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# Who is this guy?

Born in 1972, raised in Marburg, Germany

MSc in Astrophysics (1996, U of London)

Diploma in Physics (1998, U of Heidelberg)

PhD in Astrophysics (2002, U of München)

Postdoc at MPE in Garching

since 2003 research fellow in Uppsala (German and Swedish funding)

since 2008 researcher/lecturer at Uppsala University

Research interests: stars from B to K, esp. at low(est) metallicity, chemical evolution of the Galaxy, quantitative spectroscopy, atomic diffusion, Gaia (coordinator for the computation of synthetic observables)



# What will be covered

## I. Theoretical background

(most of the basics covered by Frank Grupp)  
how lines depend on  $T_{\text{eff}}$ ,  $\log g$ ,  $\log \varepsilon(X)$  etc.

## II. Methods of stellar-parameter and chemical-abundance determination

fundamental stellar parameters  
photometry (in a nutshell)  
spectroscopy (a practical selection)  
abundances (some examples)

## III. Exercise (afternoon)



# What are stellar parameters?

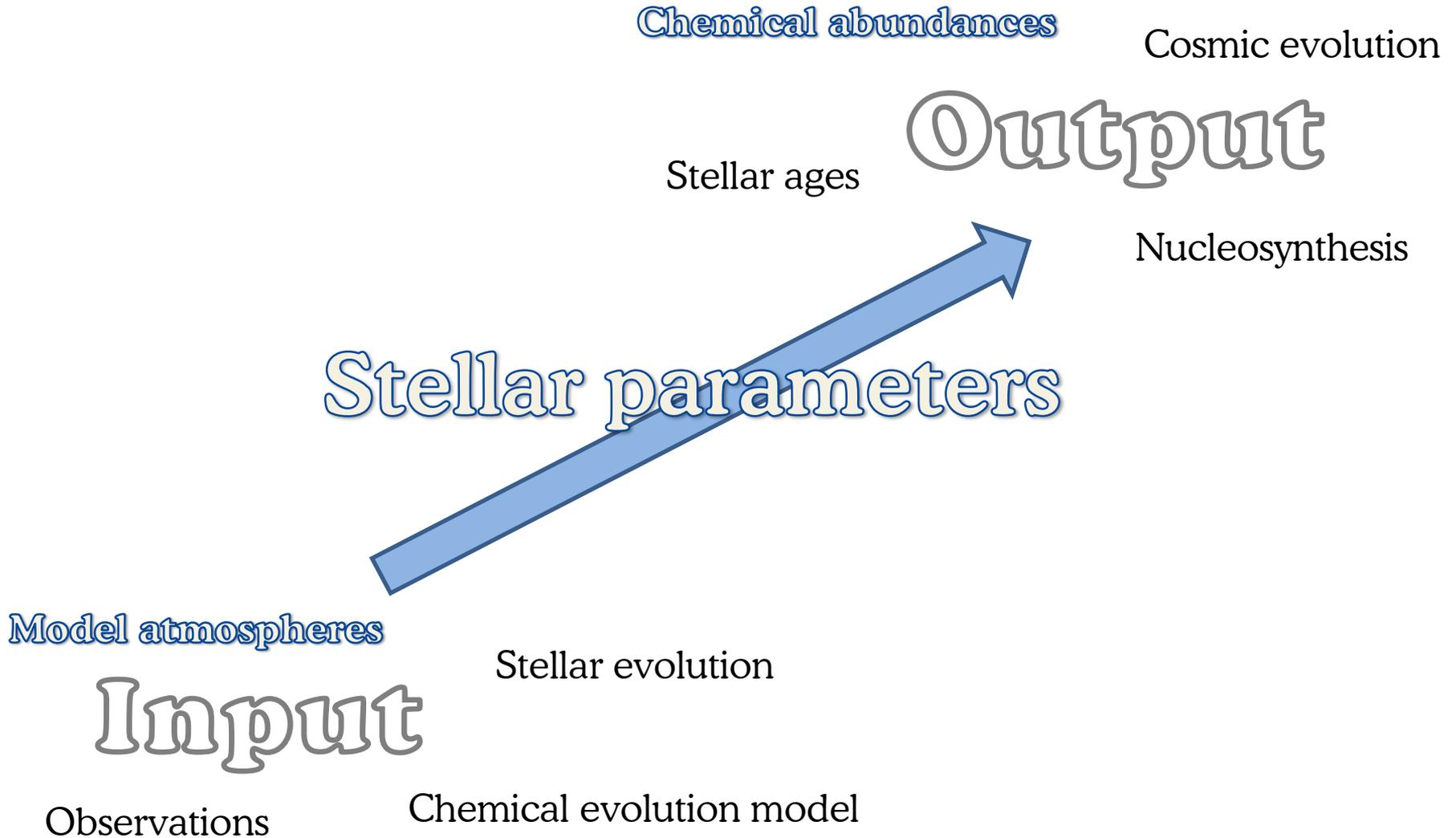
There are different ways of looking at what defines a stars:

