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Renormalization of B-meson distribution amplitudes

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We summarize a recent calculation of the evolution kernels of the two-particle B-meson distribution amplitudes ϕ_+ and ϕ_- taking into account three-particle contributions. In addition to a few phenomenological comments, we give as a new result the evolution kernel of the combination of three-particle distribution amplitudes $\Psi_A - \Psi_V$ and confirm constraints on ϕ_+ and ϕ_- derived from the light-quark equation of motion.

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1. Introduction

Exclusive decays of B-mesons provide important tools to test the Standard Model and to search for physics beyond it. Hadronic inputs encoding soft physics are not only form factors but also light-cone distribution amplitudes (LCDAs). In particular the B-meson LCDAs enter the parametrization of the hard-scattering part of hadronic matrix elements of bilocal current operators where large momentum is tranferred to the soft spectator quark [1-10]. Impressive progress has been made in the calculation of the hard scattering amplitudes entering factorization theorems, see e.g. [11-15] for the $B \to PP$ case, but one limiting factor for the extraction of fundamental parameters is the uncertainty coming from the hadronic input. Recent years have seen several analyses concerning the renormalization properties [16, 17, 18] and the shape of the B-meson LCDAs [4, 18, 19, 20, 21, 22, 23]. Up to now these analyses were restricted to the two-particle case or to leading order with the exception of [23]. Here we present the results of [24] for the renormalization of the two-particle B-meson LCDAs taking into account mixing with three-parton LCDAs and in section (2.3) the results of a new calculation for the combination of three-particle LCDAs $\Psi_A - \Psi_V$ entering the equations of motion.

2. One-loop calculation with three-parton external state

The relevant two- and three-parton distribution amplitudes are defined as B to vacuum matrix-elements of a non-local heavy-to-light operator, which reads in the two-particle case [4]

$$\langle 0|\bar{q}_\beta(z)[z,0](h_v)_\alpha(0)|B(p)\rangle = -i\frac{\hat{f}_B(\mu)}{4}\left[(1+\not{v})\left(\tilde{\phi}_+(t)+\frac{\not{z}}{2t}[\tilde{\phi}_-(t)-\tilde{\phi}_+(t)]\right)\gamma_5\right]_{\alpha\beta} \tag{2.1}$$

and in the three-particle case [21] (the most general decomposition without contraction with a light-like vector is given in [25]):

$$\begin{aligned}
&\langle 0|\bar{q}_\beta(z)[z,uz]gG_{\mu\nu}(uz)z^\nu[uz,0](h_v)_\alpha(0)|B(p)\rangle \qquad (2.2)\\
&\quad = \frac{\hat{f}_B(\mu)M}{4}\Big[(1+\not{v})\Big[(v_\mu\not{z}-t\gamma_\mu)\left(\tilde{\Psi}_A(t,u)-\tilde{\Psi}_V(t,u)\right)-i\sigma_{\mu\nu}z^\nu\tilde{\Psi}_V(t,u)\\
&\qquad\qquad -z_\mu\tilde{X}_A(t,u)+\frac{z_\mu\not{z}}{t}\tilde{Y}_A(t,u)\Big]\gamma_5\Big]_{\alpha\beta}.
\end{aligned}$$

We use light-like vectors $n_\pm$ so that every vector can be decomposed as

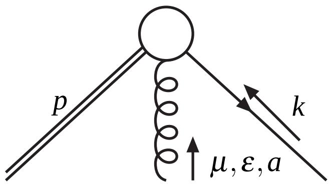
$$\begin{aligned}
q_\mu &= (n_+\cdot q)\frac{n_{-,\mu}}{2}+(n_-\cdot q)\frac{n_{+,\mu}}{2}+q_{\perp\mu} = q_+\frac{n_{-,\mu}}{2}+q_-\frac{n_{+,\mu}}{2}+q_{\perp\mu},\\
n_+^2 &= n_-^2 = 0 \qquad n_+\cdot n_- = 2 \qquad v = (n_++n_-)/2.
\end{aligned} \tag{2.3}$$

