PROCEEDINGS OF SCIENCE

V_{us} and V_{ud} determination

M. Antonelli*
Laboratori Nazionali di Frascati dell'INFN, Frascati, Italy
E-mail: Mario.Antonelli@lnf.infn.it

I present the analysis of leptonic, and semileptonic kaon decays, and nuclear beta decays between 0^+ states. These analysis lead to a very accurate determination of V_{us} and V_{ud} and allows us to perform several stringent tests of the Standard Model.

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*Speaker.

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1. Introduction

In the Standard Model, SM, transition rates of semileptonic processes such as $d^i \to u^j \ell \nu$, with d^i (u^j) being a generic down (up) quark, can be computed with high accuracy in terms of the Fermi coupling G_F and the elements V_{ji} of the Cabibbo-Kobayashi Maskawa (CKM) matrix [1, 2]. Measurements of the transition rates provide therefore precise determinations of the fundamental SM couplings.

A detailed analysis of semileptonic decays offers also the possibility to set stringent constraints on new physics scenarios. While within the SM all $d^i \to u^j \ell \nu$ transitions are ruled by the same CKM coupling V_{ji} (satisfying the unitarity condition $\sum_k |V_{ik}|^2 = 1$) and G_F is the same coupling appearing in the muon decay, this is not necessarily true beyond the SM. Setting bounds on the violations of CKM unitarity, violations of lepton universality, and deviations from the $V-A$ structure, allows us to put significant constraints on various new-physics scenarios (or eventually find evidences of new physics).

An illustration of the importance of semileptonic decays in testing the SM is provided by the unitarity relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \varepsilon_{\rm NP} \; . \tag{1.1}$$

Here the V_{ji} are the CKM elements determined from the various $d^i \to u^j$ processes, having fixed G_F from the muon life time: $G_\mu = 1.166371(6) \times 10^{-5} \mathrm{GeV}^{-2}$ [3] $\varepsilon_{\rm NP}$ parametrizes possible deviations from the SM induced by dimension-six operators, contributing either to the muon decay or to the $d^i \to u^j$ transitions. By dimensional arguments we expect $\varepsilon_{\rm NP} \sim M_W^2/\Lambda_{\rm NP}^2$, where $\Lambda_{\rm NP}$ is the effective scale of new physics. The present accuracy on $|V_{ux}|$, allows to set bounds on $\varepsilon_{\rm NP}$ around 0.1% or equivalently to set bounds on the new physics scale well above 1 TeV[4].

1.1 V_{ud} from nuclear decays

Nuclear beta decays between 0^+ states sample only the vector component of the hadronic weak interaction. This is important because the conserved vector current (CVC) hypothesis protects the vector coupling constant $G_V = G_F V_{ud}$ from renormalization by background strong interactions. To date, precise measurements of the beta decay between isospin analog states of spin, $J^\pi = 0^+$, and isospin, $T = 1$, provide the most precise value of V_{ud}. A survey of the relevant experimental data has recently been completed by Hardy and Towner [5].

For each transition, three experimental quantities have to be determined: the decay energy, $Q_{\rm ec}$; the half-life of the decaying state, $t_{1/2}$; and the branching ratio, R, for the particular transition under study. The decay energy is used to calculate the phase space integral, f, where it enters as the fifth power. The partial half-life is defined as $t = t_{1/2}/R$ and the product ft is

$$ft = \frac{K}{2\, G_F^2 V_{ud}^2}, \tag{1.2}$$

where $K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2/(m_e c^2)^5 = 8120.2787(11) \times 10^{-10}$ GeV^{-4} s. According to CVC the ft value is a constant independent of the nucleus under study. In practice, however, isospin is always a broken symmetry in nuclei, and beta decay occurs in the presence of radiative corrections, so a 'corrected' ft value is defined by

$$\mathscr{F}t \equiv ft(1+\delta_R')\,(1-(\delta_C-\delta_{NS})) = \frac{K}{2\,G_F^2 V_{ud}^2\,(1+\Delta_{\mathrm{R}}^{\mathrm{V}})}; \tag{1.3}$$

so it is this corrected $\mathscr{F}t$ that is a constant. Here the radiative correction has been separated into three components: (i) $\Delta_{\mathrm{R}}^{\mathrm{V}}$ is a nucleus-independent part that includes the universal short-distance component S_{EW} affecting all semi-leptonic decays. Being a constant, $\Delta_{\mathrm{R}}^{\mathrm{V}}$ is placed on the right-hand-side of Eq. (1.3); (ii) δ_R' is transition dependent, but only in a trivial way, since it just depends on the nuclear charge, Z, and the electron energy, E_e; while δ_{NS} is a small nuclear-structure dependent term that requires a shell-model calculation for its evaluation. (iii) Lastly, δ_C is an isospin-symmetry breaking correction, typically of order 0.5%, that also requires a shell-model calculation for its evaluation.

In Fig. 1 are shown the experimental ft values from the survey of Hardy and Towner [5] for 13 transitions, of which 10 have an accuracy at the 0.1% level, and three at up to the 0.4% level. The corrected $\mathscr{F}t$ values are also given.

The weighted average of the 13 data is

$$\overline{\mathscr{F}t} \;=\; 3071.83 \pm 0.79_{\mathrm{stat}} \pm 0.32_{\mathrm{syst}}\ \mathrm{s} \tag{1.4}$$

with a corresponding chi-square per degree of freedom of $\chi^2/\nu = 0.31$. Isospin-symmetry-breaking correction, δ_C, are taken from an average [5] of two determinations obtained with the Hartree-Fock and Saxon-Woods potential. Their differnece is taken as systematic.

