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Layer World: Living on a layer in 5D SU(3)

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We suggest another approach to five-dimensional non-isotropic gauge theory. Using non-perturbative technique we show that already modest interaction anisotropy confines heavy bound states to four-dimensional layers, while free quarks propagate with almost no penalty into the fifth dimension.

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1. Introduction

The question of dimensionality of our world is closed and re-opened with clockwork regularity. Relative silence since the days of Kaluza and Klein turned into a roar with the advent of string theory and competing field theories. Since then string theorists are trying to decrease number of dimensions to come to four, and field theorists are trying to increase them to explain all particles. Many string and all field theories of this kind are a reiteration of the Kaluza-Klein mantra - "compact external dimensions lead to new massive particles". None of these theories have been successful so far.

The reason for that may be a certain lack of attention, as the extra-dimensional gauge theories are believed to be non-renormalizable, which makes them somewhat exotic and therefore less studied. However, to our best knowledge, such non-renormalizability for the lattice regularization has not been proven.

Yet, there is a way out of this contradiction. We accepted already that time is different from the other dimensions, and though we managed to formulate the four dimensional field theory via an effective euclidian theory, it still remains different. So without much effort we may imagine that the coupling in higher dimensions is either weaker or stronger than in four. Here we consider the former possibility. This changes the rules of the game and gives us more possibilities for renormalization at the cost of extra parameter.

It was pioneered in the classical work of Nielsen and Fu [1], who, after destroying possibility of having a discrete field theory in four dimensions went on to study higher dimensional theories. Their hope was to find a high-dimensional space which would, at certain values of couplings turn into weakly or non-interacting layers of lesser dimension. Let us see how that worked out and how we can extend and reinterpret their findings using modern warfare of lattice simulations [6].

2. Brave New World

Let us assume that the world is either fundamentally or at least effectively five-dimensional and its strong dynamics is determined, in the first approximation, by a simple gauge theory. Furthermore, let us assume that the coupling in the fifth direction is different from the couplings in other directions. It has an obvious disadvantage of introducing an extra parameter, but so do Kaluza-Klein-like theories. The action, in terms of plaquette variables then reads:

$$\mathscr{S} = \beta \sum_{\mu'<\nu'=0..D-1} \sum_x \Re e^{i\theta_{\mu'\nu'}(x)} + \beta \sum_{\mu'<D} \sum_x \Re e^{i\theta_{\mu' D}(x)} \tag{2.1}$$

where $\theta_{\mu\nu}$ is the usual product of the fields around plaquette[1]. This action describes a sort of five-dimensional compact electrodynamics. As we venture into new territory it is useful to establish a glossary:

- **pentaquum** is the ground state of the five-dimensional space
- **fifth**, noun - fifth direction, or width/depth in that direction
- **pentark** is a fermi-like massive particle living in five dimensions

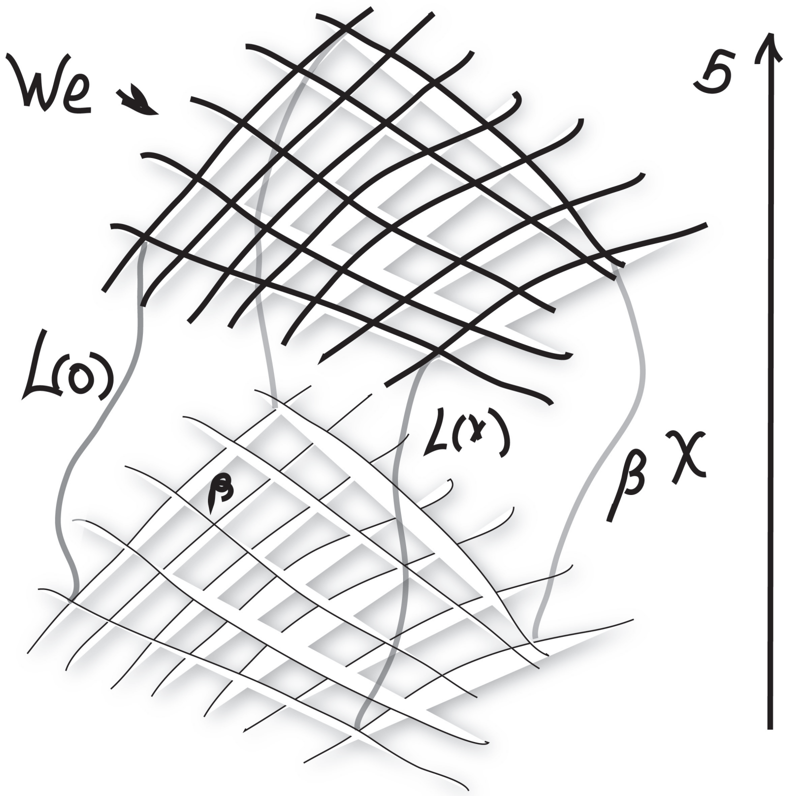
- **pluon** is a counterpart of gluon, massless coloured particle
- **penton** is a bound state of pentark and anti-pentark.

This model exhibits a rich phase structure, with the traditional Coulomb and confined phases being joined by the Layer phase, which has following properties:

- it exists for $D > 5$, $d = D - 1$,
- but not for $D = 5$ for non-Abelian theories
- in cQED massless "photons" propagate into fifth
- yet propagators are supressed by γ^{4N}, where N is the distance between layers and γ is the coupling in the fifth dimension
- wilson loops follow the area law of the d-dimensional projection
- thus charged particles are confined to the layer, always

These results have been confirmed and extended in [2], [3] and others. Unfortunately the case we find most compelling, the $SU(3)$, does not exhibit the desired behaviour. However, in [4] the authors use variational cumulant expansion to show the existence of the layer phase at finite temperature. In another work, [5] an interesting approach to the continuum limit of the five-dimensional gauge theory is suggested. But we prefer to start from another viewpoint, and worry about the continuum limit later.

