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Renormalization of B-meson distribution amplitudes

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We summarize recent calculations concerning the evolution kernels of the two-particle B-meson distribution amplitudes ϕ_+ and ϕ_- taking into account three-particle contributions, as well as the evolution kernel of the combination of three-particle distribution amplitudes $\Psi_A - \Psi_V$. We exploit these results to confirm constraints on ϕ_+ and ϕ_- derived from the light-quark equation of motion.

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Exclusive decays of B-mesons provide important tools to test the Standard Model and to search for physics beyond it. In this game, B-meson light-cone distribution amplitudes (LCDAs) have been shown to play a prominent role, since these hadronic inputs encode a part of soft physics that is not covered by the usual form factors. Recent years have seen several analyses concerning the renormalization properties [1, 2, 3] and the shape of the B-meson LCDAs [3, 4, 5, 6, 7, 8, 9]. Up to now most of these analyses were restricted to the two-particle case. Here we present the results of for the renormalization of the two-particle B-meson LCDAs taking into account mixing with three-parton LCDAs [10] as well as for the combination of three-particle LCDAs $\Psi_A - \Psi_V$ entering the equations of motion [11].

The relevant two- and three-parton distribution amplitudes are defined as B to vacuum matrix-elements of a non-local heavy-to-light operator, which reads in the two-particle case [4]:

$$\langle 0|\bar{q}_\beta(z)[z,0](h_v)_\alpha(0)|B(p)\rangle = -i\frac{\hat{f}_B(\mu)}{4}\left[(1+\not{v})\left(\tilde{\phi}_+(t)+\frac{\not{z}}{2t}[\tilde{\phi}_-(t)-\tilde{\phi}_+(t)]\right)\gamma_5\right]_{\alpha\beta}, \quad (1)$$

and in the three-particle case [7]:

$$\begin{aligned}&\langle 0|\bar{q}_\beta(z)[z,uz]gG_{\mu\nu}(uz)z^\nu[uz,0](h_v)_\alpha(0)|B(p)\rangle \qquad (2)\\ &\quad=\frac{\hat{f}_B(\mu)M}{4}\Big[(1+\not{v})\Big[(v_\mu\not{z}-t\gamma_\mu)\left(\tilde{\Psi}_A(t,u)-\tilde{\Psi}_V(t,u)\right)-i\sigma_{\mu\nu}z^\nu\tilde{\Psi}_V(t,u)\\ &\qquad -z_\mu\tilde{X}_A(t,u)+\frac{z_\mu\not{z}}{t}\tilde{Y}_A(t,u)\Big]\gamma_5\Big]_{\alpha\beta}.\end{aligned}$$

We use light-like vectors $n_\pm$ so that $n_+^2 = n_-^2 = 0$, $n_+\cdot n_- = 2$, $v = (n_+ + n_-)/2$. The computation of the renormalisation properties of the distribution amplitudes requires us to consider matrix elements of the relevant operators on the light cone:

$$O_\pm^H(\omega) = \frac{1}{2\pi}\int dt e^{i\omega t}\langle 0|\bar{q}(z)[z,0]\not{n}_\pm\Gamma h_v(0)|H\rangle \quad (3)$$

$$O_3^H(\omega,\xi) = \frac{1}{(2\pi)^2}\int dt e^{i\omega t}\int du e^{i\xi ut}\langle 0|\bar{q}(z)[z,uz]g_s G_{\mu\nu}(uz)z^\nu[uz,0]\Gamma h_v(0)|H\rangle, \quad (4)$$

with z parallel to n_+, i.e. $z_\mu = tn_{+,\mu}$, $t = v\cdot z = z_-/2$ and the path-ordered exponential in the n_+ direction: $[z,0] = P\exp\left[ig_s\int_0^z dy_\mu A^\mu(y)\right]$. The Fourier transforms of the different distribution amplitudes are then defined as

$$\phi_\pm(\omega) = \frac{1}{2\pi}\int dt e^{i\omega t}\tilde{\phi}_\pm(t), \qquad F(\omega,\xi) = \frac{1}{(2\pi)^2}\int dt\int du\, t\, e^{i(\omega+u\xi)t}\tilde{F}(t,u), \quad (5)$$

where $F = \Psi_V, \Psi_A, X_A, Y_A$. Since the renormalization of the operators is independent of the infrared properties of the matrix-elements, we can choose an on-shell partonic external state consisting of a light quark, a heavy quark and a gluon in equation (4). The resulting leading-order diagrams are shown in fig. 1 for $O_\pm$ (for $O_{3\mu}$, there is only one diagram, similar to the left diagram in fig. 1). Next-to-leading order (NLO) diagrams are obtained by adding a gluon or a quark loop (or a ghost loop) in all possible places (for a complete list of diagrams, see [10]). The evaluation of these diagrams yields the corresponding anomalous dimensions at one loop.