Using $\Delta_{\mathrm{R}}^{\mathrm{V}} = (2.631 \pm 0.038)\%$ from [6] and $\overline{\mathscr{F}t}$ from Eq. (1.4), the value of V_{ud} becomes

$$V_{ud} = 0.97425 \pm 0.00022. \tag{1.5}$$

The error is dominated by theoretical uncertainties; experiment only contributes 0.00008 to the error budget. Currently the largest contribution to the error budget comes from the nucleus-independent radiative correction $\Delta_{\mathrm{R}}^{\mathrm{V}}$.

1.2 Determination of $|V_{us}|$ from $K_{\ell 2}$ and $K_{\ell 3}$

1.2.1 $P_{\ell 2}$ ($P = \pi, K$) rates within the SM

Including all known short- and long-distance electroweak corrections, and parameterizing the hadronic effects in terms of a few dimensionless coefficients, the inclusive $P \to \ell \overline{\nu}_\ell(\gamma)$ decay rate can be written as [7, 8]

$$\Gamma_{P_{\ell 2}} = \frac{G_F^2 |V_P|^2 f_P^2}{4\pi} M_P m_\ell^2 \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2 S_{\mathrm{EW}}[1 + \frac{\alpha}{\pi} F(M_P, m_\ell, M_\rho, c_i)] \tag{1.6}$$

where $V_\pi = V_{ud}, \quad V_K = V_{us}$. The factor S_{EW} describes the short-distance electromagnetic correction [9, 10] which is universal for all semileptonic processes. Including also the leading QCD corrections [7], it assumes the numerical value $S_{\mathrm{EW}} = 1.0232$. The most recent calculation of $F(M_P, m_\ell, M_\rho, c_i)$ is described in ref. [8].

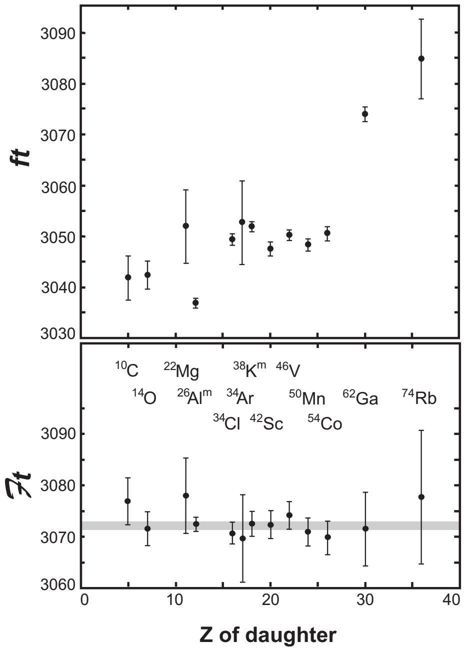

Figure 1: In the top panel are plotted the uncorrected experimental ft values as a function of the charge on the daughter nucleus. In the bottom panel, the corresponding $\mathscr{F}t$. The horizontal grey band in the bottom panel gives one standard deviation around the average of $\mathscr{F}t$.

As suggested by Marciano [11], a determination of $|V_{us}/V_{ud}|$ can be obtained by combining the experimental values for the decay rates with the lattice determination of f_K/f_π via

$$\frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.23872(30)\left(\frac{\Gamma_{K_{\ell 2(\gamma)}}}{\Gamma_{\pi_{\ell 2(\gamma)}}}\right)^{1/2} . \tag{1.7}$$

The small error is an estimate of unknown electromagnetic contributions arising at order e^2p^4.

In the standard model, the ratios $R^{(P)}_{e/\mu} = \Gamma_{P\to e\bar{\nu}_e(\gamma)}/\Gamma_{P\to\mu\bar{\nu}_\mu(\gamma)}$ are helicity suppressed as a consequence of the $V-A$ structure of the charged currents, constituting sensitive probes of new physics. In a first systematic calculation to order e^2p^4, the radiative corrections to $R^{(P)}_{e/\mu}$ have been obtained with an unprecedented theoretical accuracy [8, 12]. The two-loop effective theory results were complemented with a matching calculation of an associated counterterm, giving

$$R_\pi = (1.2352\pm 0.0001)\times 10^{-4} , \qquad R_K = (2.477\pm 0.001)\times 10^{-5} . \tag{1.8}$$

1.2.2 $K_{\ell 3}$ rates within the SM

The photon-inclusive $K_{\ell 3}$ decay rates are conveniently decomposed as [14]

$$\Gamma_{K_{\ell 3(\gamma)}} = \frac{G_F^2 M_K^5}{192\pi^3} C_K^2 S_{\rm EW} \left|V_{us} f_+^{K^0\pi^-}(0)\right|^2 I_K^\ell(\lambda_{+,0}) \left(1+\delta_{\rm EM}^{K\ell}+\delta_{\rm SU(2)}^{K\pi}\right) , \tag{1.9}$$

where $C_K^2 = 1$ $(1/2)$ for the neutral (charged) kaon decays, S_{EW} is the short distance electroweak correction, $f_+^{K^0\pi^-}(0)$ is the $K\to\pi$ vector form factor at zero momentum transfer, and $I_K^\ell(\lambda_{+,0})$ is the phase space integral which depends on the (experimentally accessible) slopes of the form factors (generically denoted by $\lambda_{+,0}$). Finally, $\delta^{K\ell}_{\rm EM}$ represent channel-dependent long distance radiative corrections and $\delta^{K\pi}_{\rm SU(2)}$ is a correction induced by strong isospin breaking.

The results of the most recent calculation [15] of the four channel-dependent long-distance electromagnetic corrections $\delta^{K\ell}_{\rm EM}$ are shown in Tab. 1.