stellar-structure view       $M, \mathcal{L}, X, Y, Z, R, v_{\text{rot}}, t, \dots$

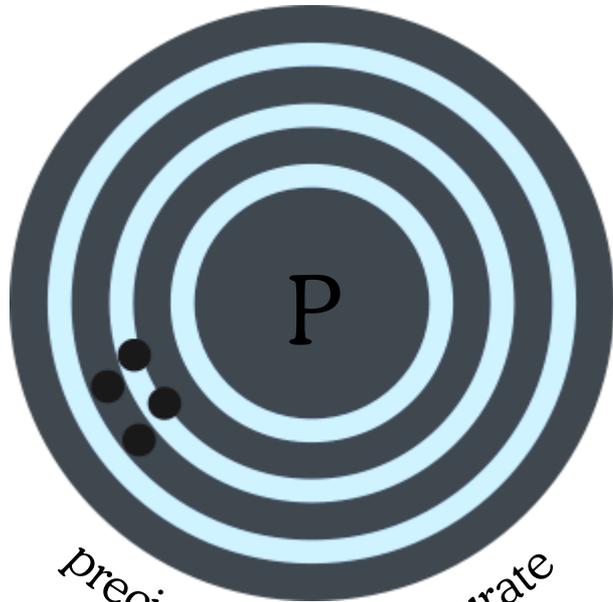
stellar-atmosphere view       $\mathcal{F}_v, T_{\text{eff}}, \log g, [X_i/H], v_{\text{rot}} \sin i, \dots$   
    $\log (G M / R^2)$

While the prior is (often) more fundamental, the latter is more directly related to observations (photospheres!) and generally speaking more applicable. In this lecture, I will follow the latter view.

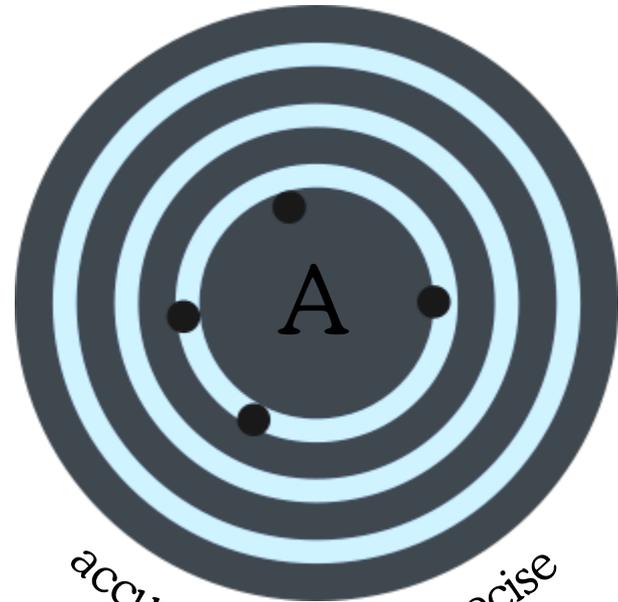
# Linking input to output



# Precision *vs.* accuracy



*precise, but not accurate*



*accurate, but not precise*

NB: Some projects may require high precision *and* accuracy, while for others it will suffice to reach some level of precision.

