

General Theoretical Introduction to Hadronic *B* Decays

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I briefly introduce the theoretical frameworks for studies of two-body hadronic *B* meson decays, which include the factorization assumption, the QCD-improved factorization, the perturbative QCD, the soft-collinear effective theory, the light-cone QCD sum rules, and the quark-diagram parametrization.

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1. Introduction

Hadronic *B* meson decays are difficult to analyze because of complicated QCD dynamics and multiple characteristic scales they involve: the *W* boson mass m_W , the *b* quark mass m_b , and the QCD scale Λ_{QCD} . The standard procedure is first to integrate out the scale m_W , such that QCD dynamics is organized into an effective weak Hamiltonian [1]. For the $B \rightarrow D\pi$ decays, the effective Hamiltonian is written as

$$\mathscr{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \right], \qquad (1.1)$$

where G_F is the Fermi coupling constant, $V_{cb}V_{ud}^*$ is the product of the Cabibbo-Kobayashi-Maskawa matrix elements, μ is the renormalization scale, $C_{1,2}$ are the Wilson coefficients, and the four-fermion operators are defined by

$$O_1 = (\bar{d}b)_{V-A}(\bar{c}u)_{V-A}, \qquad O_2 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}. \tag{1.2}$$

To derive $B \rightarrow D\pi$ decay amplitudes, one evaluates the hadronic matrix elements $\langle D\pi | O_i(\mu) | B \rangle$. Different theoretical approaches have been developed for this evaluation, which include the factorization assumption, the QCD-improved factorization, the perturbative QCD, the soft-collinear effective theory, the light-cone QCD sum rules, and the quark-diagram parametrization. In this talk I briefly introduce the basic ideas of these approaches [2].

2. Factorization Assumption

Intuitively, decay products from a heavy *b* quark move fast without further interaction between them. This naive picture is supported by the color-transparency argument [3]: the Lorentz contraction renders energetic final states emitted from the weak vertex have small longitudinal color dipoles, which can not be resolved by soft gluons. Therefore, the hadronic matrix element $\langle O(\mu) \rangle$ is factorized into a product of two matrix elements of single currents, governed by decay constants and form factors, without soft gluon exchanges between them. This factorization assumption (FA) [4] was first proved in the framework of large energy effective theory [5], and justified in the large N_c limit [6], N_c being the number of colors. For the $B \rightarrow D\pi$ decays, the color-allowed (colorsuppressed) amplitude, involving the $B \rightarrow D$ ($B \rightarrow \pi$) transition form factor, is proportional to the Wilson coefficient $a_1 = C_2 + C_1/N_c$ ($a_2 = C_1 + C_2/N_c$).

In spite of its simplicity, the FA encounters three principal difficulties. First, a hadronic matrix element under the FA is independent of the renormalization scale μ , as the vector or axial-vector current is partially conserved. Consequently, the amplitude $C(\mu)\langle O \rangle_{\text{fact}}$ is not truly physical as the scale dependence of the Wilson coefficient does not get compensation from the matrix element. This problem may not be serious for color-allowed modes, because the parameter a_1 is roughly independent of μ . It is then not a surprise that the simple FA gives predictions in relatively good agreement with data of these modes. However, the parameter a_2 depends strongly on the renormalization scale and on the renormalization scheme, because of the similar magnitude and different sign of the $C_1(\mu)$ and $C_2(\mu)/N_c$ terms (calculated in the NDR scheme and for $\Lambda_{\overline{MS}}^{(5)} = 225$ GeV, the Wilson coefficients have the values $C_1(m_B) = -0.185$ and $C_2(m_B) = 1.082$ [1], m_B being the

B meson mass). This may be the reason the FA fails to accommodate data of color-suppressed modes. It also means that a_2 is more sensitive to subleading contributions.

The second difficulty is related to the first one: nonfactorizable effects have been neglected in the FA. This neglect may be justified for color-allowed modes due to the large and roughly μ -independent value of a_1 , but not for color-suppressed modes, such as $B \rightarrow J/\psi K^{(*)}$. The J/ψ meson emitted from the weak vertex is not energetic, and the color-transparency argument does not apply. To circumvent this difficulty, nonfactorizable contributions were parameterized into the parameters χ_i [7, 8],

$$a_{1}^{\text{eff}} = C_{2}(\mu) + C_{1}(\mu) \left[\frac{1}{N_{c}} + \chi_{1}(\mu) \right],$$

$$a_{2}^{\text{eff}} = C_{1}(\mu) + C_{2}(\mu) \left[\frac{1}{N_{c}} + \chi_{2}(\mu) \right].$$
(2.1)

The μ dependence of the Wilson coefficients is assumed to be exactly compensated by that of $\chi_i(\mu)$ [9]. It is obvious that the introduction of χ_i does not really resolve the scale problem in the FA.