The computation of the renormalisation properties of the distribution amplitudes requires us to consider matrix elements of the relevant operators

$$O_+^H(\omega) = \frac{1}{2\pi}\int dt e^{i\omega t}\langle 0|\bar{q}(z)[z,0]\not{n}_+\Gamma h_v(0)|H\rangle \tag{2.4}$$

$$O_-^H(\omega) = \frac{1}{2\pi}\int dt e^{i\omega t}\langle 0|\bar{q}(z)[z,0]\not{n}_-\Gamma h_v(0)|H\rangle \tag{2.5}$$

$$O_3^H(\omega,\xi) = \frac{1}{(2\pi)^2}\int dt e^{i\omega t}\int du e^{i\xi ut}\langle 0|\bar{q}(z)[z,uz]g_sG_{\mu\nu}(uz)z^\nu[uz,0]\Gamma h_v(0)|H\rangle \tag{2.6}$$



$$A: -g_s \frac{\varepsilon_+}{q_+}\left[\delta\left(\omega-k_+-q_+\right)-\delta\left(\omega-k_+\right)\right]\bar{v}\not{n}_-\Gamma T^a u$$

$$B: -g_s \frac{v\cdot\varepsilon}{v\cdot q}\delta\left(\omega-k_+\right)\ \bar{v}\not{n}_-\Gamma T^a u \qquad C: g_s\frac{1}{(k+q)^2}\delta\left(\omega-k_+-q_+\right)\ \bar{v}\not{\varepsilon}(\not{k}+\not{q})\not{n}_-\Gamma T^a u$$

Figure 1: The three leading-order contributions to the matrix element of $O_\pm$ with a three-parton external state.

with z parallel to n_+, i.e. $z_\mu = tn_{+,\mu}$, $t = v\cdot z = z_-/2$ and the path-ordered exponential in the n_+ direction:

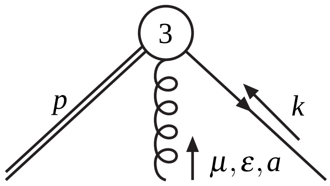
$$[z,0] = P\exp\left[ig_s\int_0^z dy_\mu A^\mu(y)\right] \tag{2.7}$$

$$= 1+ig_s\int_0^1 d\alpha\, z_\mu A^\mu(\alpha z)-g_s^2\int_0^1 d\alpha\int_0^\alpha d\beta z_\mu\, z_\nu\, A^\mu(\alpha z)\, A^\nu(\beta z)+\ldots \tag{2.8}$$

The Fourier-transforms of the different distribution amplitudes are then defined via

$$\phi_\pm(\omega) = \frac{1}{2\pi}\int dt e^{i\omega t}\tilde{\phi}_\pm(t) \qquad F(\omega,\xi) = \frac{1}{(2\pi)^2}\int dt\int du te^{i(\omega+u\xi)t}\tilde{F}(t,u) \tag{2.9}$$

where $F = \Psi_V, \Psi_A, X_A, Y_A$. Since the renormalization of the operators is independent of the infrared properties of the matrix-elements, we can choose an on-shell partonic external state consisting of a light quark, a heavy quark and a gluon in equation (2.6). The resulting leading-order diagrams are shown in figures 1 and 2. Next-to-leading order (NLO) diagrams are obtained by adding a gluon or a quark loop (a ghost loop) in all possible places (for a complete list of diagrams, see [24]). Since the operators give rise to δ-distributions in the $+$-component of the momenta, we chose to proceed via the theorem of residues. To be more explicit, we decomposed the loop momentum l in light-cone components, picked up the poles in the l_--integral and performed the $l_\perp$-integration in dimensional regularization with $D = 2-2\varepsilon$ dimensions. Additional $\frac{1}{\varepsilon}$-poles arise through the l_+-integration for diagrams where a gluon is exchanged between the Wilson-line from the operator and the heavy-quark field. These are related to the cusp anomalous dimension, see e.g [26, 27], stemming from the intersection of one light-like Wilson line from the path ordered exponential in the operator and one time-like Wilson line from the interaction of soft gluons with the heavy quark. The additional poles give rise to $\frac{1}{\varepsilon^2}$-terms as well as Sudakov logarithms.



$$A_{3\mu} : ig_s(q_+\varepsilon_\mu - q_\mu\varepsilon_+)\ \bar{v}\Gamma T^a u\delta(\omega - k_+)\delta(\xi - q_+)$$

Figure 2: Leading-order contribution to the matrix element of $O_{3\mu}$ with a three-parton external state.

