

Towards pp \rightarrow WW at NNLO

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We report on the recent progress in calculating the two-loop virtual QCD corrections to the hadronic W pair production. In particular, we describe how one covers all the kinematically allowed regions by using a combination of a deep mass expansion in the W mass around the high energy limit and of numerical integration of differential equations.

*RADCOR 2009 - 9th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology)
October 25-30 2009
Ascona, Switzerland*

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1. Introduction

The Large Hadron Collider (LHC) is expected to have a great impact on particle physics phenomenology. The increase of the centre-of-mass energy at the LHC with respect to the Tevatron will result in a huge boost of the available data. Most of the processes which will be studied at the LHC need to be calculated at least to next-to-leading order (NLO) in QCD, whereas there are some for which a theoretical prediction is needed to next-to-next-to-leading order (NNLO). Electroweak gauge boson pair production falls into the latter category.

The discovery of the Higgs boson is undoubtedly one of the primary goals for the LHC. The elusive, so far, Higgs boson is responsible for the fermions and gauge bosons mass and also part of the mechanism of dynamical breaking of the Electroweak (EW) symmetry. Another aim for the LHC is the investigation of the non-Abelian gauge structure of the Standard Model (SM). A detailed study of the hadronic production of gauge boson pairs, WW , WZ , ZZ , $W\gamma$, $Z\gamma$, will allow the measurement of the vector boson trilinear couplings and therefore, a comparison between data and SM predictions.

Seen in this context, W pair production,

$$q\bar{q} \rightarrow W^+W^-, \quad (1.1)$$

plays an important two-fold role: it serves as a signal process in the search for New Physics and also is the dominant irreducible background to the promising Higgs discovery channel

$$pp \rightarrow H \rightarrow W^*W^* \rightarrow l\bar{\nu}l'\nu' \quad (1.2)$$

in the mass range M_{Higgs} between 140 and 180 GeV [1].

The process is currently known to NLO accuracy [2, 3, 4, 5, 6, 7, 8]. The NLO corrections are large enhancing the tree-level by almost 70% which falls to a still large 30% after imposing a jet veto. Therefore, one is bound to go one order higher in the perturbative expansion, namely, to NNLO, in order to have a theoretical estimate with an accuracy of around 10%, which would allow comparison against experimental measurements at the LHC.

The same process, hadronic W pair production, requires high accuracy theoretical estimates when studied as background to Higgs production in order to match accuracies between signal and background. The signal process for the Higgs discovery via gluon fusion, $gg \rightarrow H$, as well as the process $H \rightarrow WW \rightarrow l\bar{\nu}l'\nu'$ are known to NNLO [9, 10, 11, 12, 13, 14, 15, 16, 17, 18], whereas the EW corrections are known beyond NLO [19]. W pair production in the loop induced gluon fusion channel,

$$gg \rightarrow W^+W^-, \quad (1.3)$$

needs also to be included in the background study. It contributes at $\mathcal{O}(\alpha_s^2)$ relative to the quark-anti-quark-annihilation channel but is nevertheless enhanced due to the large gluon flux at the LHC [20, 21].

The first main difficulty in studying W pair production to NNLO in QCD is the calculation of the two-loop virtual amplitude since this is a $2 \rightarrow 2$ process with massive external particles. The virtual corrections in the high energy limit, $M_W^2 \ll s, t, u$, where M_W^2 is the W mass and s, t and u the Mandelstam variables, have already been computed in Refs. [22, 23, 24]. However, this is not

enough as one cannot thus cover the whole kinematically interesting range. Therefore, in order to cover all kinematical regions we proceed as follows. We perform a deep expansion in the W mass around the high energy limit which in combination with the method of numerical integration of differential equations [25, 26, 27] allows us the numerical computation of the two-loop amplitude with full mass dependence over the whole phase space.

2. The high energy limit result

The methodology for obtaining the massive amplitude in the high energy limit, namely the limit where all the invariants are much larger than the W mass, is similar to the one followed in Refs. [28, 29]. The amplitude is reduced to an expression that only contains a small number of integrals (master integrals) with the help of the Laporta algorithm [30]. In this calculation for the two-loop amplitude there are 71 master integrals. Next step is the construction, in a fully automatised way, of the Mellin-Barnes (MB) representations [31, 32] of all the master integrals by using the **MBrepresentation** package [33]. The representations are then analytically continued in the number of space-time dimensions by means of the **MB** package [34], thus revealing the full singularity structure. An asymptotic expansion in the mass parameter (W mass) is performed by closing contours and the integrals are finally resummed, either with the help of **XSummer** [35] or the **PSLQ** algorithm [36]. The result is expressed in terms of harmonic polylogarithms.

3. Power corrections and numerical evaluation

It was mentioned in the Introduction that the high energy limit result by itself is not enough. The next step, following the methods applied in Ref. [37], is to compute power corrections in the W mass. A calculation with power corrections to a high enough order is sufficient for covering most of the phase space apart from the region near threshold as well as the regions corresponding to small angle scattering.

We repeat here some of the notation of Ref. [24] for completeness. The charged vector-boson production in the leading partonic scattering process corresponds to

$$q(p_1) + \bar{q}(p_2) \rightarrow W^-(p_3, m) + W^+(p_4, m), \quad (3.1)$$

where p_i denote the quark and W momenta and m is the mass of the W boson.

