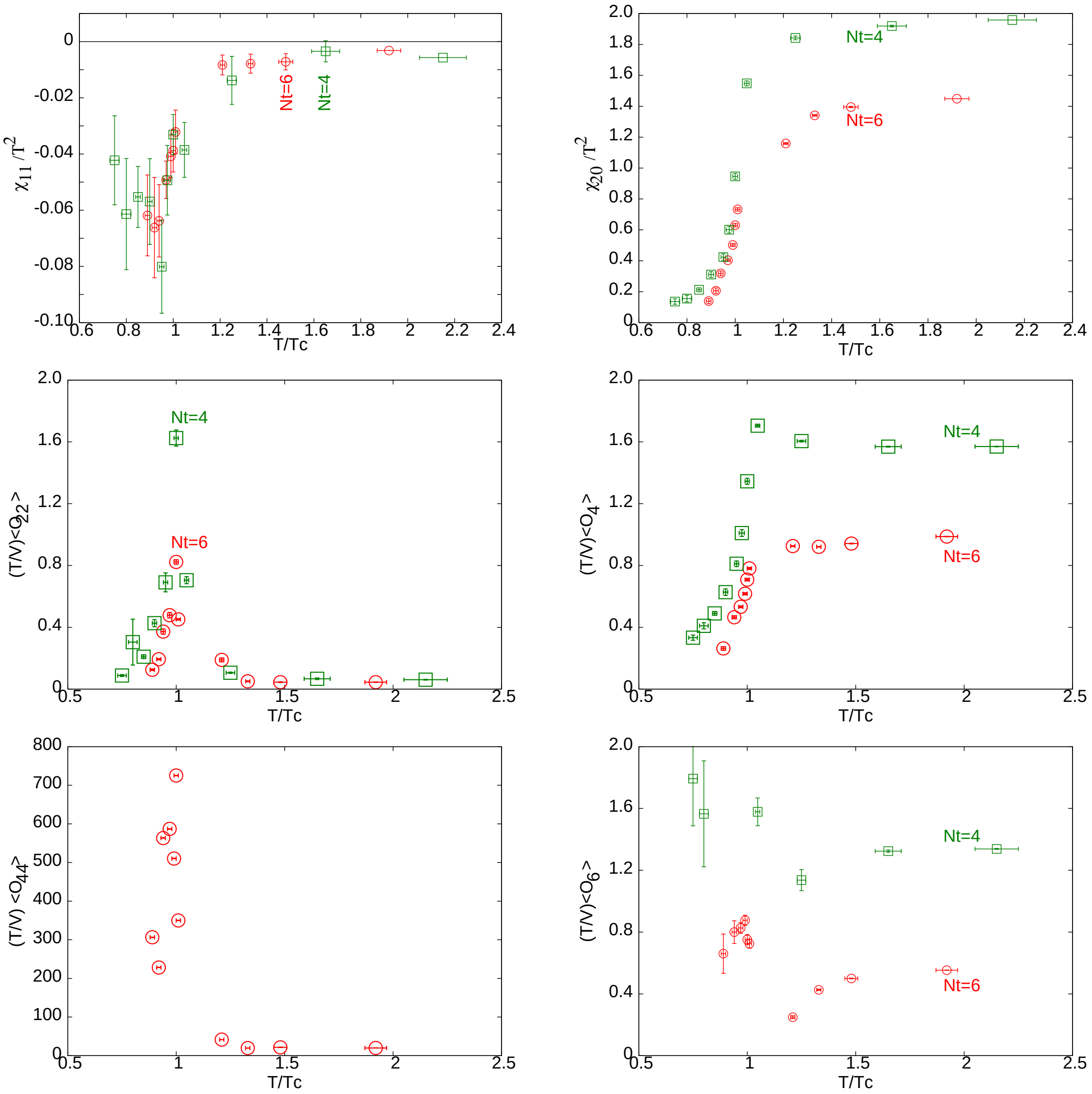

Figure 2: The off-diagonal and diagonal quark number susceptibilities on $N_t \times (4N_t)^3$ lattices.