2.1 ϕ_+-case

In the ϕ_+-case there is no mixing from three-particle distribution amplitudes. $\gamma_{+,3} = 0$ at order α_s. We confirm the result for the anomalous-dimension matrix found in [16]

$$\gamma_+^{(1)}(\omega,\omega';\mu) = \left(\Gamma_{\rm cusp}^{(1)}\log\frac{\mu}{\omega} + \gamma^{(1)}\right)\delta(\omega-\omega') - \Gamma_{\rm cusp}^{(1)}\omega\left(\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right)_+ \quad (2.10)$$

with

$$\Big[f(\omega,\omega')\Big]_+ = f(\omega,\omega') - \delta(\omega-\omega')\int d\omega' f(\omega,\omega') \qquad \Gamma_{\rm cusp}^{(1)} = 4 \qquad \gamma^{(1)} = -2$$

2.2 ϕ_--case

The ϕ_- case is more involved. After coupling-constant and external leg renormalization there remains a genuine three-particle term. The renormalization group equation to order α_s can be written as

$$\begin{aligned}\frac{\partial\phi_-(\omega;\mu)}{\partial\log\mu} = -\frac{\alpha_s(\mu)}{4\pi}\bigg(&\int d\omega'\,\gamma_-^{(1)}(\omega,\omega';\mu)\phi_-(\omega';\mu)\\ &+\int d\omega' d\xi'\gamma_{-,3}^{(1)}(\omega,\omega',\xi';\mu)[\Psi_A-\Psi_V](\omega',\xi';\mu)\bigg)\end{aligned} \quad (2.11)$$

where $\gamma_-^{(1)}$ is the result from [17]

$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)} - \Gamma_{\rm cusp}^{(1)}\frac{\theta(\omega'-\omega)}{\omega'} \quad (2.12)$$

and $\gamma_{-,3}^{(1)}$ from [24]

$$\begin{aligned}\gamma_{-,3}^{(1)}(\omega,\omega',\xi') = 4\Bigg[\frac{\Theta(\omega)}{\omega'}\bigg\{&(C_A-2C_F)\left[\frac{1}{\xi'^2}\frac{\omega-\xi'}{\omega'+\xi'-\omega}\Theta(\xi'-\omega) + \frac{\Theta(\omega'+\xi'-\omega)}{(\omega'+\xi')^2}\right]\\ &- C_A\left[\frac{\Theta(\omega'+\xi'-\omega)}{(\omega'+\xi')^2} - \frac{1}{\xi'^2}\left(\Theta(\omega-\omega')-\Theta(\omega-\omega'-\xi')\right)\right]\bigg\}\Bigg]_+\end{aligned} \quad (2.13)$$

where we defined the $+$-distribution with three variables as

$$\Big[f(\omega,\omega',\xi')\Big]_+ = f(\omega,\omega',\xi') - \delta(\omega-\omega'-\xi')\int d\omega f(\omega,\omega',\xi'') \quad (2.14)$$

2.3 $\Psi_A - \Psi_V$-renormalization and equation of motion constraints

Here we report on an up to now unpublished calculation of the renormalization of the three-particle LCDAs $\Psi_A - \Psi_V$. We project on the relevant distribution amplitudes in equation (2.3) using $\Gamma = \gamma_\perp^\mu \not{n}_+ \not{n}_- \gamma_5$ (although a γ^μ instead of $\gamma_\perp^\mu$ yields the same result). The calculations go along the same lines as in the previous two cases, even though there is only one leading-order structure (shown in figure 2) and NLO diagrams must have one gluon attached to the vertex in order not to vanish trivially. For convenience the result is splitted into C_F- and C_A-colour structures.

