PROCEEDINGS OF SCIENCE

Standard Model Theory for collider physics

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In this talk we review some of the latest theoretical developments in Standard Model collider physics.

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1. Introduction

The timing of this presentation is rather awkward, since the LHC has progressed slower than anticipated. We cannot yet relish the excitement to discuss measurements of Standard Model cross-sections at world record collision energies. On the other hand, we have the great fortune of breath-taking theoretical developments, which have created great expectations for the future, when high quality LHC data will become available. A modern "theory revolution" has arrived, with new insights about the perturbative structure of gauge theories and theoretical predictions for collider experiments. In this talk new methods for one-loop calculations and their applications to multi-particle production cross-sections at the Tevatron and the LHC will be discussed. Experimental measurements of precision observables from LEP, HERA, TEVATRON and the LHC, for which theory predictions at next-to-next-to-leading are available, will also be analyzed. The outline of the talk is as follows:

- One-loop amplitudes,
- Final states with many particles at the Tevatron and the LHC,
- Jet Algorithms,
- NNLO theory,
- Jet physics at LEP, and the determination of the strong coupling,
- Deep Inelasting Scattering at HERA,
- Parton distribution functions for the Tevatron and the LHC,
- Drell-Yan and Higgs production at the Tevatron and the LHC.

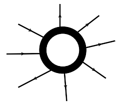
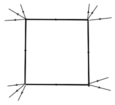
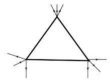
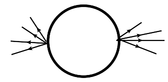
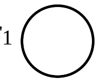
Many other topics are worth to be added to the above list. They have been ommitted due to lack of time or the speaker's ignorance.

2. One-loop amplitudes from trees and "masters"

Quantum field theories are tested for their mathematical self-consistency by studying loop effects. At a practical level, trustworthy quantitative theoretical predictions for the rates of particle processes can only be derived when loop corrections are taken into account. Almost all phenomenological comparisons between theory and data at modern collider experiments are performed with using at least the next-to-leading order approximation in perturbation theory.

For decades we believed that different and more complicated mathematical structures emerge at each higher order in perturbation theory. Consequently, the methods for tree level computations would be insufficient for one-loop amplitudes. Recently, an extraordinary discovery has been made: *The one-loop integrals which are needed to determine an one-loop amplitude form a basis of master integrals which has already been computed in a scalar field theory. The coefficients of the master integrals are sums of pro*
The existence of a scalar basis of master integrals was proven already three decades ago, by Passarino and Veltman [1]. The linear combinations which correspond to one-loop amplitudes remained until recently mysterious.

Symbolically, we write

 $= c_4$  $+c_3$  $+c_2$  $+c_1$  (2.1)

where on the right side of the equation appear scalar integrals with one, two, three, and four propagators (master integrals). Integrals with a number of propagators greater than four never appear in four dimensions, irrespective of the number of external legs of the amplitude [2, 3]. Passarino and Veltman and later other authors [4] presented algorithms to reduce the tensor integrals emerging in gauge theory one-loop amplitudes to scalar integrals, determining the master integral coefficients c_i.

These methods have been steadily improved over the years and are very powerful [5]. However, they are very difficult to apply to one-loop amplitudes of increasing complexity, since their computational cost scales as $n!$, where n is the external states of the amplitude. Not only the number of diagrams increases very rapidly with increasing the number of external legs, but, in addition, each diagram requires a large number of integrals to be computed. Recall the complexity of

- *the Feynman rules in gauge theory*

$$\mathscr{V}_{ggg} = f^{abc}\left[g_{\mu_1\mu_2}(p_1-p_2)^{\mu_3} + g_{\mu_2\mu_3}(p_2-p_3)^{\mu_1} + g_{\mu_3\mu_1}(p_3-p_1)^{\mu_2}\right]$$

- *and the algebra of γ matrices, colour algrebra, etc.*

$$Tr(\gamma^{\mu_1}\gamma^{\mu_2}) = \text{1 term}$$

$$Tr(\gamma^{\mu_1}\cdots\gamma^{\mu_8}) = \text{105 terms}$$

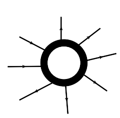
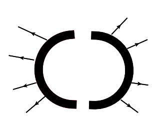
$$Tr(\gamma^{\mu_1}\cdots\gamma^{\mu_{14}}) = \text{26931 terms}$$

The large algebraic complexity of the problem is humbling.