	K^0_{e3}	$K^{\pm}_{e3}$	$K^0_{\mu 3}$	$K^{\pm}_{\mu 3}$
$\delta^{K\ell}_{\rm EM}(\%)$	0.99(22)	0.10(25)	1.40(22)	0.016(25)

Table 1: Summary of the electromagnetic corrections to the fully-inclusive $K_{\ell 3(\gamma)}$ rate [15].

The strong isosipn breaking correction has been recently updated to $\delta^{K^{\pm}\pi^0}_{\rm SU(2)} = 0.058(8)$ [16].

The hadronic $K \to \pi$ matrix element of the vector current is described by two form factors (FFs), $f_+(t)$ and $f_-(t)$

$$\langle \pi^-(p_\pi)|\bar{s}\gamma^\mu u|K^0(p_K)\rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t) \tag{1.10}$$

where $t = (p_K - p_\pi)^2 = (p_\ell + p_\nu)^2$. The vector form factor $f_+(t)$ represents the P-wave projection of the crossed channel matrix element $\langle 0|\bar{s}\gamma^\mu u|K\pi\rangle$ whereas the S-wave projection is described by the scalar form factor defined as

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t) \ . \tag{1.11}$$

By construction, $f_0(0) = f_+(0)$.

In order to compute the phase space integrals appearing in Eq. (1.9) we need experimental or theoretical inputs about the t-dependence of $f_{+,0}(t)$. The t-dependence of the FFs at present is better determined by measurements and by combining measurements and dispersion relations. To that aim, we introduce the normalized FFs

$$\tilde{f}_+(t) = \frac{f_+(t)}{f_+(0)} \ , \ \tilde{f}_0(t) = \frac{f_0(t)}{f_0(0)} \ , \ \tilde{f}_+(0) = \tilde{f}_0(0) = 1 \ . \tag{1.12}$$

Whereas $\tilde{f}_+(t)$ is accessible in the K_{e3} and $K_{\mu 3}$ decays, $\tilde{f}_0(t)$ is more difficult to measure since it is only accessible in $K_{\mu 3}$ decays, being kinematically suppressed in K_{e3} decays, and is strongly correlated with $\tilde{f}_+(t)$. The scalar form factor is of special interest due to the existence of the Callan-Treiman (CT) theorem [17] which predicts the value of the scalar form factor at the so-called CT point, namely $t \equiv \Delta_{K\pi} = m_K^2 - m_\pi^2$,

$$C \equiv \tilde{f}_0(\Delta_{K\pi}) = \frac{f_K}{f_\pi}\frac{1}{f_+(0)} + \Delta_{CT}, \tag{1.13}$$

where $\Delta_{CT} \sim \mathscr{O}(m_{u,d}/4\pi F_\pi)$ is a small correction. ChPT at NLO in the isospin limit [18] gives $\Delta_{CT} = (-3.5 \pm 8) \times 10^{-3}$. The measurement of C provide a powerful consistency check of the lattice QCD calculations of f_K/f_π and $f_+(0)$, as will be discussed in Sec. 1.3.2.

If the FF are expanded in powers of t up to t^2 as

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0}\,\frac{t}{m^2} + \frac{1}{2}\,\lambda''_{+,0}\left(\frac{t}{m^2}\right)^2 \tag{1.14}$$

four parameters: λ'_+, λ''_+, λ'_0 and λ''_0 need to be determined from the decay spectrum in order to be able to compute the phase space integral which appears in the formula for the partial decay width. The problems with the four parameters above is the large correlations, in particular -99.96% between λ'_0 and λ''_0 and -97.6% between λ'_+ and λ''_+. It is not therefore possible to obtain meaningful results for the scalar FF parameters.

It is experimentally well established in K_{Le3} decays that the vector form factor is equally described by a pole form:

$$\tilde{f}_+ = \frac{M_V^2}{M_V^2 - t}. \tag{1.15}$$

which expands to $1 + t/M_V^2 + (t/M_V^2)^2$, neglecting power of t greater than 2.

Recent results on K_{e3} show that the vector form factor is dominated by the closest vector $(q\bar{q})$ state with one strange and one light quark (or K-π resonance in an older language).

$K_{\mu 3}$ decay pion spectrum measurements, have no sensitivity to λ''_0. Therefore, all authors have fitted for a linear scalar form factor $\tilde{f}_0 = 1 + \lambda_0 \frac{t}{m^2}$.

Because of correlation this leads to incorrect answers for the value of λ'_0 which comes out of the fit increased by $\sim$3.5 the coefficient of the t^2 term. To clarify this situation it is necessary to obtain a form for $\tilde{f}_0$ with t and t^2 terms but with only one parameter.

A recent parametrization for the scalar form factor [19] allow to take into account the constraint given by the Callan-Treiman relation:

$$\tilde{f}_0(t) = \exp\left(\frac{t}{\Delta_{K\pi}} \log C - G(t)\right) \tag{1.16}$$

where $G(t)$ is obtained using a dispersion relation subtracted at $t = \Delta_{K\pi}$, such that $C = \tilde{f}_0(\Delta_{K\pi})$.

1.2.3 Lattice determinations of $f_+(0)$ and f_K/f_π

Within SU(3) ChPT one can perform a systematic expansion of $f_+(0)$ of the type $f_+(0) = 1 + f_2 + f_4 + \ldots$. The first term is equal to unity due to the vector current conservation in the SU(3) limit. Because of the Ademollo-Gatto (AG) theorem [20], the first non-trivial term f_2 does not receive contributions from the local operators of the effective theory and can be computed unambiguously in terms of the kaon and pion masses (M_K and M_π) and the pion decay constant f_π. It takes the value $f_2 = -0.023$ at the physical point [21]. The task is thus reduced to the problem of finding a prediction for the quantity Δf, defined as $\Delta f \equiv f_4 + f_6 + \ldots = f_+(0) - (1 + f_2)$, which depends on the low-energy constants (LECs) of the effective theory and cannot be deduced from other processes.