Third, strong phases are essential for predicting CP asymmetries in exclusive B meson decays. These phases, arising from the Bander-Silverman-Soni (BSS) mechanism [10], are ambiguous in the FA: the c quark loop contributes an imaginary piece proportional to

$$\int du u (1-u) \theta(q^2 u (1-u) - m_c^2) , \qquad (2.2)$$

where q^2 is the invariant mass of the gluon emitted from the penguin. Since q^2 is not precisely defined in the FA, one can not obtain definite information of strong phases from Eq. (2.2). Moreover, it is legitimate to question whether the BSS mechanism is an important source of strong phases in *B* meson decays. Viewing the above difficulties, the FA is not a complete model, and it is necessary to go beyond the FA by developing reliable and systematic theoretical approaches.

3. QCD-improved Factorization

The color-transparency argument allows the addition of hard gluons between the energetic mesons emitted from the weak vertex and the *B* meson transition form factors. These hard gluon exchanges lead to higher-order corrections in the coupling constant α_s to the FA. By means of Feynman diagrams, they appear as the vertex corrections in the first two rows of Fig. 1 [11]. It has been shown that soft divergences cancel among them, when computed in the collinear factorization theorem. These $O(\alpha_s)$ corrections weaken the μ dependence in Wilson coefficients, and generate strong phases. Besides, hard gluons can also be added to form the spectator diagrams in the last row of Fig. 1. Feynman rules of these two diagrams differ by a minus sign in the soft region resulting from the involved quark and anti-quark propagators. Including the above nonfactorizable corrections to the FA leads to the QCD-improved factorization (QCDF) approach [11]. The gluon invariant mass q^2 in the BSS mechanism can be unambiguously defined and related to parton momentum fractions in QCDF. Hence, the theoretical difficulties in the FA are resolved. This is an important step towards a rigorous framework for two-body hadronic *B* meson decays in the heavy quark limit.

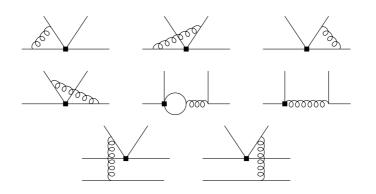


Figure 1: $O(\alpha_s)$ corrections to the FA in the QCDF approach.

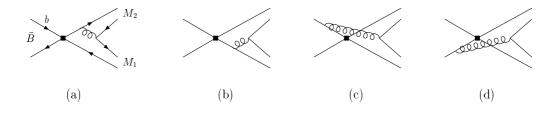


Figure 2: Annihilation contributions.

Corrections in higher powers of $1/m_b$ to the FA can also be included into QCDF, such as those from the annihilation topology in Fig. 2, and from twist-3 contributions to the spectator amplitudes. However, it has been found that endpoint singularities exist in these high-power contributions, which arise from the divergent integral $\int_0^1 dx/x$, *x* being a momentum fraction. Similar singularities are developed, when applying the collinear factorization to *B* meson transition form factors. Because of the endpoint singularities, the annihilation and twist-3 spectator contributions must be parameterized as [11]

$$\ln \frac{m_B}{\Lambda_h} \left(1 + \rho_A e^{i\delta_A} \right) , \quad \ln \frac{m_B}{\Lambda_h} \left(1 + \rho_H e^{i\delta_H} \right) , \tag{3.1}$$

respectively, with the hadronic scale Λ_h . A QCDF formula then contains the arbitrary parameters $\rho_{A,H}$ and $\delta_{A,H}$. Setting these parameters to zero, one obtains predictions in the "default" scenario, and the variation of the arbitrary parameters gives theoretical uncertainties. If tuning these parameters to fit data, one obtains results in the scenarios "S", "S2",...[12].

4. Perturbative QCD

The endpoint singularities signal the breakdown of the collinear factorization for two-body hadronic *B* meson decays. Motivated by removing these singularities, the perturbative QCD (PQCD) approach based on the k_T factorization theorem was developed [13, 14, 15, 16]. A parton transverse momentum k_T is produced by gluon radiations, before hard scattering occurs. The endpoint singularities from the small *x* region indicate that the parton transverse momentum k_T is not negligible.