At the Tevatron, we have an abundance of events with three or larger number of jets. At LHC energies, there is room in the phase-space for an even larger number of jets. Such processes will be of special importance for the discovery of novel heavy particles, which are pair produced. With traditional methods, the evaluation of many of the required one-loop amplitudes is prohibitive. Besides their practical limitations, traditional methods have a disturbing theoretical handicap. The coefficients c_i emerge at the end of a long calculation as mysterious mathematical quantities with unclear physical meaning.

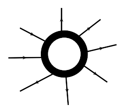
In the 90s, Bern, Dixon, Dunbar and Kosower developed a visionary idea. Tree amplitudes should not only determine the leading order approximation in pertubation theory, but they can be used also to obtain more complicated amplitudes at higher orders. In their approach, they would unravel the mathematical mechanism by which a gauge theory amplitude satisfied the conditions of unitarity, and turn it into a powerful method to simplify its analytic evaluation [6].

An ansatz for the integrand of one-loop amplitudes as a product of two tree amplitudes,

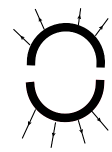
 $\simeq \int d^dk \frac{1}{k^2(k+p)^2}$  (2.2)

is consistent with the Cutkosky rules when two of the propagators are cut. Many advantages are offered when tree amplitudes are used as input for the integrand. Unlike expressions for one-loop Feynman diagrams, gauge invariance is manifest. The expression for the input product of tree amplitudes could be simple, by using spinor variables, spinor identities, and tree-level recursion relations. The task of integrating out the unitarity tailored integrands was often simple, leading to impressive NLO computations [7].

An important issue in this approach was to ascertain that a unitarity inspired integrand would yield the full result for an one-loop amplitude. Some potentially missing terms could be captured by considering all other double cuts of the amplitude,



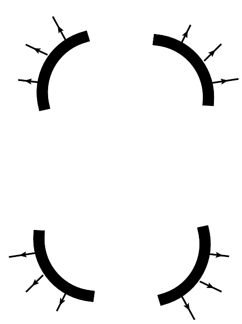
$$\simeq \int d^d k \frac{1}{k^2(k+p)^2} \tag{2.3}$$



Other terms which originated from a mismach of dimensions in the integrand, where tree amplitudes were usually taken in four dimensions, and the integration measure which, in dimensional regularization, is required to be in D dimensions. Clever theory input from the factorized limit of the one-loop amplitude in the limit of collinear external legs would provide additional information to reconstruct the full result.

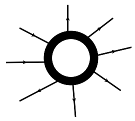
The advent of unitarity method in the 90s was constituted a very significant progress and offered a largely orthogonal view of the promlem of calculating one-loop amplitudes. In this approach, tree amplitudes were an essential ingredient of the next order in the perturbative expansion. However, a direct relation of tree amplitudes to master integral coefficients was lacking. Such a connection was first discovered by Britto, Cachazo and Feng in 2004 [8]. They realized that the coefficent c_4 of the box master integral in Eq. 2.1 is simply the product of four tree amplitudes, evaluated at complex momenta:

$$c_4 = \tag{2.4}$$



Two of the external momenta in the tree-amplitudes of the coefficient correspond to values of the loop momenta for which all four propagators of the box master integral are on-shell. This discovery was suggestive that the coefficients of the remaining master integrals, tadpoles, bubbles, and triangles, could also be related to tree amplitudes. However, the explicit relation remained for a little longer uncovered and required an important breaktrhough.