The original estimate made by Leutwyler and Roos [21] was based on the quark model yielding $\Delta f = -0.016(8)$.

During the recent years various collaborations have provided new results for $f_+(0)$ using unquenched gauge configurations with both 2 and 2+1 dynamical flavors. They are shown graphically in Fig. 2 (left).

In contrast to $f_+(0)$, the pseudoscalar decay constants are not protected by the AG theorem [20] against corrections linear in the SU(3) breaking. Moreover the first non-trivial term (of order (p^4)) in the chiral expansion of f_K/f_π depends on the LECs and therefore it cannot be predicted

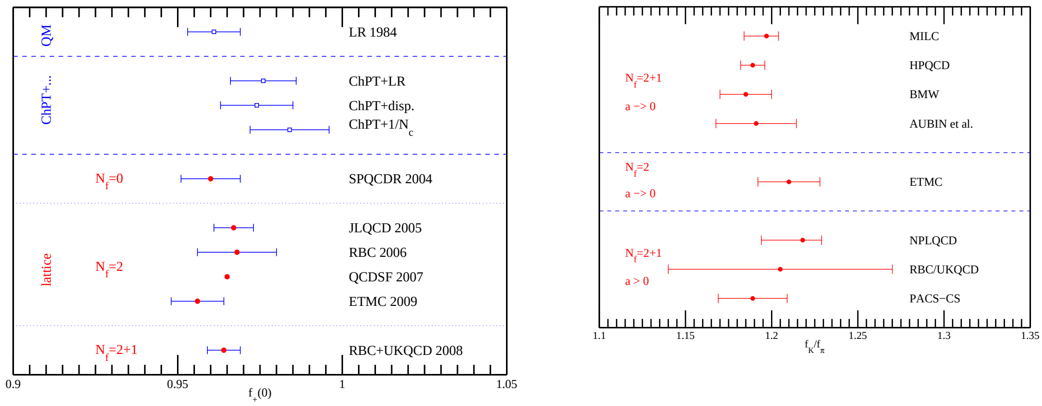

Figure 2: Left: Results of model (squares) and lattice (dots) calculations of $f_+(0)$. Right: Results of lattice calculations of f_K/f_π.

unambiguously within ChPT. This is the reason why the most precise determinations of f_K/f_π come from lattice QCD simulations.

During the recent years various collaborations have provided new results for f_K/f_π using unquenched gauge configurations with both 2 and 2+1 dynamical flavors. They are shown graphically in Fig. 2(Right).

For $f_+(0)$ the 2+1 flavor result by the RBC+UKQCD [22] collaboration is the most advanced calculation.

For f_K/f_π with $N_f = 2+1$ dynamical quarks, the currently most precise predictions are by MILC [23] which has been recently updated to $f_K/f_\pi = 1.199(^{+6}_{-8})$ [24] and HPQCD [25] both using the same set of staggered sea quark configurations.

1.2.4 Data Analysis

The FlaviaNet kaon working group performs fits to world data on the BRs and lifetimes for the K_L and $K^\pm$, with the constraint that BRs add to unity. A detailed description of the fit is given in Ref [26]. The present version of our fits uses only published measurements.

K_L leading branching ratios and τ_L

Numerous measurements of the principal K_L BRs, or of various ratios of these BRs, have been published recently. For the purposes of evaluating $|V_{us}|f_+(0)$, these data can be used in a PDG-like fit to the K_L BRs and lifetime, so all such measurements are interesting.

KTeV has measured five ratios of the six main K_L BRs [27]. The six channels involved account for more than 99.9% of the K_L width and KTeV combines the five measured ratios to extract the six BRs. We use the five measured ratios in our analysis: $\mathscr{B}(K_{\mu 3})/\mathscr{B}(K_{e3}) = 0.6640(26)$, $\mathscr{B}(\pi^+\pi^-\pi^0)/\mathscr{B}(K_{e3}) = 0.3078(18)$, $\mathscr{B}(\pi^+\pi^-)/\mathscr{B}(K_{e3}) = 0.004856(28)$, $\mathscr{B}(3\pi^0)/\mathscr{B}(K_{e3}) = 0.4782(55)$, and $\mathscr{B}(2\pi^0)/\mathscr{B}(3\pi^0) = 0.004446(25)$. The errors on these measurements are correlated; this is taken into account in our fit.

NA48 has measured the ratio of the BR for K_{e3} decays to the sum of BRs for all decays to two tracks, giving $\mathscr{B}(K_{e3})/(1-\mathscr{B}(3\pi^0)) = 0.4978(35)$ [28].

Parameter	Value	S
$\mathscr{B}(K_{e3})$	0.4056(9)	1.3
$\mathscr{B}(K_{\mu3})$	0.2704(10)	1.5
$\mathscr{B}(3\pi^0)$	0.1952(9)	1.2
$\mathscr{B}(\pi^+\pi^-\pi^0)$	0.1254(6)	1.1
$\mathscr{B}(\pi^+\pi^-)$	$1.967(7)\times10^{-3}$	1.1
$\mathscr{B}(\pi^+\pi^-\gamma)$	$4.15(9)\times10^{-5}$	1.6
$\mathscr{B}(\pi^+\pi^-\gamma)$ DE	$2.84(8)\times10^{-5}$	1.3
$\mathscr{B}(2\pi^0)$	$8.65(4)\times10^{-4}$	1.4
$\mathscr{B}(\gamma\gamma)$	$5.47(4)\times10^{-4}$	1.1
τ_L	51.16(21) ns	1.1

Table 2: Results of fit to K_L BRs and lifetime.