**It is a good idea to be aware of the needs of your project.**

# Stellar parameters: typical figures

## The Sun

$$M = 2 \times 10^{33} \text{ g} = M_{\odot}$$

$$R = 7 \times 10^{10} \text{ cm} = R_{\odot}$$

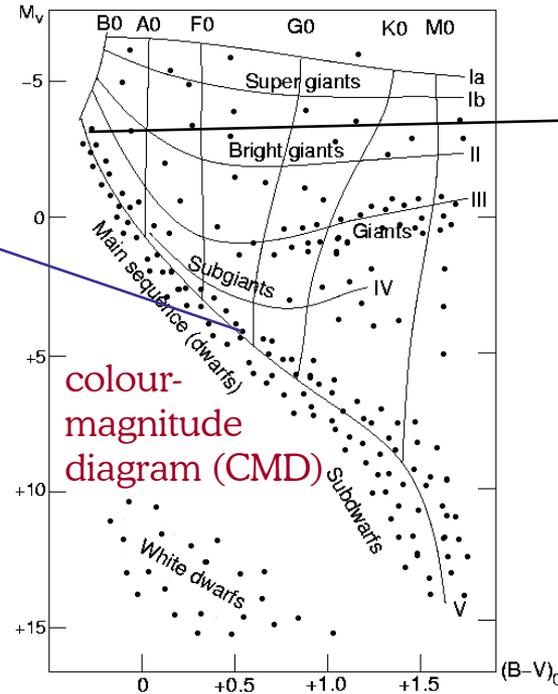
$$\mathcal{L} = 4 \times 10^{33} \text{ erg/s} = L_{\odot}$$

photosphere:

$$\Delta R \approx 200 \text{ km} < 10^{-3} R_{\odot}$$

$$n \approx 10^{15} \text{ cm}^{-3}$$

$$T \approx 6000 \text{ K}$$



## an O star

$$M \approx 50 M_{\odot}$$

$$R \approx 20 R_{\odot}$$

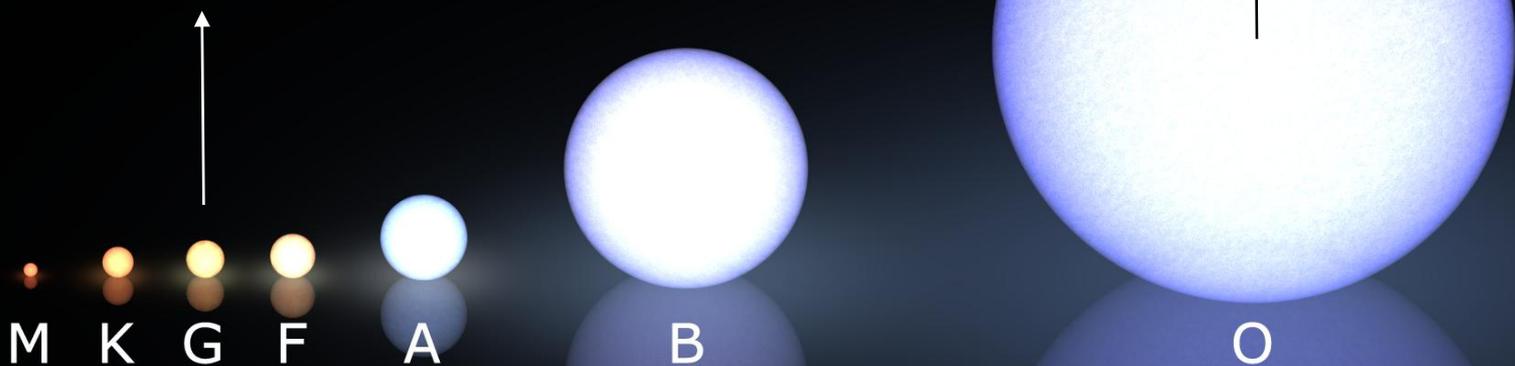
$$\mathcal{L} \approx 10^6 L_{\odot} (\propto M^3)$$