$$
\begin{aligned}
\gamma^{(1)}_{3,3,C_A}(\omega,\xi,\omega',\xi') &= 2\Bigg[\delta(\omega-\omega')\left\{\frac{\xi}{\xi'^2}\Theta(\xi'-\xi) - \left[\frac{\Theta(\xi-\xi')}{\xi-\xi'}\right]_+ - \left[\frac{\xi}{\xi'}\frac{\Theta(\xi'-\xi)}{\xi'-\xi}\right]_+\right\} \\
&+ \delta(\xi-\xi')\left\{\left[\frac{\Theta(\omega-\omega')}{\omega-\omega'}\right]_+ + \left[\frac{\omega}{\omega'}\frac{\Theta(\omega'-\omega)}{\omega'-\omega}\right]_+\right\} + \delta(\omega+\xi-\omega'-\xi') \\
&\times \left\{\frac{1}{\xi'}\Theta(\omega-\omega') - \left[\frac{\Theta(\omega-\omega')}{\omega-\omega'}\right]_+ - \left[\frac{\omega}{\omega'}\frac{\Theta(\omega'-\omega)}{\omega'-\omega}\right]_+\right\} \\
&+ \delta(\omega+\xi-\omega'-\xi')\frac{1}{\xi'(\omega'+\xi')}\Bigg\{\frac{\omega-\xi'}{\xi'}(\omega'+\xi'-\omega)\Theta(\omega-\omega') \\
&- \frac{\omega}{\omega'}(\omega'+2\xi'-\omega)\Theta(\omega'-\omega)\Theta(\omega) + \frac{\omega}{\xi'}(\omega-\xi')\Theta(\xi'-\omega)\Theta(\omega) \\
&+ \frac{\omega-\xi'}{\omega'}(\omega'+\xi'-\omega)\Theta(\omega-\xi')\Theta(\xi)\Bigg\}\Bigg]
\end{aligned}
\tag{2.15}
$$

$$
\begin{aligned}
\gamma^{(1)}_{3,3,C_F}(\omega,\xi,\omega',\xi';\mu) &= \gamma^{(1)}_+(\omega,\omega';\mu)\delta(\xi-\xi') + \gamma^{(1)}_{R3,3}(\omega,\xi,\omega',\xi') \\
\gamma^{(1)}_{R3,3}(\omega,\xi,\omega',\xi') &= 4\delta(\omega+\xi-\omega'-\xi') \\
&\times \left[\frac{\xi^2}{\omega'}\frac{\Theta(\omega'-\xi)}{(\omega+\xi)^2}\Theta(\xi) + \frac{\omega}{\xi'}\frac{\Theta(\xi-\omega')}{\omega+\xi}\Theta(\omega)\left(\frac{\xi}{\omega+\xi} - \frac{\omega-\xi'}{\xi'}\right)\right]
\end{aligned}
\tag{2.16}
$$

with $\gamma^{(1)}_+$ the same as in equation (2.10) and $\gamma^{(1)}_{3,3}$ defined as in (2.11) with obvious changes. Part of this calculation, namely the light-quark-gluon part, has been calculated in a different context and a different scheme, e.g. in [29, 28].

In [21] two equations from the light- and heavy-quark equations of motion were derived

$$
\omega\phi'_-(\omega;\mu) + \phi_+(\omega;\mu) = I(\omega;\mu), \qquad (\omega - 2\bar{\Lambda})\phi_+(\omega;\mu) + \omega\phi_-(\omega;\mu) = J(\omega;\mu), \tag{2.17}
$$

where $I(J)(\omega;\mu)$ are integro-differential expressions involving the three-particle LCDAs $\Psi_A - \Psi_V$ ($\Psi_A + X_A$ and Ψ_V) respectively. While the second equation was shown not to hold beyond leading order in [17, 23] we have checked that the first one is valid once renormalization is taken into account. Taking the derivative of the first equation with respect to $\log\mu$, inserting

$$
I(\omega;\mu) = 2\frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\frac{\partial}{\partial\xi}\left[\Psi_A(\rho,\xi;\mu) - \Psi_V(\rho,\xi;\mu)\right] \tag{2.18}
$$

