

Effective Field Theories for the X(3872)

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I summarize the properties of the X(3872) and explain why they lead inevitably to the conclusion that it is a weakly-bound charm meson molecule. The tiny binding energy of the X(3872) makes this system an ideal application of effective field theory. A good first approximation is provided by a field theory in which the charm mesons have contact interactions only. An effective field theory called X-EFT that includes explicit pions can be used to systematically improve the accuracy of the description of this remarkable resonance.

International Workshop on Effective Field Theories: from the pion to the upsilon February 2-6 2009 Valencia, Spain

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[†]A footnote may follow.

1. Introduction

In August 2003, the Belle Collaboration discovered a new $c\bar{c}$ meson that they named the X(3872) [1]. It was produced by the decays $B^\pm \to K^\pm X$ and observed through the decay mode $J/\psi \pi^+\pi^-$. The existence of the X(3872) was quickly confirmed in September 2003 by the CDF Collaboration through inclusive production of $J/\psi \pi^+\pi^-$ in $p\bar{p}$ collisions [2]. Thus this resonance is dramatic enough to stand out in a hadron collider experiment.

Below, I summarize the properties of the X(3872) and explain why they lead inevitably to the conclusion that it is a weakly-bound charm meson molecule. The separation of scales associated with the tiny binding energy of the X(3872) makes this an ideal system for applying effective field theory. I describe a field theory of charm mesons with contact interactions that provides a good first approximation. I then describe briefly an effective field theory called X-EFT that includes explicit pions and can be used to systematically improve the accuracy of the description of the X(3872) resonance.

2. Properties of the X(3872)

The mass of the X(3872) has been measured in both the discovery channel $J/\psi \pi^+\pi^-$ and in the decay channel $D^0\bar{D}^0\pi^0$. In the $J/\psi \pi^+\pi^-$ channel, the mass obtained by combining the most recent measurements by the BABAR, Belle, and CDF Collaborations [3, 4, 5] is

$$M_X = 3871.55 \pm 0.20 \,\text{MeV}.$$
 (2.1)

In the $D^0\bar{D}^0\pi^0$ channel, the mass obtained by combining the most recent measurements by the BABAR and Belle Collaborations [6, 7] is

$$M_X = 3873.49 \pm 0.51 \text{ MeV}.$$
 (2.2)

There is a 3σ discrepancy between these measurements of M_X . There are also results for the width of the X(3872) from both the $J/\psi \pi^+\pi^-$ and $D^0\bar{D}^0\pi^0$ channels. In the $J/\psi \pi^+\pi^-$ channel, the Belle Collaboration obtained an upper bound [1]:

$$\Gamma_X < 2.3 \text{ MeV}$$
 at 90% C.L.. (2.3)

In the $D^0\bar{D}^0\pi^0$ channel, the BABAR and Belle Collaborations [6, 7] measured the width to be

$$\Gamma_X = 3.4^{+1.6}_{-1.0} \text{ MeV}.$$
 (2.4)

The measurements in Eq. (2.4) was actually carried out under the assumption that the $D^0\bar{D}^0\pi^0$ events come from $D^{*0}\bar{D}^0$ or $D^0\bar{D}^{*0}$. As pointed out in Section 4, it is a mistake to interpret them as measurements of the mass and width of the X(3872). This mistake was also made in the 2008 edition of the Review of Particle Physics [8]. The Particle Data Group determined their average for the mass of the X(3872) by combining four values from $J/\psi\pi^+\pi^-$ decays with two values from $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$ decays. The 3.5 sigma discrepancy between the two sets of measurements was taken into account by increasing the error by a scale factor of 2.5.

The J^{PC} quantum numbers of the X(3872) are not yet definitely established. Observations of decays into $J/\psi \gamma$ and $\psi(2S) \gamma$ by the Belle and Babar Collaborations [9, 10, 11] imply that X is even under charge conjugation. The spin and parity quantum numbers have been studied by the Belle and CDF Collaborations [12, 13] using the angular distributions in $J/\psi \pi^+\pi^-$ decays. The CDF analysis is compatible only with $J^{PC}=1^{++}$ and 2^{-+} . The possibility 2^{-+} is disfavored by the observations of both decays into $\psi(2S) \gamma$ [11], which would have multipole suppression, and decays into $D^{*0}\bar{D}^0$ [6, 7], which would have angular-momentum suppression. I assume from here on that the quantum numbers of the X(3872) are 1^{++} .