2. Quark number susceptibilties

The off-diagonal QNS, χ_{11} is shown in Figure 2. There is no evidence of lattice spacing effects, nor of a crossover at T_c. The diagonal QNS is also shown in the same figure, and shows finite lattice spacing effects. There is a rapid crossover near T_c. This is due to the operator O_2 which appears in the diagonal QNS but not in the off-diagonal.

Hence the fourth order QNS which contains O_{22} should peak. This is exactly what we see. Also, the operator O_4, which is another contribution to the fourth order QNS do not peak, but show a cross over. These two kinds of behaviour are also exhibited in Figure 2.

By an extension of this argument, the eighth order QNS which contain O_{44} shopuld peak, and the peak is shown in Figure 2. The operator O_6 which is one of the contributions to the sixth order QNS, however, does not show a simple crossover, but exhibits interesting structure below T_c,

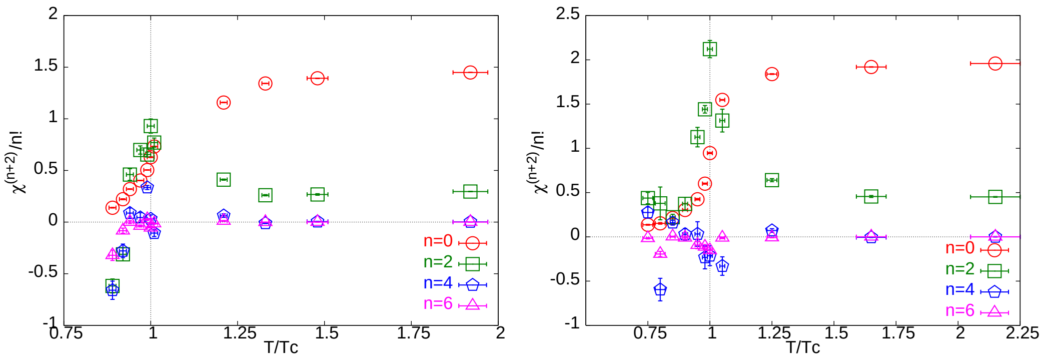

Figure 3: The series coefficients in the Taylor expansion of χ_B (normalized by appropriate powers of T to make them dimensionless) on lattice sizes $LT = 4$ for $N_t = 6$ (first panel) and $N_t = 4$ (second panel). Note that there is a small window of temperature, a little below T_c, in which all the measured series coefficients are positive.

as also shown in Figure 2. The same is true of the operator O_8, which is not shown. Hence the susceptibilities of these operators need not peak at T_c. These patterns were noticed earlier in studies with $N_t = 4$ lattices [4].

Figure 2 also shows that the peaks of O_{22} and O_{44} occur at the same value of T where the peak of χ_L occurs (which was used to set the temperature scale used here). Hence with these three measurements we find the same cross over coupling, at least within the precision of our measurements.

These figures also show that measurements of the most important matrix elements which contribute to the QNS up to the eighth order are under control when 500 random vectors are used for the measurements.

Finally, in Figure 3 we collect together the series coefficients in the Taylor expansion of μ_B. For both $N_t = 6$ and 4, we find that there is a small window of temperatures where all the measured series coefficients are positive. Also note that at high temperatures only the first two coefficients are appreciable.

3. The critical end point

Identifying a critical point through finite lattice computations is hard even when direct simulations can be performed, and there is a well-known theory of critical phenomena. In our case direct simulations are impossible and series expansions have to be resorted to.

The radius of convergence of the series is the point at which it breaks down: the critical point must be located at one such point of break down. Since the series in question is in powers of μ_B^2, a QCD critical point occurs only when the series breaks down at a positive value of μ_B^2, implying that all coefficients must be positive. We find that at the lowest temperatures which we study, the series coefficients are not all positive. Also at high temperatures the coefficients change sign. As a result, there is only a small window of temperatures where the critical end point can lie.