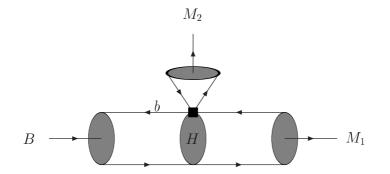


Figure 3: Perturbative QCD factorization.

Taking into account k_T , a particle propagator does not diverge as $x \to 0$. The *B* meson transition form factors, and the spectator and annihilation contributions are then all calculable in the framework of the k_T factorization. It has been shown that a $B \to M_1M_2$ decay amplitude is factorized into the convolution of the six-quark hard kernel, the jet function and the Sudakov factor with the bound-state wave functions as shown in Fig. 3,

$$A(B \to M_1 M_2) = \phi_B \otimes H \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2} . \tag{4.1}$$

The jet function *J* comes from the threshold resummation, which exhibits suppression in the small *x* region [17]. The Sudakov factor *S* comes from the k_T resummation, which exhibits suppression in the small k_T region [18, 19]. Therefore, these resummation effects guarantee the removal of the endpoint singularities. *J* (*S*), organizing double logarithms in the hard kernel (meson wave functions), is hidden in *H* (the three meson states) in Fig. 3. The arbitrary parameters introduced in QCDF [11] are not necessary, and PQCD involves only universal and controllable inputs.

The theoretical difficulties in the FA are also resolved in PQCD but in a different way. The FA limit of the PQCD approach at large m_b , which is not as obvious as in QCDF, has been examined [17]. It was found that the factorizable emission amplitude decreases like $m_b^{-3/2}$, if the *B* meson decay constant f_B scales like $f_B \propto m_b^{-1/2}$. This power-law behavior is consistent with that obtained in [11, 20]. The higher-order corrections to the FA have been included in PQCD, which moderate the dependence on the renormalization scale μ . The ratio of the spectator contribution over the factorizable emission contribution decreases with m_b in PQCD, showing a behavior close to that in QCDF. The gluon invariant mass q^2 in the BSS mechanism is clearly defined and related to parton momentum fractions. The penguin annihilation amplitude is almost imaginary in PQCD [15], whose mechanism is similar to the BSS one [10]: in the annihilation topology, the loop is formed by the internal particles in the LO hard kernel and by infinitely many Sudakov gluons exchanged between two partons in a light meson. A sizable strong phase is generated, when the internal particles go on mass shell. In terms of the principal-value prescription for the internal particle propagator, the strong phase is given by [15]

$$\frac{1}{xm_B^2 - k_T^2 + i\varepsilon} = \frac{P}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2).$$
(4.2)

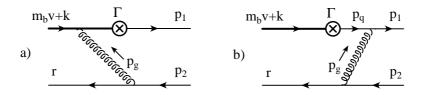


Figure 4: Diagrams for the $B \rightarrow \pi$ form factor in QCD.

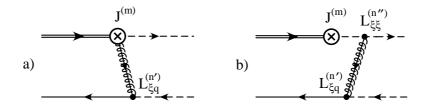


Figure 5: Diagrams for the $B \rightarrow \pi$ form factor in SCET_I.

5. Soft-Collinear Effective Theory

The soft-collinear effective theory (SCET) based on the collinear factorization is formulated in the framework of operator product expansion (OPE) [21, 22, 23, 24]. The matching at different scales involved in *B* meson decays has been carefully handled in SCET. Take the simple $B \rightarrow \pi$ transition form factor in Fig. 4 as an example. The soft spectator in the *B* meson carries the momentum $r \sim O(\Lambda_{\rm QCD})$, because it is dominated by soft dynamics. If the spectator in the energetic pion carries the momentum $p_2 \sim O(m_b)$, the virtual gluon in Fig. 4 is off-shell by $p_g^2 = (p_2 - r)^2 =$ $-2p_2 \cdot r \sim O(m_b \Lambda_{\rm QCD})$. Then the virtual quark in Figs. 4(a) is off-shell by $(m_b v + k + p_g)^2 - m_b^2 \sim$ $O(m_b^2)$, where v is the b quark velocity and $k \sim O(\Lambda_{\rm QCD})$ denotes the Fermi motion of the b quark. Hence, *B* meson decays contain three scales below m_W : m_b , $\sqrt{m_b \Lambda_{\rm QCD}}$, and $\Lambda_{\rm QCD}$.