There are measurements of the branching ratios for several of the decay modes of the X(3872). It is convenient to use the branching fraction for the discovery mode $J/\psi \pi^+\pi^-$ as the normalizing factor in the branching ratios. The branching ratio for $J/\psi \pi^+\pi^-\pi^0$ was measured by the Belle Collaboration [9]:

$$\frac{\text{Br}[X \to J/\psi \,\pi^+ \pi^- \pi^0]}{\text{Br}[X \to J/\psi \,\pi^+ \pi^-]} = 1.0 \pm 0.5. \tag{2.5}$$

In the decays into $J/\psi \pi^+\pi^-\pi^0$, the $\pi^+\pi^-\pi^0$ come predominantly from a virtual ω meson, so this decay mode has isospin 0. In the decays into $J/\psi \pi^+\pi^-$, the $\pi^+\pi^-$ come predominantly from a virtual ρ meson, so this decay mode has isospin 1. The comparable branching fractions into these two decay modes with isospins 0 and 1 implies a dramatic violation of isospin symmetry. The violation is not quite as large as it might first appear, because there are significant differences between the couplings of ω to three pions and ρ to two pions. The branching ratio in Eq. (2.5) can be used to deduce the relative strengths of the couplings of X to J/ψ and the two vector mesons [16]:

$$\frac{|G_{XJ/\psi\omega}|^2}{|G_{XJ/\psi\rho}|^2} = 12 \pm 6. \tag{2.6}$$

The stronger coupling to the isospin 0 state suggests that the X(3872) is predominantly isospin 0. The largest branching ratios that have been measured are for the decay modes $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$, which can be expressed as [6, 7]

$$\frac{\text{Br}[X \to D^{*0}\bar{D}^0, D^0\bar{D}^{*0}]}{\text{Br}[X \to J/\psi \pi^+ \pi^-]} = 11 \pm 3. \tag{2.7}$$

The branching ratio for the decays into $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$ is consistent with the branching ratio for decays of D^{*0} into $D^0\pi^0$ and $D^0\gamma$. Finally, the branching ratios for $J/\psi\gamma$ and $\psi(2S)\gamma$ have also been measured [9, 10, 11]:

$$\frac{\text{Br}[X \to J/\psi \gamma]}{\text{Br}[X \to J/\psi \pi^+ \pi^-]} = 0.29 \pm 0.07,$$
(2.8)

$$\frac{\text{Br}[X \to \psi(2S)\,\gamma]}{\text{Br}[X \to J/\psi\,\pi^{+}\pi^{-}]} = 1.3 \pm 0.4. \tag{2.9}$$

3. What is the X(3872)?

There are two crucial pieces of experimental information that determine the nature of the X(3872) unambiguously: its quantum numbers and its mass. The quantum numbers 1^{++} imply

that the X(3872) has an S-wave coupling to the charm meson pairs $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$. The mass in Eq. (2.1), which as measured in the $J/\psi\pi^+\pi^-$ decay mode, implies that its energy relative to the $D^{*0}\bar{D}^0$ threshold is

$$M_X - (M_{D^{*0}} + M_{D^0}) = -0.25 \pm 0.40 \text{ MeV}.$$
 (3.1)

This tiny mass difference implies that the X(3872) has a resonant S-wave coupling to the charm meson pairs $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$. This system is therefore governed by the universal properties of S-wave threshold resonances that are predicted by nonrelativistic quantum mechanics [14]. The universal properties apply to any nonrelativistic particles with short-range interactions that are tuned so that there is an S-wave bound state very close to the threshold. Because of this S-wave threshold resonance, the particles have a scattering length a that is large compared to the range of their interactions. The universal features depend on the scattering length, but they are otherwise insensitive to the short distance scales such as the range. The fact that the X(3872) resonance is in the 1^{++} channel allows us to conclude unambiguously that it is a charm meson molecule whose constituents are a superposition of $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$:

$$X = \frac{1}{\sqrt{2}} \left(D^{*0} \bar{D}^0 + D^0 \bar{D}^{*0} \right). \tag{3.2}$$

This state is a mixture of isospin 0 and isospin 1, which explains the violation of isospin symmetry implied by the branching ratio in Eq. (2.5).