Using $\phi\to K_LK_S$ decays in which the K_S decays to $\pi^+\pi^-$, providing normalization, KLOE has directly measured the BRs for the four main K_L decay channels [29]. The errors on the KLOE BR values are dominated by the uncertainty on the K_L lifetime τ_L; since the dependence of the geometrical efficiency on τ_L is known, KLOE can solve for τ_L by imposing $\sum_x \mathscr{B}(K_L\to x)=1$ (using previous averages for the minor BRs), thereby greatly reducing the uncertainties on the BR values obtained. Our fit makes use of the KLOE BR values before application of this constraint: $\mathscr{B}(K_{e3})$ = 0.4049(21), $\mathscr{B}(K_{\mu3})$ = 0.2726(16), $\mathscr{B}(3\pi^0)$ = 0.2018(24), and $\mathscr{B}(\pi^+\pi^-\pi^0)$ = 0.1276(15). The dependence of these values on τ_L and the correlations between the errors are taken into account. KLOE has also measured τ_L directly, by fitting the proper decay time distribution for $K_L\to3\pi^0$ events, for which the reconstruction efficiency is high and uniform over a fiducial volume of $\sim0.4\lambda_L$. They obtain $\tau_L=50.92(30)$ ns [30].

There are also two recent measurements of $\mathscr{B}(\pi^+\pi^-)/\mathscr{B}(K_{\ell3})$, in addition to the KTeV measurement of $\mathscr{B}(\pi^+\pi^-)/\mathscr{B}(K_{e3})$ discussed above. The KLOE collaboration obtains $\mathscr{B}(\pi^+\pi^-)/\mathscr{B}(K_{\mu3})$ = $7.275(68)\times10^{-3}$ [31], while NA48 obtains $\mathscr{B}(\pi^+\pi^-)/\mathscr{B}(K_{e3})$ = $4.826(27)\times10^{-3}$ [32]. All measurements are fully inclusive of inner bremsstrahlung. The KLOE measurement is fully inclusive of the direct-emission (DE) component, DE contributes negligibly to the KTeV measurement, and a residual DE contribution of 0.19% has been subtracted from the NA48 value to obtain the number quoted above.

We fit the 13 recent measurements listed above, together with eight additional ratios of the BRs for subdominant decays. The fit gives $\chi^2/ndf=19.8/12(P=7.1\%)$.

K_S leading branching ratios and τ_S

KLOE has measured the ratio $\mathscr{B}(K_S\to\pi e\nu)/\mathscr{B}(K_S\to\pi^+\pi^-)$ with 1.3% precision [33], making possible an independent determination of $|V_{us}|\,f_+(0)$ to better than 0.7%. In [34], KLOE combines the above measurement with their measurement $\mathscr{B}(K_S\to\pi^+\pi^-)/\mathscr{B}(K_S\to\pi^0\pi^0)=2.2459(54)$. Using the constraint that the K_S BRs sum to unity and assuming the universality of lepton couplings, they determine the BRs for $\pi^+\pi^-$, $\pi^0\pi^0$, K_{e3}, and $K_{\mu3}$ decays. We perform a fit to the data on the K_S BRs to $\pi^+\pi^-$, $\pi^0\pi^0$, and K_{e3} that uses, in addition to the above two measure-

Parameter	Value	S
$\mathscr{B}(K_{\mu 2})$	63.47(18)%	1.3
$\mathscr{B}(\pi\pi^0)$	20.61(8)%	1.1
$\mathscr{B}(\pi\pi\pi)$	5.573(16)%	1.2
$\mathscr{B}(K_{e3})$	5.078(31)%	1.3
$\mathscr{B}(K_{\mu 3})$	3.359(32)%	1.9
$\mathscr{B}(\pi\pi^0\pi^0)$	1.757(24)%	1.0
$\tau_{\pm}$	12.384(15) ns	1.2

Table 3: Results of fit to $K^{\pm}$ BRs and lifetime.

ments: the measurement from NA48, $\Gamma(K_S \to \pi e \nu)/\Gamma(K_L \to \pi e \nu)$ [35], where the denominator is obtained from the results of our K_L fit, the measurement of τ_S (not assuming CPT) from NA48 [14], 89.589(70) ps, the measurement of τ_S (not assuming CPT) from KTeV [14], 89.58(13) ps, and the result $\mathscr{B}(K_{\mu 3})/\mathscr{B}(K_{e3}) = 0.66100(214)$, obtained from the assumption of universal lepton couplings.

The free parameters are the four BRs listed above plus τ_S. With six inputs and one constraint (on the sum of the BRs), the fit has one degree of freedom and gives $\chi^2 = 0.0038$ ($P = 95\%$). The results of the fit give $\mathscr{B}(K_{e3}) = 7.05(8) \times 10^{-4}$, and $\tau_S = 89.58(5)\ ps$.

$K^{\pm}$ leading branching ratios and $\tau^{\pm}$

There are several new results providing information on $K^{\pm}_{\ell 3}$ rates. The NA48/2 collaboration has published measurements of the three ratios $\mathscr{B}(K_{e3}/\pi\pi^0)$, $\mathscr{B}(K_{\mu 3}/\pi\pi^0)$, and $\mathscr{B}(K_{\mu 3}/K_{e3})$ [36]. These measurements are not independent; in our fit, we use the values $\mathscr{B}(K_{e3}/\pi\pi^0) = 0.2470(10)$ and $\mathscr{B}(K_{\mu 3}/\pi\pi^0) = 0.1637(7)$ and take their correlation into account.