The separate matching at the two scales m_b and $\sqrt{m_b\Lambda_{QCD}}$ is briefly explained below [25]. The first step is to integrate out the lines off-shell by m_b^2 in QCD, and the resultant effective theory is called SCET_I. One then derives the zeroth-order effective current $J^{(0)}$ from the $b \rightarrow u$ weak vertex, and the first-order effective current $J^{(1)}$ by shrinking the virtual *b* quark line in Fig. 4(a). The next step is to integrate out the lines off-shell by $m_b\Lambda_{QCD}$ in SCET_I, arriving at SCET_{II}. The relevant diagrams to start with are displayed in Fig. 5. Shrinking all the lines off-shell by $m_b\Lambda_{QCD}$, one derives the corresponding Wilson coefficients, i.e., the jet functions, and the effective fourfermion operators. Sandwiching these four-fermion operators by the initial *B* meson state and the final pion state leads to the *B* meson and pion distribution amplitudes. The $B \rightarrow \pi$ transition form factor is then factorized as depicted in Fig. 6. The factorization of two-body hadronic *B* meson decays is constructed in a similar way, and the result is also shown in Fig. 6.

At leading power in $1/m_b$, there is no large source of strong phases in SCET (the annihilation contribution is parametrically power-suppressed). To acquire strong phases, it has been argued that $c\bar{c}$ (charming) penguins could give long-distance effects at leading power [26]. This contribution is nonperturbative, so it must be parameterized as an arbitrary amplitude $A^{c\bar{c}}$. Including the

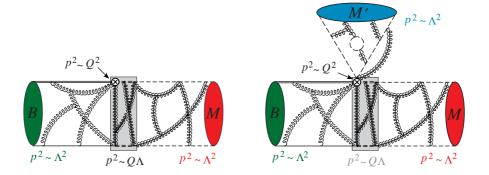


Figure 6: Factorization of the $B \rightarrow \pi$ form factor and of the $B \rightarrow M_1 M_2$ decay in SCET.

charming penguin, SCET has been applied as an QCD-improved parametrization, and $A^{c\bar{c}}$ is determined together with other hadronic inputs from data. It should be mentioned that the long-distance charming-penguin contribution is power-suppressed according to QCDF, PQCD and light-cone sum rules [27].

6. Light-Cone Sum Rules

QCD sum rules [28, 29] are based on the quark-hadron duality, which is very different from a factorization theorem. A simple argument of the quark-hadron duality has been presented in [30, 31]: consider evaluation of a correlation function by means of the dispersion relation. One can choose either a contour along the real axis, which may be close to a physical pole, or a contour far away from the physical pole. When moving along the former contour, one picks up nonperturbative contributions to the correlation function from the pole. When moving along the latter, the correlation function can be evaluated in the framework of OPE. If there is no other pole inside the combined contour of the former and the latter, the above two choices should give the same result. This explains the idea of the quark-hadron duality.

QCD sum rules have been applied to various problems in heavy flavor physics. Take the *B* meson decay constant f_B as an example [32, 33, 34], which is defined via the matrix element $\langle 0|m_B\bar{q}i\gamma_5 b|\bar{B}\rangle = f_B m_B^2$, q = u, d. Start with the correlation function of two heavy-light currents,

$$\Pi(q^2) = i \int d^4 y e^{iq \cdot y} \langle 0|T[m_B \bar{q} i \gamma_5 b(y), m_B \bar{b} i \gamma_5 q(0)]|0\rangle , \qquad (6.1)$$

which can be treated by OPE at the quark level, if q^2 is far below m_b^2 , or parameterized as a sum over hadronic states including the ground-state *B* meson for $q^2 \ge m_B^2$. The quark-hadron duality relates the expressions in these two regions. Therefore, on the left-hand (hadron) side of the sum rule, one has

$$\Pi(q^2) = \frac{f_B^2 m_B^4}{m_B^2 - q^2} + \cdots,$$
(6.2)

where the contribution of the ground-state B meson has been singled out, and \cdots represents those from the excited resonances and from the continuum of hadronic states with the B meson quantum

numbers. On the right-hand (quark) side of the sum rule, one has the expansion including the perturbative series in α_s and the quark, gluon and quark-gluon condensates. Evaluating the right-hand side of the sum rule, one is able to estimate f_B .