photosphere:

$$\Delta R \approx 0.1 R_{\odot}$$

$$n \approx 10^{14} \text{ cm}^{-3}$$

$$T \approx 40\,000 \text{ K}$$



# Abundance nomenclature

**Mass fractions:** let  $X$ ,  $Y$ ,  $Z$  denote the mass-weighted abundances of H, He and all other elements (“metals”), respectively, normalized to unity ( $X + Y + Z = 1$ ).

example:  $X = 0.7381$ ,  $Y = 0.2485$ ,  $Z = 0.0134$  for the Sun according to Asplund *et al.* (2009)

**The 12 scale:**  $\log \varepsilon(X) = \log (n_X / n_H) + 12$  ( $\log \varepsilon(H) \equiv 12$ )

example:  $\log \varepsilon(O)_\odot \approx 8.7$  dex, i.e., oxygen, the most abundant metal, is 2000 times less abundant than H in the Sun (the exact value is currently hotly debated!)

**Square-bracket scale:**  $[X/H] = \log (n_X / n_H)_\star - \log (n_X / n_H)_\odot$

example:  $[Fe/H]_{HE0107-5240} = -5.3$  dex, i.e., this star has an iron abundance a factor of 200 000 below the Sun (Christlieb *et al.* 2002)

# Opacities

## Continuous opacity

Caused by *bf* or *ff* transitions

In the optical and near-IR of cool stars,  $H^-$  ( $I = 0.75$  eV) dominates:

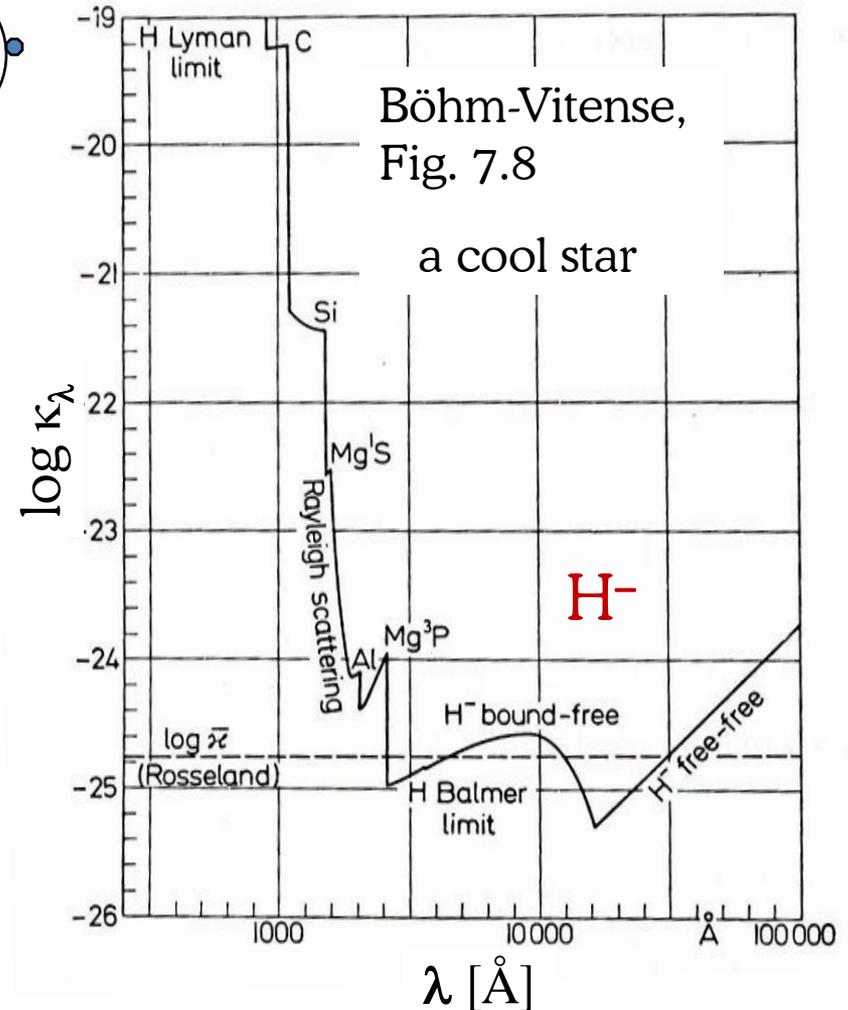
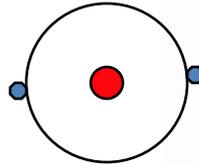
$$\kappa_{\nu}(H^-_{bf}) = \text{const. } T^{-5/2} P_e \exp(0.75/kT)$$