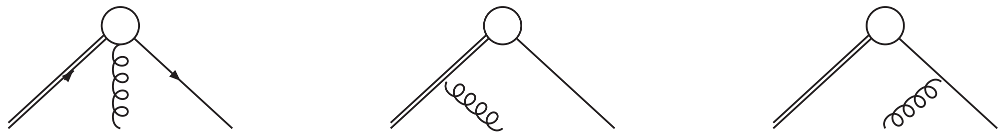

Figure 1: The three leading-order contributions to the matrix element of $O_\pm$ with a three-parton external state. The white circle represents the operator and the double line corresponds to the heavy quark.

For both two-parton distribution amplitudes, the renormalization group equation to order α_s can then be written as:

$$\frac{\partial \phi_\pm(\omega;\mu)}{\partial \log\mu} = -\frac{\alpha_s(\mu)}{4\pi}\left(\int d\omega'\, \gamma_-^{(1)}(\omega,\omega';\mu)\phi_\pm(\omega';\mu) + \int d\omega' d\xi' \gamma_{\pm,3}^{(1)}(\omega,\omega',\xi';\mu)\Psi_3(\omega',\xi';\mu)\right), \quad (6)$$

where Ψ_3 denotes the combination of three-parton distribution amplitudes mixing with the two-parton distribution amplitude of interest.

In the ϕ_+-case there is no mixing from three-particle distribution amplitudes: $\gamma_{+,3}=0$ at order α_s. We confirm the result for the anomalous-dimension matrix found in ref. [1]

$$\gamma_+^{(1)}(\omega,\omega';\mu) = \left(\Gamma_{\rm cusp}^{(1)}\log\frac{\mu}{\omega}+\gamma^{(1)}\right)\delta(\omega-\omega') - \Gamma_{\rm cusp}^{(1)}\omega\left(\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)}+\frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right)_+, \quad (7)$$

with $[f(\omega,\omega')]_+ = f(\omega,\omega')-\delta(\omega-\omega')\int d\omega' f(\omega,\omega')$, $\Gamma_{\rm cusp}^{(1)}=4$ and $\gamma^{(1)}=-2$.

The ϕ_- case is more involved. After including the renormalisation of the coupling constant and the wave functions there remains a genuine three-particle term, which corresponds to $\Psi_3 = \Psi_A-\Psi_V$. In eq. (6), the anomalous dimensions are $\gamma_-^{(1)}$, from ref. [2], and $\gamma_{-,3}^{(1)}$, from ref. [10]:

$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)} - \Gamma_{\rm cusp}^{(1)}\frac{\theta(\omega'-\omega)}{\omega'} \quad (8)$$

$$\gamma_{-,3}^{(1)}(\omega,\omega',\xi') = 4\left[\frac{\Theta(\omega)}{\omega'}\left\{(C_A-2C_F)\left[\frac{1}{\xi'^2}\frac{\omega-\xi'}{\omega'+\xi'-\omega}\Theta(\xi'-\omega)+\frac{\Theta(\omega'+\xi'-\omega)}{(\omega'+\xi')^2}\right] - C_A\left[\frac{\Theta(\omega'+\xi'-\omega)}{(\omega'+\xi')^2}-\frac{1}{\xi'^2}\left(\Theta(\omega-\omega')-\Theta(\omega-\omega'-\xi')\right)\right]\right\}\right]_+, \quad (9)$$

where we defined the $+$-distribution with three variables as

$$\Big[f(\omega,\omega',\xi')\Big]_+ = f(\omega,\omega',\xi')-\delta(\omega-\omega'-\xi')\int d\omega\, f(\omega,\omega',\xi''). \quad (10)$$

A similar result can be derived concerning the three-particle LCDAs $\Psi_A-\Psi_V$ which arises in the renormalization-group equation of ϕ_-. We project on the relevant distribution amplitudes in equation (3) using $\Gamma=\gamma_\perp^\mu \not n_+ \not n_- \gamma_5$ (taking γ^μ instead of $\gamma_\perp^\mu$ yields the same result). The result cqn be cqst into C_F- and C_A-colour structures

$$\begin{aligned}
\gamma^{(1)}_{3,3,C_A}(\omega,\xi,\omega',\xi') &= 2\Bigg[\delta(\omega-\omega')\left\{\frac{\xi}{\xi'^2}\Theta(\xi'-\xi)-\left[\frac{\Theta(\xi-\xi')}{\xi-\xi'}\right]_+-\left[\frac{\xi}{\xi'}\frac{\Theta(\xi'-\xi)}{\xi'-\xi}\right]_+\right\}\\
&+\delta(\xi-\xi')\left\{\left[\frac{\Theta(\omega-\omega')}{\omega-\omega'}\right]_++\left[\frac{\omega}{\omega'}\frac{\Theta(\omega'-\omega)}{\omega'-\omega}\right]_+\right\}+\delta(\omega+\xi-\omega'-\xi')\\
&\times\left\{\frac{1}{\xi'}\Theta(\omega-\omega')-\left[\frac{\Theta(\omega-\omega')}{\omega-\omega'}\right]_+-\left[\frac{\omega}{\omega'}\frac{\Theta(\omega'-\omega)}{\omega'-\omega}\right]_+\right\}\\
&+\delta(\omega+\xi-\omega'-\xi')\frac{1}{\xi'(\omega'+\xi')}\Bigg\{\frac{\omega-\xi'}{\xi'}(\omega'+\xi'-\omega)\Theta(\omega-\omega')\\
&-\frac{\omega}{\omega'}(\omega'+2\xi'-\omega)\Theta(\omega'-\omega)\Theta(\omega)+\frac{\omega}{\xi'}(\omega-\xi')\Theta(\xi'-\omega)\Theta(\omega)\\
&+\frac{\omega-\xi'}{\omega'}(\omega'+\xi'-\omega)\Theta(\omega-\xi')\Theta(\xi)\Bigg\}\Bigg], \qquad (11)
\end{aligned}$$