We proceed to list some of the universal properties of an S-wave threshold resonance for the case of a large scattering length a that is positive:

• The cross section for low-energy scattering is

$$\sigma = 4\pi a^2. \tag{3.3}$$

• There is a bound state with a small binding energy given by

$$E_X = \frac{1}{2\mu a^2},\tag{3.4}$$

where μ is the reduced mass of the constituents.

• The constituents in the bound state have a large root-mean square separation:

$$\langle r^2 \rangle^{1/2} = \frac{a}{\sqrt{2}}. (3.5)$$

We now apply these results to the X(3872). Taking the central value 0.26 MeV of the binding energy E_X from Eq. (3.1) as our input, we can use Eq. (3.4) to determine the large scattering length a of the charm mesons in the 1^{++} channel. Inserting the result into Eq. (3.5), we find that the RMS separation of the charm mesons should be about 6 fermis. This is more than an order of magnitude larger than that of most hadrons.

It should be emphasized that our answer to the question "What is the X(3872)?" does not answer the question "Why is the X(3872)?". In particular, it does not identify the mechanism for binding of the charm mesons. Several possible mechanisms have been proposed:

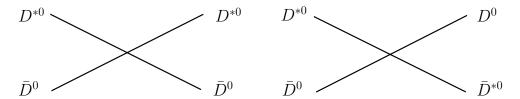


Figure 1: Contact interactions for the scattering of $D^{*0}\bar{D}^0$ into $D^{*0}\bar{D}^0$ (left panel) and into $D^0\bar{D}^{*0}$ (right panel). There are similar vertices for the time-reversed processes.

- The potential betwen D^{*0} and \bar{D}^0 could be just deep enough to form a bound state near the threshold.
- The P-wave charmonium state $\chi_{c1}(2P)$, which has quantum numbers 1^{++} , could be fortuitously close to the $D^{*0}\bar{D}^0$ threshold.
- There could be a tetraquark state with constituents $cq\bar{c}q$ and quantum numbers 1^{++} that is fortuitously close to the $D^{*0}\bar{D}^0$ threshold.

The universal properties of S-wave threshold resonances allow us to predict certain properties of the X(3872) without understanding the binding mechanism.

4. Contact Field Theory

The X(3872) resonance is an ideal system for the applications of effective field theories, because the tiny binding energy provides excellent separation of scales. The most important energy scale is the binding energy E_X , which according to Eq. (3.1) is a fraction of an MeV. This is very small compared to the pion mass: $m_{\pi} \approx 135$ MeV. The most important momentum scale is the inverse scattering length $\gamma = (2\mu E_X)^{1/2}$ which, if we use the central value of E_X , is approximately 20 MeV. This is also small compared to m_{π} . The problem is however complicated by several other small energy scales:

• The energy released in the decay of D^{*0} into $D^0\pi^0$ is

$$M_{D^{*0}} - (M_{D^0} + M_{\pi^0}) = 7.1 \text{ MeV}.$$
 (4.1)

• The energy difference between the thresholds for the charged charm mesons $D^{*+}D^-$ and the neutral charm mesons $D^{*0}\bar{D}^0$ is

$$(M_{D^{*+}} + M_{D^{+}}) - (M_{D^{*0}} + M_{D^{0}}) = 8.1 \text{ MeV}.$$
 (4.2)

• The total decay width of the D^{*0} can be determined from measurements of the D^{*+} width and the D^{*+} and D^{*0} branching fractions using isospin symmetry:

$$\Gamma_{*0} = 0.066 \pm 0.015 \text{ MeV}.$$
 (4.3)

The simplest field theory that can be used to describe the X(3872) is a nonrelativistic quantum field theory with complex scalar fields for the spin-0 charm mesons D^0 and \bar{D}^0 and complex vector fields for the spin-1 charm mesons D^{*0} and \bar{D}^{*0} . The charm mesons interact through a contact interaction in the C=+ channel, which gives diagonal and off-diagonal scattering of the charm mesons $D^{*0}\bar{D}^0$, as illustrated in Fig. 1. The large scattering length of the charm mesons implies that the geometric series of amplitudes illustrated in Fig. 2 must be summed to all orders. After renormalization, the scattering amplitude in the C=+ channel has the simple form

$$f(E) = \frac{1}{-\gamma + \sqrt{-2\mu(E + i\varepsilon)}},$$
(4.4)

where γ is the inverse scattering length and μ is the reduced mass for $D^{*0}\bar{D}^0$. This scattering amplitude for an S-wave threshold resonance encodes many of its universal properties.