Light-cone sum rules (LCSR) [35] are a simplified version of QCD sum rules. Consider the $B \rightarrow \pi$ transition form factors [31, 36, 37], for which the correlation function is chosen as

$$i \int d^4 y e^{iq \cdot y} \langle \pi^+(P_2) | T[\bar{u}\gamma_{\lambda}b(y), m_b\bar{b}i\gamma_5 d(0)] | 0 \rangle, \qquad (6.3)$$

with the current $\bar{u}\gamma_{\lambda}b(y)$ representing the weak vertex. Compared to Eq. (6.1), the final state has been specified as a pion, and the twist expansion has been applied to the local current associated with the pion. The LCSR approach has been extended to the analysis of two-body hadronic *B* meson decays in [38], and interesting observations were obtained.

7. Quark-diagram Parametrization

The quark-diagram parametrization is a widely adopted approach to two-body hadronic *B* meson decays [39]. Various quark diagrams are defined in Fig. 7, with the color-allowed tree amplitude *T*, the color-suppressed tree amplitude *C*, the penguin amplitude *P*, the *W*-exchange amplitude *E*, the annihilation amplitude *A*, and the penguin annihilation amplitude *PA*. One also defines the electroweak penguin amplitude P_{ew} and the color-suppressed electroweak penguin amplitude P_{ew}^c . According to the above definitions, the quark-diagram parametrization for the $B \rightarrow \pi\pi$ decays is given by

$$\begin{split} \sqrt{2}A(B^+ \to \pi^+ \pi^0) &= -T \left[1 + \frac{C}{T} + \frac{P_{ew}}{T} e^{i\phi_2} \right] ,\\ A(B^0_d \to \pi^+ \pi^-) &= -T \left(1 + \frac{P}{T} e^{i\phi_2} \right) ,\\ \sqrt{2}A(B^0_d \to \pi^0 \pi^0) &= T \left[\left(\frac{P}{T} - \frac{P_{ew}}{T} \right) e^{i\phi_2} - \frac{C}{T} \right] , \end{split}$$
(7.1)

with the weak phase ϕ_2 . The parametrization for other decays can be written down in a similar way.

Predictive power of this approach arises from flavor symmetry, such as SU(3) [40] and U-spin [41], which relate the amplitudes among relevant modes. For example, the former relates the colorallowed tree amplitudes with the light quark q = u, d, s and with the light quark q' = d, s in Fig. 7(a). The latter relates the $B_d^0 \rightarrow K^+\pi^-$ and $B_s^0 \rightarrow \pi^+K^-$ decay amplitudes, and the $B_s^0 \rightarrow K^+K^-$ and $B_d^0 \rightarrow \pi^+\pi^-$ decay amplitudes. Hence, one can determine the quark amplitudes (together with the weak phases sometimes) from data of some modes, and then use them to make predictions for other modes. However, it is difficult to estimate symmetry breaking effects in this approach [42].

8. Summary

We have been able to go beyond the factorization assumption by including QCD corrections. Different approaches have been developed: in QCDF the higher-order corrections are computed in the collinear factorization, but the high-power corrections must be parameterized due to the existence of the endpoint singularities. There are no endpoint singularities in PQCD, which is based

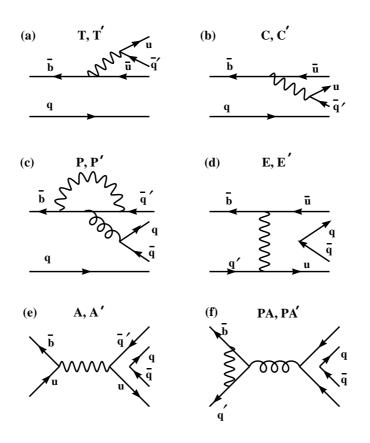


Figure 7: Quark diagrams for the $B \rightarrow M_1 M_2$ decays.

on the k_T factorization, and in SCET, which employs the zero-bin subtraction [43]. Therefore, *B* meson transition form factors and $1/m_b$ power-suppressed contributions are calculable in both approaches. The difference is that SCET involves more arbitrary parameters, such as the charming penguins, which are the main source of strong phases. The annihilation contribution is the main source of strong phases in PQCD. Since external lines are off-shell in the dispersion relation on the OPE side of QCD sum rules or LCSR, a soft contribution has a definition different from those in factorization theorems. Hence, the dominance of the soft contribution in the former does not apply to the latter. At last, predictive power of the factorization approaches comes from universality of nonperturbative inputs, such as meson wave functions, but that of the quark-diagram parametrization comes from flavor symmetry.

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