In a remarkable paper [9], Ossola, Pittau and Papadopoulos provided a most general expression for the integrand of arbitrary one-loop amplitudes in terms of a small number of known rational functions $f_i, \tilde{f}_i$ of the loop momentum,



$$= \int \frac{d^d k}{(2\pi)^d} \Big[c_4 f_4(k) + c_3 f_3(k) + c_2 f_2(k) + c_1 f_1(k)$$

$$\tilde{c}_4 \tilde{f}_4(k) + \tilde{c}_3 \tilde{f}_3(k) + \tilde{c}_2 \tilde{f}_2(k) + \tilde{c}_1 \tilde{f}_1(k) \Big]. \tag{2.5}$$

The independent functions of the integrand are constrained to a small number by simple power counting arguments. The functions $\tilde{f}_i$ integrate to zero. The remaining functions integrate to scalar master integrals: f_4 integrates to the box master integral, f_3 yields the triangle master integral, and so on. The knowledge of the basis functions for the integrand renders unnecessary any integration or integral recurrence relations in order to find the coefficients of the master integrals. The $c_i, \tilde{c}_i$ can be determined algebraically by evaluating the integrand at a sufficient number of values for the loop momentum k, and inverting a system of equations. In the same publication [9], Ossola, Pittau and Papadopoulos suggested to choose values for the loop momenta which set four, three, two, or one propagator at a time on their shell. This results to a system of equations for the coefficients $c_i, \tilde{c}_i$ which is very easy to diagonalize.

Ellis, Giele and Kunszt observed that the expression for the one-loop integrand becomes a product of tree amplitudes when it is evaluated for momenta where loop propagators are on-shell [11]. The master integral coefficients are therefore linear combinations of products of tree amplitudes which are derived by an easy to solve, almost diagonal ab initio, system of equations.

A naive application of the above ideas leads to incomplete results for the majority of one-loop amplitudes which exhibit ultraviolet divergences. The old problem of a mismatch in the D-dimensional loop integration and tree amplitudes being evaluated in four dimensions is present here as well, and it results to missing some non-logarithmic terms from the result of the amplitude. These require a second calculation. A few approaches have been devised, deriving specialized tree-like recursion relations [12] or Feynman rules [13] for computing the rational part of one-loop amplitudes. Ellis, Giele, Kunszt and Melnikov developed an elegant method to reconstruct the full dependence of an one-loop amplitude in D-dimensions [14, 15], by carrying out the evaluation of its master integral coefficients from tree-amplitudes in five and six dimensions.

The discovery of an explicit cross-order relation for the amplitudes at the leading order and the next-to-leading order in perturbation theory is breaking new ground. As we shall describe shortly, it gives great promise for performing precise simulations of complicated collider processes. An important promise for a deeper understanding of the structure of the perturbative series in gauge theories is also made. The follow up theoretical breakthroughs could be equally or even more important.

Tree amplitudes in gauge theories can be obtained very efficiently from simpler amplitudes of smaller multiplicity with recursion methods (Berends, Giele [16]; Britto, Cachazo, Feng, Witten [17]). With the discovery of cross-order relations where tree amplitudes and known master integral functions make up the full result for one-loop amplitudes, the same recursion relations enable the computation of extraordinarily difficult one-loop amplitudes. Powerful programs have been written which are able to compute one-loop amplitudes for the scattering of as many as 20 or more gluons [18–20]. Such complicated mathematical objects required computations that they could last longer than the age of the universe with traditional methods.

For the needs of the LHC a number of $2 \to 4$ and even $2 \to 5$ processes are required with next-to-leading order accuracy. A list of processes of interest has been identified by the theoetical and experimental community in a series of conferences at Les Houches [21]. A few years ago, the Les Houches NLO wish-lists appeared to be an enormous challenge to the theory community, leaving most theorists sceptical about the feasibility of the untertaking. In a recent "proof of principle" paper, numerical results for all one-loop amplitudes of the Les Houches $2 \to 4$ processes ($q\bar{q} \to t\bar{t}b\bar{b}, b\bar{b}b\bar{b}, W^+W^-b\bar{b}, t\bar{t}gg$, $q\bar{q}' \to Wggg, Zggg$) were published [22].