We have chosen to express the amplitude in terms of the kinematic variables x and m_s , which are defined to be

$$x = -\frac{t}{s}, \quad m_s = \frac{m^2}{s}, \quad (3.2)$$

where

$$s = (p_1 + p_2)^2 \quad \text{and} \quad t = (p_1 - p_3)^2 - m^2. \quad (3.3)$$

The variation then of x within the range $[1/2(1 - \beta), 1/2(1 + \beta)]$, where $\beta = \sqrt{1 - 4m^2/s}$ is the velocity, corresponds to angular variation between the forward and backward scattering.

¹Note that the definition of t here is shifted with respect to the usual Mandelstam t .

It is evident that any master integral M_i can be expressed as

$$M_i = M_i(m_s, x, \varepsilon) = \sum_{j=k}^l \varepsilon^j I_{ij}(m_s, x), \quad (3.4)$$

where the lowest power of ε in the sum can be -4 .

The interesting point now is that the derivative of any Feynman integral with respect to any kinematical variable is again a Feynman integral with possibly higher powers of denominators or numerators. These new Feynman integrals can also be reduced into masters using the Laporta algorithm anew. This means that one can construct a partially triangular system of differential equations in the mass, which can subsequently be solved in the form of a power series expansion, with the expansion parameter in our case being m_s , following the conventions above.

Let us therefore, differentiate with respect to m_s and x , we will then have respectively

$$m_s \frac{d}{dm_s} M_i(m_s, x, \varepsilon) = \sum_j C_{ij}(m_s, x, \varepsilon) M_j(m_s, x, \varepsilon) \quad (3.5)$$

and

$$x \frac{d}{dx} M_i(m_s, x, \varepsilon) = \sum_j C'_{ij}(m_s, x, \varepsilon) M_j(m_s, x, \varepsilon). \quad (3.6)$$

We use Eq. (3.5) to obtain the mass corrections for the masters calculating the power series expansion up to order m_s^{11} (see also Ref. [37] for more details on the method). This deep expansion in m_s should be sufficient for most of the phase space but still not enough to cover the whole allowed kinematical region. The way to proceed from this point is to numerically integrate the system of differential equations.

In particular, we choose to have the masters expressed in the form of Eq. (3.4), where the ε dependence is explicit. We can then work with the coefficients of the ε terms and accordingly have

$$m_s \frac{d}{dm_s} I_i(m_s, x) = \sum_j J_{ij}^M(m_s, x) I_j(m_s, x) \quad (3.7)$$

and

$$x \frac{d}{dx} I_i(m_s, x) = \sum_j J_{ij}^X(m_s, x) I_j(m_s, x), \quad (3.8)$$

where the Jacobian matrices J^M and J^X have rational function elements.

By using this last system of differential equations, one can obtain a full numerical solution to the problem. What we are essentially dealing now with is an initial value problem and the main requirement is to have the initial conditions to proper accuracy. The initial conditions would be the values of the masters at a proper kinematical point which we call initial point. The initial point has to be chosen somewhere in the high energy limit region, where m_s is small and therefore, the values obtained by the power series expansion are very accurate. Starting from there, one can evolve to any other point of the phase space by numerically integrating the system of differential equations Eqs. (3.7) and (3.8).

We parametrise with a suitable grid of points the region close to threshold and then we calculate the masters for all points of the grid by evolving as described previously. Given that the master

integrals have to be very smooth one can use, after having obtained the values for the grid points, interpolation to get the values at any point of the region. We use 1600 points for the grid and take as initial conditions the values of the master integrals at the point $m_s = 5 \times 10^{-3}$, $x = 1/4$. The relative errors at that point were estimated not to exceed 10^{-18} .

The numerical integration is performed by using one of the most advanced software packages implementing the variable coefficient multistep method (ODEPACK) [38]. We use quadruple precision to maximise accuracy. The values at any single grid point can be obtained in about 15 minutes in average (with a typical 2GHz Intel Core 2 Duo system). The compilation is done with the Intel Fortran compiler. The accuracy is around 10 digits for most of the points of the grid. It is interesting to note that in order to perform the numerical integration one needs to deform the contour in the complex plane away from the real axis. This is due to the fact that along the real axis there are spurious singularities. We use an elliptic contour and we achieve a better estimate of the final global error by calculating more than once for each point of the grid, using each time different eccentricities.

There will be no results presented as we only report on work in progress. The aim here was to describe the numerical method, the results of the study will be presented in detail in a future publication [39].

4. Conclusions

W pair production via quark-anti-quark-annihilation is an important process for the LHC seen both as signal in the search for New Physics and as the dominant irreducible background for the Higgs discovery channel: $H \rightarrow WW \rightarrow 4$ leptons. Therefore, the need for accurate study of this process at the LHC is beyond dispute. After having calculated the two-loop and the one-loop-squared virtual QCD corrections to the W boson pair production in the limit where all kinematical invariants are large compared to the mass of the W boson we proceed to the next step. Namely, we use a combination of a deep expansion in the W mass around the high energy limit and of numerical integration of differential equations that allows the computation of the two-loop amplitude with full mass dependence over the whole phase space.

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