NB: There is only 1  $H^-$  per  $10^8$  H atoms in the Solar photosphere!

## Line opacity (*all the lines you see!*)

Caused by *bb* transitions

Need to know  $\log gf$ , damping and assume an abundance



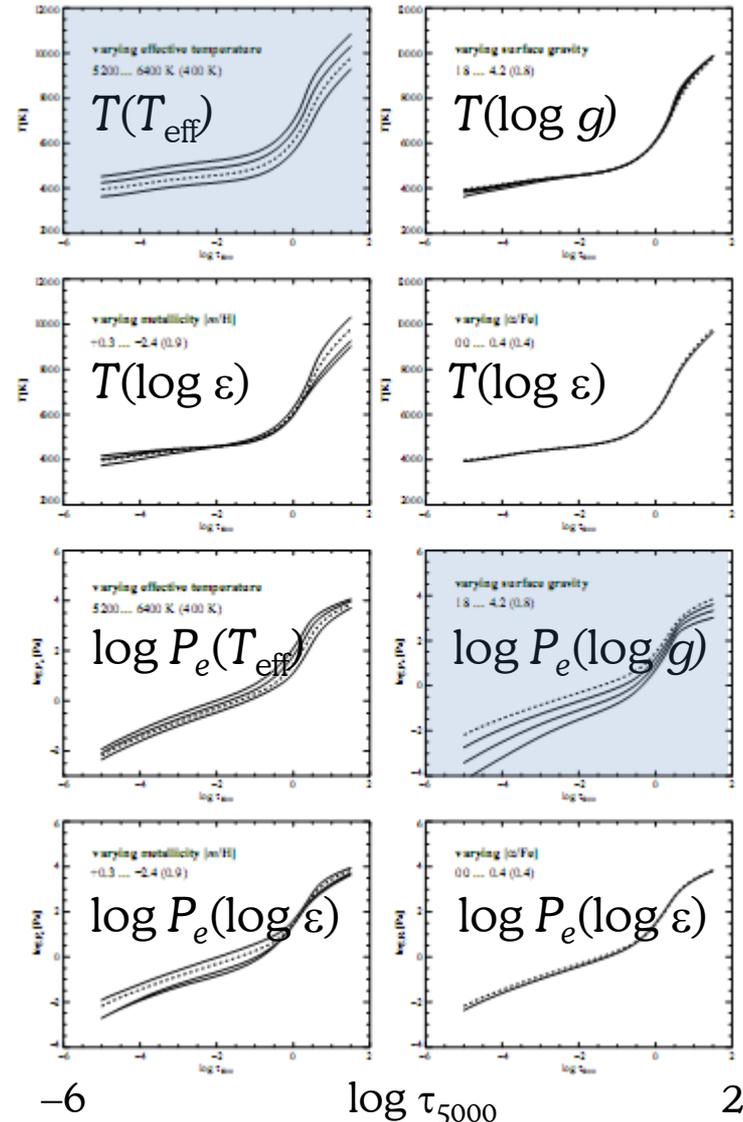
# Model atmosphere output

A 1D model atmosphere is a tabulation of various quantities as a function of (optical) depth:

- $T$  (temperature)
- $P_g$  (gas pressure)
- $P_e$  (electron pressure)
- $\mathcal{F}_v$  (esp. *surface flux*) etc.

as computed under certain input assumptions:

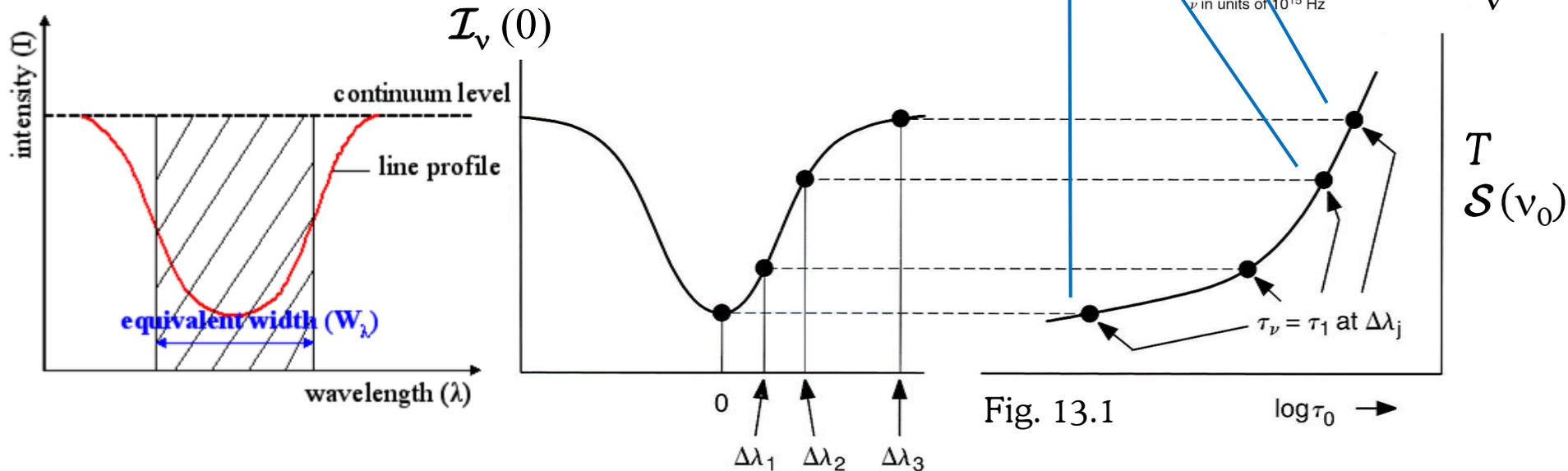
- $T_{\text{eff}}$  (effective temperature)
- $\log g$  (surface gravity)
- $\log \varepsilon(X_i)$  (chemical composition)
- hydrostatic equilibrium
- LTE (local thermodynamic equilibrium)
- MLT (mixing-length theory) and
- a statistical representation of opacities (either via opacity distribution functions, ODF, or opacity sampling, OS).



# How spectral lines originate

The formation of absorption lines can be qualitatively understood by studying how  $\mathcal{S}_\nu$  changes with depth.