and using the relation from [17]

$$
-\omega\frac{d}{d\omega}\int_0^\eta \frac{d\omega'}{\eta}\gamma^{(1)}_-(\omega,\omega';\mu) = \gamma^{(1)}_+(\omega,\eta;\mu) \tag{2.19}
$$

one arrives at the following equation

$$
\begin{aligned}
& \omega\frac{d}{d\omega}\int d\omega' d\xi' \gamma^{(1)}_{-,3}(\omega,\omega',\xi')\left(\Psi_A(\omega',\xi';\mu)-\Psi_V(\omega',\xi';\mu)\right) \\
&+ 2\int d\omega' \gamma^{(1)}_{+}(\omega,\omega';\mu)\frac{d}{d\omega'}\int_0^{\omega'} d\rho \int_{\omega'-\rho}^{\infty}\frac{d\xi}{\xi}\frac{\partial}{\partial\xi}\left(\Psi_A(\rho,\xi;\mu)-\Psi_V(\rho,\xi;\mu)\right) \\
&= 2\int d\omega' d\xi' \frac{d}{d\omega}\int_0^{\omega} d\rho \int_{\omega-\rho}^{\infty}\frac{d\xi}{\xi}\frac{\partial}{\partial\xi}\gamma^{(1)}_{3,3}(\rho,\xi,\omega'\xi';\mu)\left(\Psi_A(\omega',\xi';\mu)-\Psi_V(\omega',\xi';\mu)\right),
\end{aligned}
\tag{2.20}
$$

which can be proven to hold at order α_s by simple insertion of the respective evolution kernels (2.10), (2.13), (2.15), (2.16). This non-trivial outcome gives us further confidence concerning the renormalization group properties of the LCDAs.

3. Conclusions

The presence of $\delta(\omega-\omega')\log(\mu/\omega)$ in the renormalization matrices gives rise to a radiative tail falling off like $(\log\omega)/\omega$ for large ω. Therefore non-negative moments of the LCDAs are not well defined and have to be considered with an ultraviolett cut-off.[16, 17, 22, 23]

$$
\langle\omega^N\rangle_{\pm}(\mu) = \int_0^{\Lambda_{UV}} d\omega\,\omega^N\,\phi_{\pm}(\omega;\mu) \tag{3.1}
$$

For ϕ_- it is interesting to examine the limit

$$
\lim_{\Lambda_{UV}\to\infty}\int_0^{\Lambda_{UV}} d\omega\,\omega^N z^{(1)}_{-,3}(\omega,\omega',\xi') = 0 \qquad N=0,1, \quad z^{(1)}_{-,3} = \frac{1}{2\varepsilon}\gamma^{(1)}_{-,3}, \tag{3.2}
$$

which is relevant for the calculation of the three-particle contributions to the moments:

$$
\begin{aligned}
\int_0^{\Lambda_{UV}} d\omega\,\omega^N\,\phi_-(\omega;\mu) \;=\; & 1+\frac{\alpha_s}{4\pi}\Bigg(\int d\omega'\,\phi_-(\omega')\int_0^{\Lambda_{UV}} d\omega\,\omega^N z^{(1)}_-(\omega,\omega';\mu) \\
& - \int d\omega' d\xi'(2-D)[\Psi_A-\Psi_V](\omega',\xi')\int_0^{\Lambda_{UV}} d\omega\,\omega^N z^{(1)}_{-,3}(\omega,\omega',\xi')\Bigg)
\end{aligned}
\tag{3.3}
$$

Therefore as stated in [17] three-particle distribution amplitudes give only subleading contribution to the first two moments ($N=0,1$) and we have explicitly checked that this statement cannot be extended to higher moments ($N\geq 2$).

The next step consists in using the renormalization properties as a guide to go beyond the existing models derived from a leading-order sum-rule calculation resulting in $\Psi_A=\Psi_V$ [20] and to analyze their influence on ϕ_-. Finally, for practical calculations involving three-particle contributions, one would need the evolution kernels for the all relevant LCDAs, which will be the subject of a future work.

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