3. Final states with many particles at the Tevatron and the LHC

A significant amount of work and further development of new computer programs are required to obtain NLO cross-sections for fully automated cross-sections of so complicated processes. Theoretically well understood ingredients of NLO calculations, such as the cancelation of infrared divergences from real radiation, need optimization and automatization. It is clear, however, that a complete automatization of NLO calculations is at sight and feasible.

Currently some of the challenging processes in the wish-list are ticked out! A beautiful demonstration of the power of the new techniques has been the "tour de force" evaluation, by two groups (the BlackHat and Rocket collaborations), of the cross-section for the production of a W-boson in association with three jets at the Tevatron and the LHC [23, 24].

The front of $2 \to 4$ NLO computations for hadron colliders has been first cracked with traditional methods which evaluate Feynman diagrams and are not based on unitarity. In a spectacular computation, Bredenstein, Denner, Dittmaier and Pozzorini computed the NLO cross-section for the process $pp \to t\bar{t}bb$ [25]. This calculation demostrates the maturity of traditional methods and the level of sophistication that they have reached.

What can we hope for in the future? Obviously we will never be able to compute more complicated processes at NLO than what we can achieve at leading order, currently processes with seven or eight particles in the final state. It is realistic to expect that forthcoming NLO programs, based on either unitarity methods or more traditional Feynman diagramatic methods, will be able to evaluate all interesting cross-sections for $2 \to 4$ processes at the Tevatron and the LHC. It is very unlikely that Feynman diagram methods can be extended to processes with higher multiplicity. Nevertheless, unitarity methods are very promising to become capable in the future of $2 \to 5$ and, perhaps $2 \to 6$ processes. We are just at the start of a new era of precise theoretical predictions for multiparticle production at the LHC.

The importance of these NLO calculations for phenomenology is great. Leading order cross-sections for high multiplicity processes are very uncertain, due to their dependence on the strong coupling at a high power. It is first at NLO, where a quantitative estimate of the cross-section may be attained. The magnitude of NLO radiative corrections turns out to be often rather large, as for example in the process $pp \to t\bar{t}b\bar{b}$ [25]. With new results at hand for NLO cross-sections it has become more obvious that a guesswork of higher order corrections is extremely dfficult and unreliable. K-factors can be variable in phase-space [23, 24]. Novel NLO computations will be a theoretical cornesrtone for the estimation of the size of background processes in comparison to new physics signals at the LHC.

4. Jet algorithms and Infrared safety

Perturbation theory is the only method available to describe phenomena at high energy collision experiments. However, its applicability is restricted to a small set of observables which are "infrared safe". Higher order calculations rely on a delicate cancelation of infrared singularities in virtual corrections against real radiation configurations of the same perturbative order which have an indistinguishable final state below a certain resolution. Jet algorithms in a perturbative calculation cluster partons into jets. Virtual and real configurations which have opposite singularities must be clustered under the same jet multiplicity for a finite result to be obtained.

Unfortunately, jet measurements at hadron colliders were only rarely performed with infrared safe algorithms. It is remarkable that the prediction of the W+3jets rate as measured at the Tevatron cannot be compared with the recently obtained NLO theoretical cross-sections due to the use of infrared unsafe algorithm. Programs for jet observables in fixed order perturbation theory literally return NAN when the cancelation of infrared singularities is not achieved. It is crucial that at the LHC era infrared safe algorithms are used.

In the last couple of years, Cacciari, Salam, and Soyez provided a fast implementation of recombination algorithms, which are infrared safe, and developed an infrared safe cone algorithm (SIScone) [26–29]. Cone algorithms have the advantage of a very simple geometry for jets, allowing easier estimates of the underlying event and hadtonization effects. However, the fast implementation of recombination algorithms allows for Monte-Carlo methods to estimate jet areas. Recently a new recombination algorithm has been developed by the same authors, the anti-k_t algorithm, with "perfect cone" jet geometry.