KLOE has measured the absolute BRs for the K_{e3} and $K_{\mu 3}$ decays [37]. In $\phi \to K^+K^-$ events, K^+ decays into $\mu\nu$ or $\pi\pi^0$ are used to tag a K^- beam, and vice versa. KLOE performs four separate measurements for each $K_{\ell 3}$ BR, corresponding to the different combinations of kaon charge and tagging decay. The final averages are $\mathscr{B}(K_{e3}) = 4.965(53)(38)\%$ and $\mathscr{B}(K_{\mu 3}) = 3.233(29)(26)\%$. KLOE has also measured the absolute branching ratio for the $\pi\pi^0$[38] and $\mu\nu$ decay[39].

Our fit takes into account the correlation between these values, as well as their dependence on the $K^{\pm}$ lifetime. The world average value for $\tau_{\pm}$ is nominally quite precise. However, the PDG error is scaled by 2.1; the confidence level for the average is 0.17%. It is important to confirm the value of $\tau_{\pm}$. The new measurement from KLOE, $\tau_{\pm} = 12.347(30)$ ns, agrees with the PDG average.

Our fit for the six largest $K^{\pm}$ branching ratios and lifetime uses some of the old measurements ans the six measurements noted above. Fit results are given in Table 3

Measurement of BR(K_{e2})/BR($K_{\mu 2}$)

Experimental knowledge of $K_{e2}/K_{\mu 2}$ was poor until recently. The current world average $R_K = \mathscr{B}(K_{e2})/\mathscr{B}(K_{\mu 2}) = (2.45 \pm 0.11) \times 10^{-5}$ dates back to three experiments of the 1970s [14] and has a precision of about 5%. Two new measurements were reported recently by NA62 and KLOE. The preliminary result $R_K = (2.493 \pm 0.031) \times 10^{-5}$ based on about 14,000 K_{e2} events, was presented at the 2009 winter conferences by the KLOE collaboration [40]. The NA62 collaboration has recently presented at KAON09 the result $R_K = (2.500 \pm 0.016) \times 10^{-5}$, based on about 50,000 K_{e2} events from the 2008 data set [41]. Both the KLOE and the NA62 measurements are inclusive with respect to final state radiation contribution due to bremsstrahlung. The small contribution of $K_{l2\gamma}$ events from direct photon emission from the decay vertex was subtracted by each of the experiments. Combining these new results with the current PDG value yields a new world average $R_K = (2.498 \pm 0.014) \times 10^{-5}$, which is in good agreement with the SM expectation of eq. 1.8 and, with a relative error of 0.56%.

Measurements of $K_{\ell 3}$ slopes

For K_{e3} decays, recent measurements of the quadratic slope parameters of the vector form factor $(\lambda'_+, \lambda''_+)$, see eq. 1.14 are available from KTeV [42], KLOE [43], ISTRA+ [44], and NA48 [45].

For $K_{\mu 3}$ decays, recent measurements of the slope parameters $(\lambda'_+, \lambda''_+, \lambda_0)$ are available from KTeV [42], KLOE [46], ISTRA+ [47], and NA48 [48]. We will not use the ISTRA+ result for the average because systematic errors have not been provided. We use the $K_{e3} - K_{\mu 3}$ averegas provided by the experiments for KTeV and KLOE. NA48 does not provide such an average, so we calculate it for inclusion in the fit.

The results of the combination are listed in Table 4.

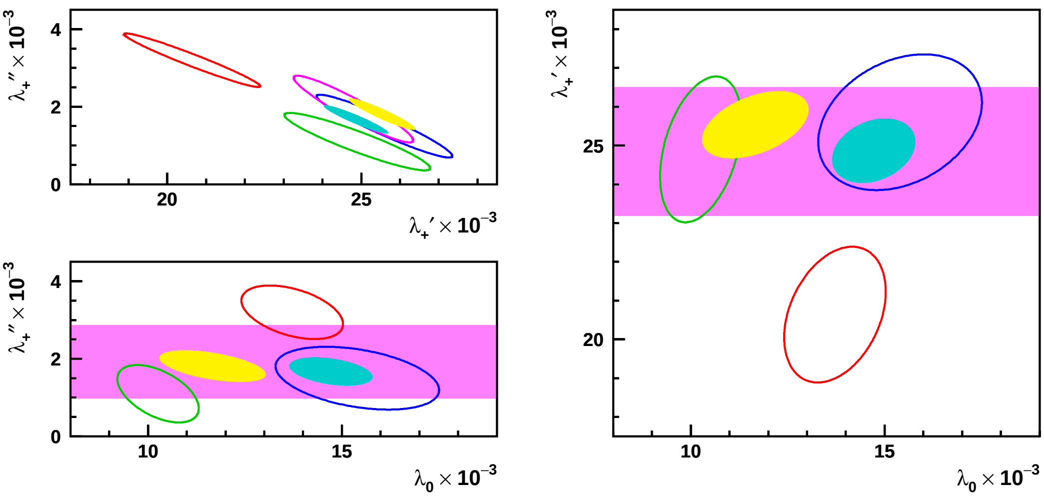

Figure 3: 1-σ contours for λ'_+, λ''_+, λ_0 determinations from KLOE(blue ellipse), KTeV(red ellipse), NA48(green ellipse), and world average with(filled yellow ellipse) and without(filled cyan ellipse) the NA48 $K_{\mu 3}$ result.

The value of χ^2/ndf for all measurements is terrible; we quote the results with scaled errors. This leads to errors on the phase-space integrals that are ~60% larger after inclusion of the new $K_{\mu 3}$ NA48 data.