3. World Overview

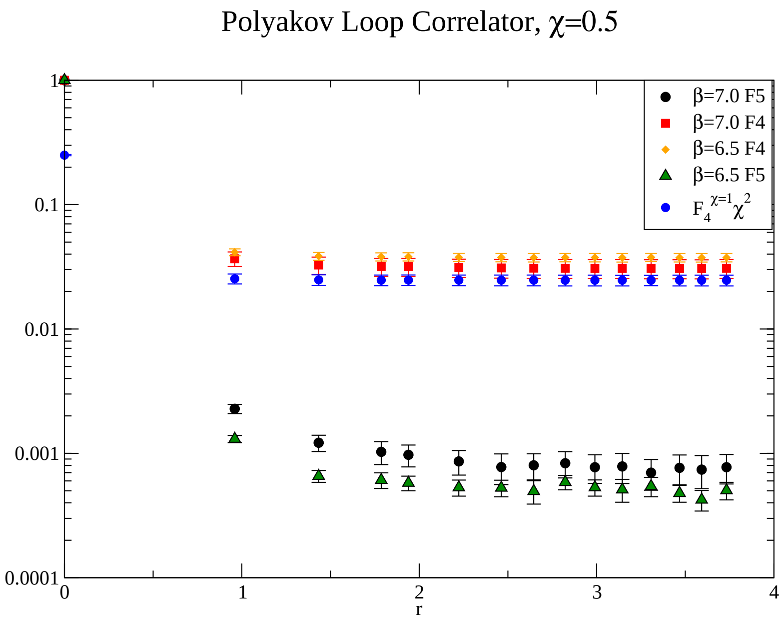


So let us consider what a desired theory should look like. It is obviously $SU(3)$-symmetric gauge theory, which would mimic the four-dimensional gauge theory that QCD is. Now, philosophically speaking, we should be able to see the effects of the fifth dimension from where we are. Otherwise we will be subject to the landscape problem which would make the model far less attractive. The Layer Phase in the original understanding does not allow such luxury, layers are self-contained and particle propagation is suppressed immensely. So the hope is that, when considered from another standpoint, five-dimensional non-isotropic theory will be not in the layer phase, yet has both required phenomenology of the real world and something else. So let us check if "traditional" non-isotropic five dimensional gauge theory will do. Consider pure gauge theory on the lattice with one coupling being smaller, the penta-coupling

$$S = -\frac{\beta}{N_c} \sum_{\mu \neq \nu = 0..3} P_{\mu\nu} - \chi \frac{\beta}{N_c} \sum_{\mu = 0..3} P_{\mu 4}$$

where P is the ordered product of the gauge fields along the plaquette. Simulating it with the standard heat-bath on a penta-cubic lattice we can confirm that, in the range of couplings, studied the Layer phase is not found. Usual Coulomb and strong-coupling phases are present, and the plaquettes (square of the field strength tensor) change only slowly with χ, while Polyakov loops, the usual order parameter, do not go anywhere near zero.

While it may sound disappointing, it actually is not. None of these variables is truly physical in four-dimensional sense. Strictly speaking, the only physical construction we can make with



pure glue is Wilson loop / Polyakov loops correlation, which (in four dimension) corresponds to the propagation of a heavy meson. So let us define

$$F_4(x) = \sum TrL_4(0)TrL_4(x) \quad F_5(x) = \sum TrL_5(0)TrL_5(x)$$

which then describe the propagation of a particle-anti-particle bound state into the 4th and 5th dimension.

And here things start to look interesting. We are obviously in the Coulomb phase. However, even at a very modest anisotropy coefficient, we observe that fifth-correlator is supressed almost by two orders of magnitude with respect to the four-correlator. The latter is reduced only by a modest amount, roughly proprotionate to the naively expected χ^2. Moreover, the situation remains the same when changing the four dimensional coupling towards the confined phase, while keeping the coupling ratio constant. This indicates that dynamics of the gauge fields in pentaquum favours layerisation for the bound states while still granting free pass for the pentarks. Such situation is actually close to the desired theory, on one hand we have obviously four-dimensional layers for the pentons, which become mesons; and on the other hand, pentarks travel freely into fifth, giving us obvious explanation for the fact that we cannot observe free quarks. At the same time as propagation is not forbidden but just supressed, one may envision that with sufficient energy we can pick up a penton stuck to another layer and have it as virtual quark-anti-quark pair. This is far more intuitive than producing them out of vacuum. Of course this is more of a conjecture than a proof. Not only we are in a wrong phase for the four-dimensional world (which is known to be confining) but also somewhat away from the continuum limit. Moreover, these results are only applicable to heavy pentarks. Yet as we are still far from the connection to phenomenology, we do not know if our “light” quarks are actually “heavy” in pentaquum.

4. Conclusion and Outlook

We found a region in $5D$ space which is not exactly Nielsen Layer phase, but confines bound states to the layer, which is even more desirable and has chances to lead to observable predictions. It provides a compactification scenario alternative to Kaluza-Klein compact dimensions. It gives straightforward interpretation of confinement and virtual quark loops, which is qialitatively consistent with the traditional picture. Our intention now is to continue studying this model, calculate meson propagators and confirm the conjecture that bound states get stuck to the layer.

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