A jet algorithm can be more useful than a mundane definition of what a jet is. A felxibility to employ diverse infrared safe algorithms is necessary, since different algorithms may have varied diagnostic for the discovery of new physics.

A beautiful example is shown in a recent paper by Butterworth, Davison, Rubin and Salam [30]. The discovery of a Higgs boson in association with electroweak gauge bosons

$$pp \to VH \to Vb\bar{b}$$

has been considered to be very difficult at the LHC. These authors exploit that a heavy jet from the decay of a Higgs boson with high transverse momentum has a characteristic substructure, containing both $b-$quarks, with a different angular dependence than the very frequent QCD splitting of a gluon to b-quarks. In that case, the Aachen-Cambridge recombination algorithm captures best the differences of the QCD versus the Higgs splitting. Now, with this method, this channel is a realistic discovery channel of the Higgs boson at the LHC.

5. The NNLO front

The precision of measurements at collider experiments is often excellent. On the other hand, perturbation theory is often slow at work, with the first correction after the leading order being too large and too uncertain. This necessitates the evaluation of radiative corrections at the next-to-next-to-leading order.

The task of computing NNLO cross-sections is herculean. One would therefore need to consider carefully their utility. What is then an NNLO wishlist? Modern collider experiments allow for superb determinations of production rates of single particles or particle pairs. All these measurements need to be confronted with accurate predictions. For hadron colliders, almost all cross-sections for $2 \to 1$ and $2 \to 2$ processes must be computed at NNLO. The LEP, HERA, the TEVATRON and the LHC call for NNLO phenomenology.

Methods for the evaluation of two-loop amplitudes are powerful, although such calculations remain a formidable task. A much more challenging task at NNLO is the cancelation of infrared divergences of real and virtual radiation. Gehrmann-de Ridder, Gehrmann, Glover and Heinrich developed a universal method for the cancelation of matrix-element singularities through NNLO for lepton collider processes [31, 32]. This method was later reviewed by Weinzierl where an intricate correction was made [33].

This effort produced the most spectacular calculations in perturbative QCD, where the three-jet rate and event shapes at LEP are evaluated through NNLO [31–34]. Jet LEP data is described excellently with a synthesis of fixed order QCD and electroweak corrections, resummation of logarithms and taking into account hadronization effects. A state of the art extraction of the value of the strong coupling by comparing LEP data with the new NNLO result and NLL resummation, by Dissertori, Gehrmann-de Ridder, Gehrmann, Glover, Heinrich, Luisoni and Stelzer [35]. This leads to the value

$$\alpha_s(M_z) = 0.1224 \pm 0.0009(\text{stat}) \pm 0.0009(\text{exp}) \pm 0.0012(\text{had}) \pm 0.0035(\text{theo}) \tag{5.1}$$

A very precise determination of α_s has also been performed by Becher and Schwartz from the NNLO fixed order calculation of the thrust distribution and very accurate resummation methods based on soft collinear effective theory [36].

6. The legacy of HERA and parton densities for the Tevatron and the LHC

HERA experiments made tremendous contributions in understanding QCD and the proton. The corresponding theory has been pushed to extreme precision. Altarelli-Parisi parton evaluation kernels have been computed thgrough NNLO, and structure functions thgrough NNNLO, in heroci calculations by Moch, Vogt and Vermaseren [37–40]. An experimental highlight at the end of the HERA era was the measurement of F_L which is directly sensitive to the gluon density.

HERA's heritage of parton distribution functions is of paramount importance for both the Tevatron and the LHC. Several collaborations have published updated parton densitites providing valuable input for precise hadron collider phenomenology. Recently, new ideas have emerged on the extraction of parton densities from experimental data, using Artificial Neural Network methods [41]. In addition, several improvements have been made on the theoretical treatment of the error estimation, leading to more realistic uncertainties. We should note that in the course of the last few years, changes in parton densities were in some important cases rather significant [42].