λ'_+	$(24.5 \pm 0.9) \times 10^{-3}$	1	-0.94	$+0.44$
λ''_+	$(1.8 \pm 0.4) \times 10^{-3}$		1	-0.52
λ_0	$(11.7 \pm 1.4) \times 10^{-3}$			1

Table 4: Averages and correlation matrix of quadratic fit results for K_{e3} and $K_{\mu 3}$ slopes.

mode	$\|V_{us}\| \times f_+(0)$	% err	BR	τ	Δ	Int
$K_L \to \pi e \nu$	0.2165(5)	0.26	0.09	0.20	0.11	0.06
$K_L \to \pi \mu \nu$	0.2175(6)	0.32	0.15	0.18	0.15	0.16
$K_S \to \pi e \nu$	0.2157(13)	0.61	0.60	0.03	0.11	0.06
$K^\pm \to \pi e \nu$	0.2162(11)	0.52	0.31	0.09	0.41	0.06
$K^\pm \to \pi \mu \nu$	0.2168(14)	0.65	0.47	0.08	0.42	0.16
average	0.2166(5)					

Table 5: Summary of $|V_{us}| \times f_+(0)$ determination from all channels.

The evaluations of the phase-space integrals for all four modes are listed in each case. Correlations are fully accounted for, both in the fits and in the evaluation of the integrals.

1.3 Physics Results

In this section we summarize the results for $|V_{us}|$ discussed in the previous sections and based on these results we give constraints on physics beyond the SM. Instead of averages for lattice results for f_K/f_π we use $f_K/f_\pi = 1.189(7)$ by HPQCD [25].

1.3.1 Determination of $|V_{us}| \times f_+(0)$ and $|V_{us}|/|V_{ud}| \times f_K/f_\pi$

This section describes the results that are independent of the theoretical parameters $f_+(0)$ and f_K/f_π.

Determination of $|V_{us}| \times f_+(0)$

The value of $|V_{us}| \times f_+(0)$ has been determined from (1.9) using the world average values reported in section 1.2.4 for lifetimes, branching ratios and phase space integrals, and the radiative and $SU(2)$ breaking corrections discussed in section 1.2.2.

The results are given in Tab. 5, and are shown in Figure 4(left) for $K_L \to \pi e \nu$, $K_L \to \pi \mu \nu$, $K_S \to \pi e \nu$, $K^\pm \to \pi e \nu$, $K^\pm \to \pi \mu \nu$, and for the combination. The average,

$$|V_{us}| \times f_+(0) = 0.2166(5), \tag{1.17}$$

has an uncertainty of about of 0.2%. The results from the five modes are in good agreement, the fit probability is 55%. In particular, comparing the values of $|V_{us}| \times f_+(0)$ obtained from $K^0_{\ell 3}$ and $K^\pm_{\ell 3}$ we obtain a value of the SU(2) breaking correction

$$\delta^K_{SU(2)_{exp.}} = 5.4(8)\%$$

in agreement with the CHPT calculation reported in sec. 1.2.2:$\delta^K_{SU(2)} = 5.8(8)\%$.

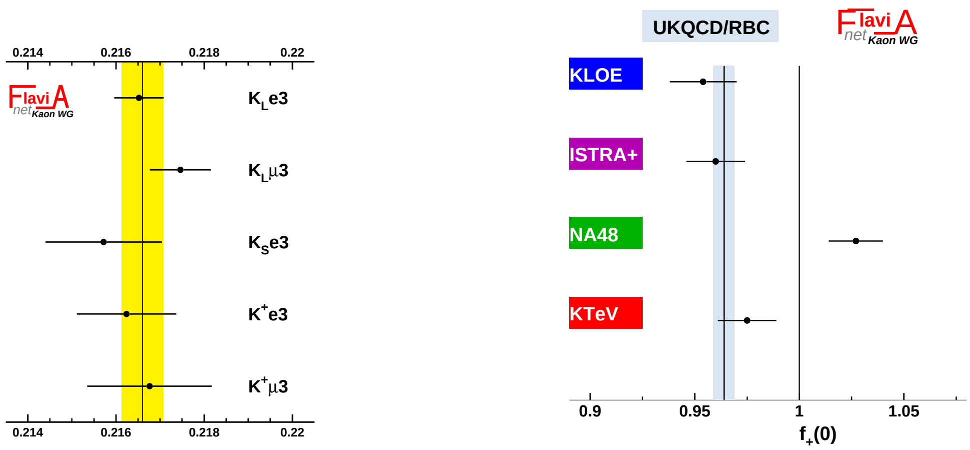

Figure 4: Left: Display of $|V_{us}| \times f_+(0)$ for all channels. Right: Values for $f_+(0)$ determined with the Callan-Treiman relation.

1.3.2 A test of lattice calculation: the Callan-Treiman relation

As described in Sect. 1.2.2 the Callan-Treiman relation fixes the value of scalar form factor at $t = m_K^2 - m_\pi^2$ (the so-called Callan-Treiman point) to the ratio $(f_K/f_\pi)/f_+(0)$. The dispersive parametrization for the scalar form factor proposed in [49] and discussed in Sect. 1.2.2 allows the available measurements of the scalar form factor to be transformed into a precise information on $(f_K/f_\pi)/f_+(0)$, completely independent of the lattice estimates.

Very recently KLOE [50], KTeV [51], ISTRA+ [52], and NA48[48] have produced results on the scalar FF behavior using the dispersive parametrization.

Fig. 4(right) shows the values for $f_+(0)$ determined from the scalar form factor slope measurements obtained using the Callan-Treiman relation and $f_K/f_\pi = 1.189(7)$. The value of $f_+(0) = 0.964(5)$ from UKQCD/RBC is also shown. As already noted in Sec. 1.2.4, the NA48 result is difficult to accommodate. Here one can see that this results is also inconsistent with the theoretical estimates of $f_+(0)$. In particular, it violates the Fubini-Furlan bound $f_+(0) < 1$ [53]. For this reason, the NA48 result is excluded when using the Callan-Treiman constraint.