$$\begin{aligned}
\gamma^{(1)}_{3,3,C_F}(\omega,\xi,\omega',\xi';\mu) &= \gamma^{(1)}_+(\omega,\omega';\mu)\delta(\xi-\xi')+\gamma^{(1)}_{R3,3}(\omega,\xi,\omega',\xi')\\
\gamma^{(1)}_{R3,3}(\omega,\xi,\omega',\xi') &= 4\delta(\omega+\xi-\omega'-\xi')\\
&\times\left[\frac{\xi^2}{\omega'}\frac{\Theta(\omega'-\xi)}{(\omega+\xi)^2}\Theta(\xi)+\frac{\omega}{\xi'}\frac{\Theta(\xi-\omega')}{\omega+\xi}\Theta(\omega)\left(\frac{\xi}{\omega+\xi}-\frac{\omega-\xi'}{\xi'}\right)\right],
\end{aligned}$$

with $\gamma^{(1)}_+$ is given in eq. (7) and $\gamma^{(1)}_{3,3}$ defined as in (2.11) with obvious changes. Part of this calculation, namely the light-quark-gluon part, has been calculated in a different context and a different scheme, e.g. in [12, 13].

We turn to two applications of our results now. In ref. [7] two equations from the light- and heavy-quark equations of motion were derived

$$\omega\phi'_-(\omega;\mu)+\phi_+(\omega;\mu) = I(\omega;\mu), \qquad (\omega-2\bar{\Lambda})\phi_+(\omega;\mu)+\omega\phi_-(\omega;\mu) = J(\omega;\mu), \qquad (12)$$

where $I(J)(\omega;\mu)$ are integro-differential expressions involving the three-particle LCDAs $\Psi_A-\Psi_V$ (Ψ_A+X_A and Ψ_V) respectively. While the second equation was shown not to hold beyond leading order in ref. [2, 9] we have checked that the first one is valid once renormalization is taken into account by taking the derivative of the first equation with respect to $\log\mu$, and exploiting the respective evolution kernels eqs. (7), (9), (11), (12). This non-trivial outcome gives us further confidence concerning the renormalization group properties of the LCDAs.

The presence of $\delta(\omega-\omega')\log(\mu/\omega)$ in the renormalization matrices gives rise to a radiative tail falling off like $(\log\omega)/\omega$ for large ω. Therefore non-negative moments of the LCDAs are not well defined and have to be considered with an ultraviolet cut-off [1, 2, 8, 9]:

$$\langle\omega^N\rangle_\pm(\mu) = \int_0^{\Lambda_{UV}} d\omega\,\omega^N\,\phi_\pm(\omega;\mu)\,. \qquad (13)$$

For ϕ_- it is interesting to examine the limit

$$\lim_{\Lambda_{UV}\to\infty}\int_0^{\Lambda_{UV}} d\omega\,\omega^N z^{(1)}_{-,3}(\omega,\omega',\xi')=0\,, \qquad z^{(1)}_{-,3}=\frac{1}{2\varepsilon}\gamma^{(1)}_{-,3}\,, \qquad (14)$$

which is relevant for the calculation of the three-particle contributions to the moments:

$$\int_0^{\Lambda_{UV}} d\omega\,\omega^N\,\phi_-(\omega;\mu) = 1+\frac{\alpha_s}{4\pi}\left(\int d\omega'\,\phi_-(\omega')\int_0^{\Lambda_{UV}} d\omega\,\omega^N z_-^{(1)}(\omega,\omega';\mu)\right.$$
$$\left.-\int d\omega' d\xi'(2-D)[\Psi_A-\Psi_V](\omega',\xi')\int_0^{\Lambda_{UV}} d\omega\,\omega^N z_{-,3}^{(1)}(\omega,\omega',\xi')\right). \tag{15}$$

Therefore as stated in ref. [2] three-particle distribution amplitudes give only subleading contribution to the first two moments ($N=0,1$). We have explicitly checked that this statement cannot be extended to higher moments ($N\geq 2$).

The next step consists in using the renormalization properties as a guide to go beyond the existing models derived from a leading-order sum-rule calculation resulting in $\Psi_A=\Psi_V$ [6] and to analyze their influence on ϕ_-. Finally, for practical calculations involving three-particle contributions, one would need the evolution kernels for the relevant LCDAs, which will be the subject of a future work.

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