Of course, any numerical method will study only a finite number of terms and therefore can give only a finite number of estimators of the radius of convergence. We find that, among the tem-

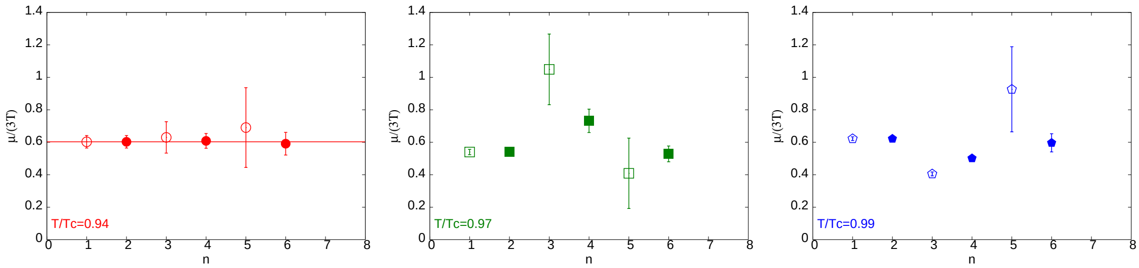

Figure 4: The radius of convergence at three neighbouring temperatures in our scan. A critical point can only be deduced from the first. The filled symbols indicate the estimator of the radius of convergence which is $(\chi^{(0)}/\chi^{(n)})^{1/n}$. Open symbols stand for a different estimator, which is $\sqrt{\chi^{(n-1)}/\chi^{(n+1)}}$. Here $\chi^{(n)}$ is the n-th coefficient in the Taylor expansion of χ_B.

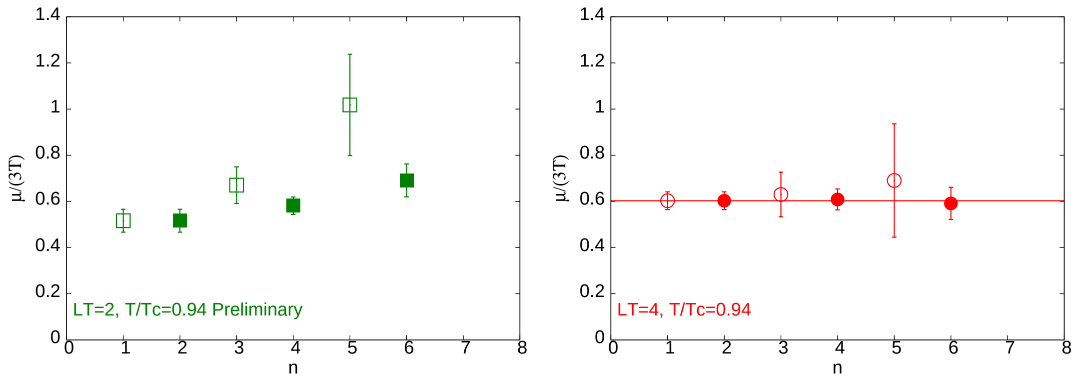

Figure 5: The volume dependence of estimators of the radius of convergence at T^E. The behaviour is consistent with expectations at a critical point. The open and filled symbols have the same meaning as in Figure 4.

peratures we studied, there is only one where the successive estimators are statistically consistent with each other 4. At this temperature the same value for the radius of convergence is also obtained with two different estimators. Therefore we identify this temperature with T^E. Of course, it is possible that at some other temperature the estimate of the radius of convergence only stabilizes after several more terms. Further work is needed to rule out such pathologies.

If our identification of the critical end point has any merit, then we should be able to check for finite volume effects. Since there is no true criticality at finite spatial size, L, we would expect that at T^E, estimates of the radius of convergence would seem to stabilize, but at some critical order of the expansion, $n_*(L)$, it would suddenly begin to increase with the order of the expansion. Of course, $n_*(L)$ should increase with L, going to infinity as L goes to infinity, so that a stable value of the radius of convergence is obtained in the thermodynamic limit. We show in Figure 5 that this behaviour is seen at the point we identify as T^E.