We combine the average of the above results, $\log C = 0.207 \pm 0.008$, with the lattice determinations of $f_K/f_\pi = 1.189(7)$ and $f_+(0) = 0.964(5)$ using the constraint given by the Callan-Treiman relation. The fit slightly improve the accuracies on $f_K/f_\pi = 1.187(6)$ and $f_+(0) = 0.964(4)$ with essentially unchanged values. New measurements of the $\log C$ are currently ongoing and better constraints on $f_+(0)/(f_K/f_\pi)$ form the Callan-Treiman relation are expected.

Determination of $|V_{us}|/|V_{ud}| \times f_K/f_\pi$

An independent determination of $|V_{us}|$ is obtained from $K_{\ell 2}$ decays. The most important mode is $K^+ \to \mu^+ \nu$, which has been measured by KLOE with a relative uncertainty of about 0.3%. Hadronic uncertainties are minimized by making use of the ratio $\Gamma(K^+ \to \mu^+ \nu)/\Gamma(\pi^+ \to \mu^+ \nu)$.

Using the world average values of $\mathrm{BR}(K^\pm \to \mu^\pm \nu)$ and of $\tau^\pm$ given in Sec. 1.2.4 and the value

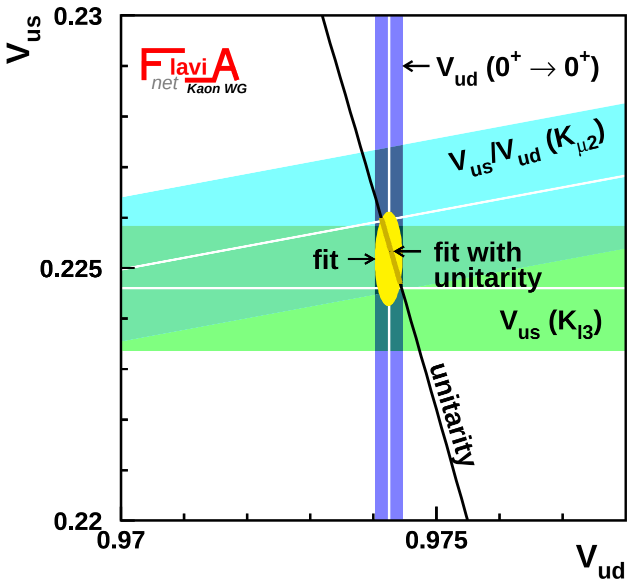

Figure 5: Results of fits to $|V_{ud}|$, $|V_{us}|$, and $|V_{us}|/|V_{ud}|$.

of $\Gamma(\pi^\pm \to \mu^\pm \nu) = 38.408(7)\ \mu s^{-1}$ from [14] we obtain:

$$|V_{us}|/|V_{ud}| \times f_K/f_\pi = 0.2758 \pm 0.0007\ . \tag{1.18}$$

1.3.3 Test of Cabibbo Universality or CKM unitarity

To determine $|V_{us}|$ and $|V_{ud}|$ we use the value $|V_{us}| \times f_+(0) = 0.2166(5)$ reported in Tab. 5, the result $|V_{us}|/|V_{ud}|f_K/f_\pi = 0.2758(7)$ discussed in Sect. 1.3.2, $f_+(0) = 0.964(5)$, and $f_K/f_\pi = 1.189(7)$. From the above we find:

$$|V_{us}| = 0.2246 \pm 0.0012 \qquad [K_{\ell 3} \text{ only}]\ , \tag{1.19}$$

$$|V_{us}|/|V_{ud}| = 0.2319 \pm 0.0015 \qquad [K_{\ell 2} \text{ only}]\ . \tag{1.20}$$

These determinations can be used in a fit together with the the evaluation of $|V_{ud}|$ from $0^+ \to 0^+$ nuclear beta decays quoted in section 1.1: $|V_{ud}|$=0.97425±0.00022. The global fit gives

$$|V_{ud}| = 0.97425(22) \qquad |V_{us}| = 0.2252(9) \qquad [K_{\ell 3,\ell 2}\ +\ 0^+ \to 0^+]\ , \tag{1.21}$$

with $\chi^2/\text{ndf} = 0.52/1$ (47%). This result does not make use of CKM unitarity. If the unitarity constraint is included, the fit gives

$$|V_{us}| = \sin\theta_C = \lambda = 0.2253(6) \qquad [\text{with unitarity}] \tag{1.22}$$

Both results are illustrated in Figure 5.

Using the (rather negligible) $|V_{ub}|^2 \simeq 1.5 \times 10^{-5}$ in conjunction with the above results leads to

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(4)_{V_{ud}}(4)_{V_{us}} = 0.9999(6) \tag{1.23}$$

The outstanding agreement with unitarity provides an impressive confirmation of Standard Model radiative corrections [54, 6](at about the 60 sigma level!). It can be used to constain "new physics" effects which, if present, would manifest themselves as a deviation from 1, *i.e.* what would appear to be a breakdown of unitarity.

A way to illustrate the above constraint is to extract the Fermi constant from nuclear, K and B decays assuming the validity of CKM unitarity without employing muon decay. Values in eq. 1.21 give

$$G_F^{\mathrm{CKM}} = 1.166279(261) \times 10^{-5}\,\mathrm{GeV}^{-2} \quad \text{CKM Unitarity} \tag{1.24}$$

which is in fact the second best determination of G_F, after G_μ The comparison between G_μ and G_F^{CKM} in eq. (1.24) is providing the constraints on "new physics", if it affects them differently. So far, they are equal to within errors.

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