There are two essential aspects of the X(3872) that are not taken into account in the simple scattering amplitude in Eq. (4.4). One aspect is the nonzero width Γ_{*0} of the D^{*0} , which is given in Eq. (4.3). Its dominant effects can be taken into account by replacing ε in Eq. (4.4) by $\Gamma_{*0}/2$. Another aspect that is not taken into account by Eq. (4.4) is the existence of inelastic scattering channels for the charm mesons, such as $J/\psi \pi^+\pi^-$. They imply that the inverse scattering length must have a positive imaginary part. The resulting expression for the scattering amplitude is

$$f(E) = \frac{1}{-(\gamma_{\rm re} + i\gamma_{\rm im}) + \sqrt{-2\mu(E + i\Gamma_{*0}/2)}}.$$
 (4.5)

The line shape of the resonance is proportional to the imaginary part of the scattering amplitude:

$$\operatorname{Im} f(E) = |f(E)|^2 \left[\gamma_{\text{im}} + \left(\mu \sqrt{E^2 + \Gamma_{*0}^2 / 4} - \mu E \right)^{1/2} \right) . \tag{4.6}$$

The line shape in the $J/\psi \pi^+\pi^-$ channel can be identified as a contribution to the $\gamma_{\rm im}$ term. If $\gamma_{\rm re}>0$, it exhibits a resonance peak below the $D^{*0}\bar{D}^0$ threshold, as illustrated in the left panel of Fig. 3. The square-root term in Eq. (4.6) is the contribution from the $D^0\bar{D}^0\pi^0$ and $D^0\bar{D}^0\gamma$ channels. If $\gamma_{\rm re}>0$, this term exhibits a resonance peak below the $D^{*0}\bar{D}^0$ threshold and a second broader peak above the threshold, as illustrated in the right panel of Fig. 3. The second peak corresponds to an enhancement in the production of $D^{*0}\bar{D}^0$ and $D^0\bar{D}^{*0}$. The line shapes illustrated in Fig. 3 explain why measurements in the $D^0\bar{D}^0\pi^0$ channel do not directly measure the mass and the width of the X(3872). The threshold enhancement biases the measurement of the mass to higher values and it also leads to an overestimate of the width. In contrast, measurements in the $J/\psi \pi^+\pi^-$ channel give unbiased determinations of the mass and the width.

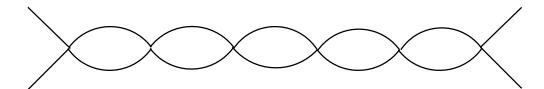


Figure 2: Class of diagrams that must be summed to all orders to obtain the scattering amplitude for an S-wave threshold resonance.

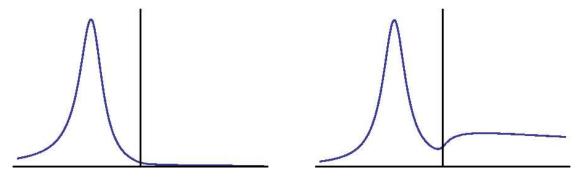


Figure 3: Qualitative behavior of the line shapes of the X(3872) in the $J/\psi \pi^+\pi^-$ decay channel (left panel) and in the $D^0\bar{D}^0\pi^0$ channel (right panel).