Our estimate of the critical end point with lattice cutoff of $a = 1/(6T^E)$, a finite lattice of spatial size $L = 4/T^E$, when the quark mass is tuned to give $m_\pi = 230$ MeV, is

$$T^E/T_c = 0.94 \pm 0.01 \qquad \text{and} \qquad \mu_B^E/T^E = 1.8 \pm 0.1. \tag{3.1}$$

In an earlier computation with $a = 1/(4T^E)$, where the lattice size and quark mass were kept at this

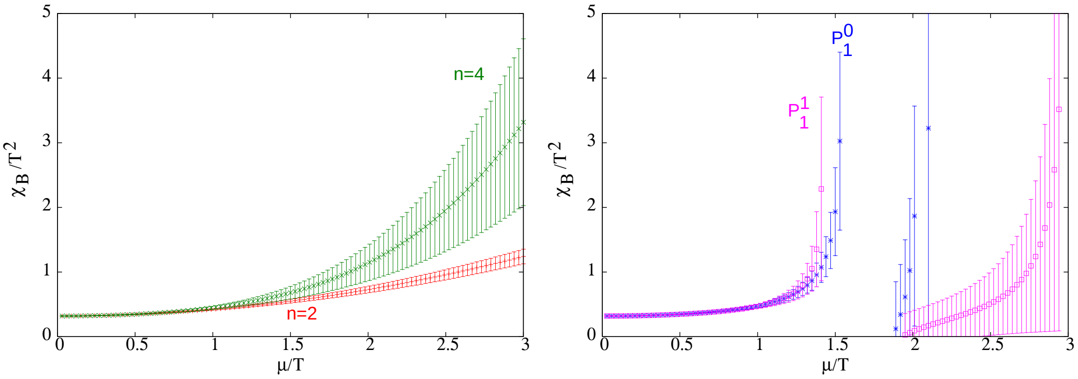

Figure 6: Series expansions to different orders fail to agree; nor do they indicate the presence of the critical point. Padé approximants constructed using the same coefficients do much better at predicting physical quantities. Both figures concern χ_B at $T - 0.94T_c$.

value [5], we had found the same T^E/T_c, but obtained $\mu_B^E/T^E = 1.3 \pm 0.3$. We had also used larger volumes in that study and on extrapolating $L \to \infty$ we found that $\mu_B^E/T^E \simeq 1.1$. If the same pattern recurs at the smaller cutoff, then the result quoted here would provide an upper bound on μ_B^E/T^E.

4. Series expansions and Padé approximants

A finite series expansion is a bad way to extrapolate results to finite μ_B, especially when the series shows signs of breaking down. One needs instead a method of resumming the series. A well-known method for doing this is to use Padé approximants [6].

The breakdown of the series expansion can be illustrated well enough by taking the expansion at $T = 0.94T_c$. The sum of the first two terms in the series is a quadratic in μ_B and hence increases monotonically. The sum of the first three terms is a quartic and increases even faster. There is no sign of a breakdown at μ_B^E (Figure 6, and a comparison of the two expansions shows that the series are only reliable for $\mu_B/T \ll 1$.

On the other hand, one could construct Padé approximants out of the same series coefficients. The two corresponding Padé approximants are shown in Figure 6. There is good agreement between the two approximants right upto the singular point. Error propagation in these computations require care, and are dealt with in detail in [1].

An interesting point about this concerns the spurious peak in χ_B at $T = T_c$. $\chi^{(2)}$ has no peak at T_c but $\chi^{(4)}$ has [7]. The truncated series $\chi_B = \chi^{(2)} + \chi^{(4)}\mu_B^2/2$ will therefore have a peak at T_c, with the peak growing infinitely high as $\mu_B \to \infty$. This is spurious. A Padé resummed extrapolation will shift the peak correctly to the critical end point.

5. Conclusions

A computation with two flavours of staggered quarks with the quark mass tuned to give $m_\pi =$ 230 MeV, with a cutoff $1/a = 6T$ on lattice sizes $LT = 2$, 3 and 4 was used to obtain series coefficients for a Taylor expansion of χ_B to sixth order (eighth order for P) and yielded the estimate of the location of the critical end point given in eq. (3.1). The series expansion coefficients can be used to resum the series into Padé approximants and obtain stable predictions for various quantities at finite chemical potential.

The computations reported here were performed on the Cray X1 of the Indian Lattice Gauge Theory Initiative.

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