$$W_\lambda \propto d \ln \mathcal{S}_\nu / d \tau_\nu$$



# Spectral lines as a function of abundance

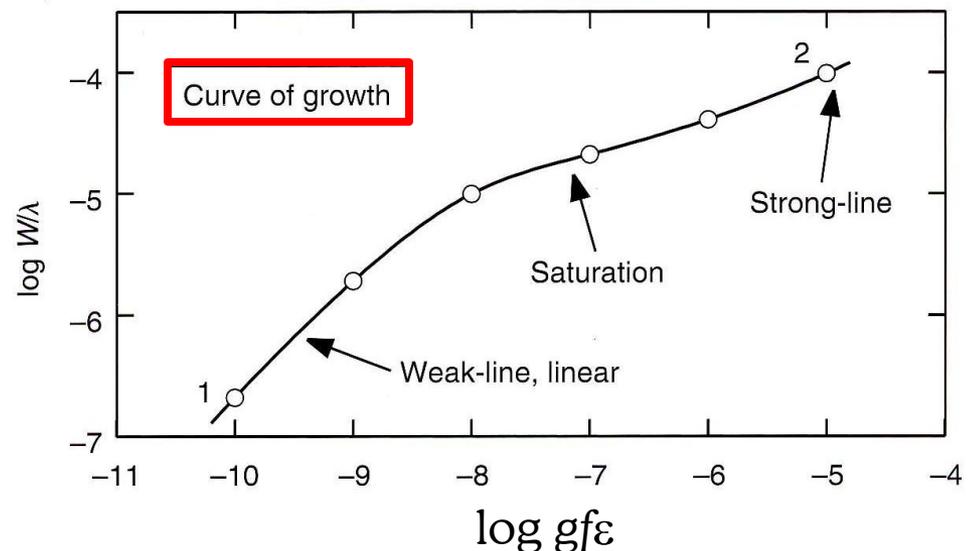
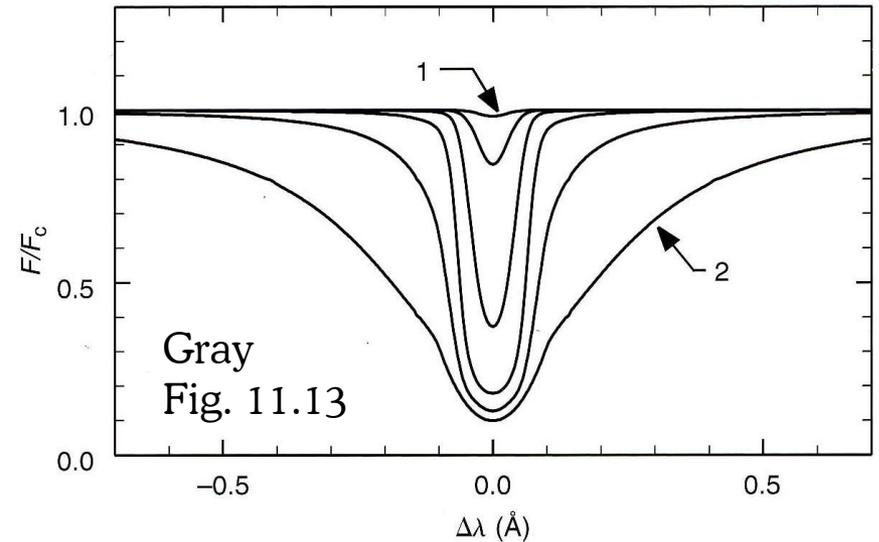
Starting from low  $\log \varepsilon$  (low  $\log gf$ ), the line strength is directly proportional to  $\log gf\varepsilon$ :

$$W_\lambda \propto gf n_X$$

When the line centre becomes optically thick, the line begins to saturate. The dependence on abundance lessens. Only when damping wings develop, the line can grow again in a more rapid fashion:

$$W_\lambda \propto \text{sqrt}(gf n_X)$$

Weak lines are thus best suited to derive the elemental composition of a star, given that they are well-observed (blending!)



# Spectral lines as a function of $T_{\text{eff}}$

The **strength of a weak line** is **proportional to** the ratio of line to continuous absorption coefficients,  $l_{\nu} / \kappa_{\nu}$ . Evaluation of this ratio can tell us about the  $T_{\text{eff}}$  sensitivity of spectral lines:

$$R = l_{\nu} / \kappa_{\nu} = \text{const. } T^{5/2} / P_e \exp-(\chi + 0.75)/kT$$

for a neutral line of an element that is mostly ionized.

$$\text{Fractional change with } T: 1/R \, dR/dT = (\chi + 0.75 - I)/kT^2$$

$\Rightarrow$  depending  $\chi$  on **neutral lines decrease with  $T_{\text{eff}}$**  by between 10 and 30% per 100 K (typically 0.07 dex per 100 K). Lines of different  $\chi$  can be used to constrain  $T_{\text{eff}}$  (**excitation equilibrium** condition).