7. From the Tevatron to the LHC

The Tevatron has single handedly carried out experimentation at the highest energies during the last decade. A large number of studies of extraordinary quality have been performed, preparing solid ground for the anticipated discoveries of the coming decade. Indicatively, we shall discuss two processes that have preoccupied the Tevatron experiments and will undoubtely be of high interest at the LHC.

The Drell-Yan process of W and Z boson production has a clean signal which allows for high precision measurements at a hadron collider environment. This process is a valuable source of information. Already at the Tevatron, it has been used in order to determine the luminosity. At the LHC, it is anticipated to be the main method for luminosity monitoring. Tevatron data for Drell-Yan production is used in global analyses for the determination of parton densities. The Drell-Yan process is our window to electroweak physics at a hadron collider, and can be used to determine electroweak parameters such as the W-mass and the Winberg angle. Amazingly, the Tevatron has produced one of the most accurate determinations of the W-mass, which is comparable in precision with the one of the LEP experiments.

The Drell-Yan process can be simulated very accurately by NNLO QCD theory. Fully differential cross-sections are computed at this perturbative order [43, 44], with a typical scale variation of less than 2%. We remark that a future calculation of mixed QCD and QED corrections, is desired for the W-mass measurement.

The Tevatron experiments have embarked into a vigorous search for the Higgs boson. Exclusion limits on the cross-section are close to the Standard Model predition, and in a very small mass region around twice the mass of the W-boson they are smaller. The sensitivity in this mass range is almost entirely due to the gluon fusion process. Sophisticated experimental analyses, exploiting efficiently kinematic differences of signal and background processes with multivariate statistical methods, have been developed and implemented by both CDF and D0.

The exclusion of the Higgs boson for $m_h \sim 2m_W$ relies substantially on signal theory predictions. Total and fully differential cross-sections are computed through NNLO. Unlike Drell-Yan production, NNLO theory is not as precise. The perturbative series converges rather slowly, and the magnitude of the corrections is sensitive to selection cuts which limit to small transverse momenta associated jet radiation in central detector regions.

The gluon fusion cross-section is very sensitive to the gluon distribution function of the proton. Only the MSTW pdf sets have an NNLO evolution and fit TEVATRON jet data, which constrains the gluon partons at high Bjorken-x. Essentially, these two conditions render MSTW pdfs the only available choice for parton densities to be used for the gluon fusion cross-section [42]. During the last few years, these sets have changed significantly in a systematic effort to estimate realistically the uncertainty of the parton densities. Recently, it became possible to estimate consistently the additional error due to the uncertainty on the value of the strong coupling constant [45]. This enlarges the uncertainty in comparison to the estimation used in the experimental analysis.

The differential nature of multivariate techniques necessitates a very careful study of the stability of experimental acceptances with higher order corrections. It is found that Jet-veto and lepton isolation efficiencies can vary with the use of different parton shower Monte-Carlo programs for the Tevatron setups [46]. In the experimental analysis, potential Higgs signal events are subjected to different selections according to the number of central jets in the event. Separate theoretical predictions for each jet multiplicity are required, since these cannot be deduced from the total cross-section and parton showers reliably. We remark, that very detailed neural network studies which are very close to the experimental setup can be performed at NNLO [46]. These studies need to be adopted as part of the official analysis from the Tevatron collaborations in the future.

8. Outlook

Our abilities in simulating precisely collider processes have grown tremendously. New computational methods at NLO are extremely powerful. A classic piece of theoretical work has been made in calculating one-loop amplitudes in gauge theories. This work will become part of future field theory books. Phenomenology has moved to a precision era, even at the most difficult conditions which prevail at hadron collider experiments. Theory is ready to take on the big challenge of finding new physics signals convincingly in hadron collider data.

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