The range of validity of this field theory is limited to energies within about 1 MeV of the $D^{*0}\bar{D}^0$ threshold. For larger energies, it is necessary to take into account the effects of 3-body states $D^0\bar{D}^0\pi^0$ that come from decays of the D^{*0} or \bar{D}^{*0} , because their threshold according to Eq. (4.1) is smaller by only about 7 MeV. It is also necessary to take into account scattering into pairs of charged charm mesons $D^{*+}D^-$ and $D^{*-}D^+$, because their threshold according to Eq. (4.2) is larger only about 8 MeV. There is also a limitation on the accuracy of this field theory. The accuracy cannot be systematically improved by adding derivative interactions, so it is not an effective field theory in the strict sense. This problem can be understood by considering the effects of pion exchange between the D^{*0} and \bar{D}^0 , which changes the charm mesons into D^0 and \bar{D}^{*0} as illustrated in Fig. 4. The product of the propagator for the virtual π^0 and the momentum factors from the pion vertices is

$$\frac{q^{i}q^{j}}{(M_{D^{*0}} - M_{D^{0}})^{2} - q^{2} - m_{\pi^{0}}^{2} + i\varepsilon} \approx \frac{q^{i}q^{j}}{-q^{2} + i\varepsilon} . \tag{4.7}$$

The tiny energy release given in Eq. (4.1) implies a near cancellation in the denominator between the $M_{D^{*0}}-M_{D^0}$ term and the m_{π^0} term. Thus π^0 exchange does not generate the usual Yukawa potential that decreases exponentially at large distances. Instead it generates a tensor force that falls off at large distances like $1/r^3$. This problem was first pointed out by Suzuki [17]. It implies that there is no range expansion for the S-wave scattering amplitude. As a consequence, pion-exchange interactions cannot be taken into account by adding improvement terms to the Lagrangian that involve derivatives of the charm meson fields.

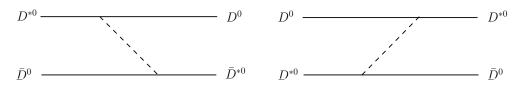


Figure 4: Diagrams for the scattering of $D^{*0}\bar{D}^0$ into $D^0\bar{D}^{*0}$ (left panel) and into $D^{*0}\bar{D}^0$ into $D^0\bar{D}^{*0}$ (right panel) via the exchange of a π^0 that is almost on its mass shell.

5. X-EFT

An effective field theory for the X(3872) that can be used to systematically improve its accuracy and extend its range of validity has been developed by Fleming, Kusunoki, Mehen, and van Kolck [18]. They named this effective field theory X-EFT. In addition to charm meson fields, it has explicit pion fields. The power-counting rules for this effective field theory guarantee that pion interactions can be treated perturbatively. The simplest version of X-EFT is a nonrelativistic quantum field theory with complex scalar fields for the charm mesons D^0 and \bar{D}^0 , complex vector fields for the charm mesons D^{*0} and \bar{D}^{*0} , and a complex scalar field for the neutral pion π^0 . This effective field theory can be used to systematically improve the accuracy of calculations at energies within about 1 MeV of the $D^{*0}\bar{D}^0$ threshold. It can also be used to extend the domain of validity to a few MeV from the threshold. X-EFT has been used to calculate the partial decay width of X(3872) into $D^0\bar{D}^0\pi^0$ at next-to-leading order [18]. It has also been applied to the decays of X into P-wave charmonium states and pions [19].

One problem that was not appreciated in Ref. [18] was the importance of taking into account the nonzero width of the D^{*0} and the existence of inelastic scattering channels for the charm mesons. To extend the domain of validity of X-EFT beyond a few MeV from the threshold, it is necessary to include fields for the charged charm mesons D^+ , D^- , D^{*+} , and D^{*-} and the charged pions π^+ and π^- . At some point, it may also be necessary to include a field for the P-wave charmonium state $\chi_{c1}(2P)$, which has quantum numbers 1^{++} .

6. Summary

Assuming the quantum numbers of the X(3872) are 1^{++} , the fact that its mass as measured in the $J/\psi \pi^+\pi^-$ channel is a fraction of an MeV from the $D^{*0}\bar{D}^0$ threshold implies beyond any reasonable doubt that the X(3872) is a weakly-bound charm meson molecule. Its extremely tiny binding energy makes it an ideal arena for the application of effective field theories. A field theory with contact interactions between the charm mesons is a good first approximation provided one takes into account the D^{*0} width and the existence of inelastic scattering channels for the charm mesons. The domain of validity of this field theory is limited to energies within about 1 MeV of the $D^{*0}\bar{D}^0$ threshold, and it cannot be used to calculate range corrections. The accuracy can be systematically improved and the range of validity can be extended by using the effective field theory X-EFT with explicit pion fields.

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