For **ionized lines** of mainly ionized elements, one finds low sensitivities to  $T_{\text{eff}}$ , except those **with a large  $\chi$** . These **become stronger with  $T_{\text{eff}}$**  by up to 20% per 100 K.

# Spectral lines as a function of $\log g$

The  $T_{\text{eff}}$  sensitivity of spectral lines may be surpassed by sensitivities with respect to other stellar parameters.

## Sensitivity to $\log g$ in cool stars?

Case 1: (weak) neutral line of an element that is mainly ionized

$W_\lambda$  is proportional to the ratio of line to continuous absorption coefficients,  $l_\nu / \kappa_\nu$ .

$$n_{r+1} / n_r = \Phi(T) / P_e \quad \Leftrightarrow \quad n_r \approx \text{const. } P_e$$

$\Rightarrow l_\nu / \kappa_\nu \neq f(P_e)$       neutral lines do not depend on  $\log g$

Case 2: ionized line of an element that is mainly ionized

**(universal)**  $\log g$  sensitivity via the continuous opacity of  $\text{H}^-$

**NB:** for strong lines, a damping-related  $\log g$  sensitivity comes into play.

# Fundamental stellar parameters

$T_{\text{eff}}$ : via  $\mathcal{F}_{\text{Bol}}$  and  $\theta$  (see IRFM below).

To get  $\theta$ , one uses interferometry and model-atmosphere theory (limb darkening!).

$\log g$ : Newton's law, needs  $M$  and  $R$ .

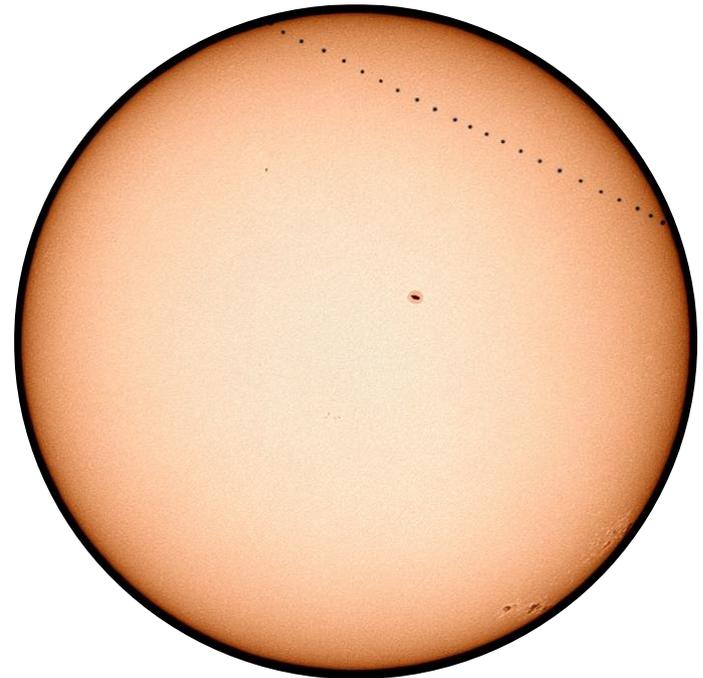
So usually one needs  $\pi$  (parallax) and  $\theta$ . Gaia is the key  $\pi$  mission (to be launched in 2012).

$M$  needs to be inferred from stellar evolution.

Exception: eclipsing binaries.

$[m/H]$ : via meteorites (only for the Sun), which lack important (volatile) elements like CNO and noble gases.

In principle, asteroseismology can provide compositions (both at the surface and in the interior).



# Photometry: pros vs. cons

Photometry is

- ✓ an efficient way of determining stellar parameters,
- ✓ can probe very deep,
- ✓ freely available (surveys!),
- ✓ comparatively cheap to obtain.

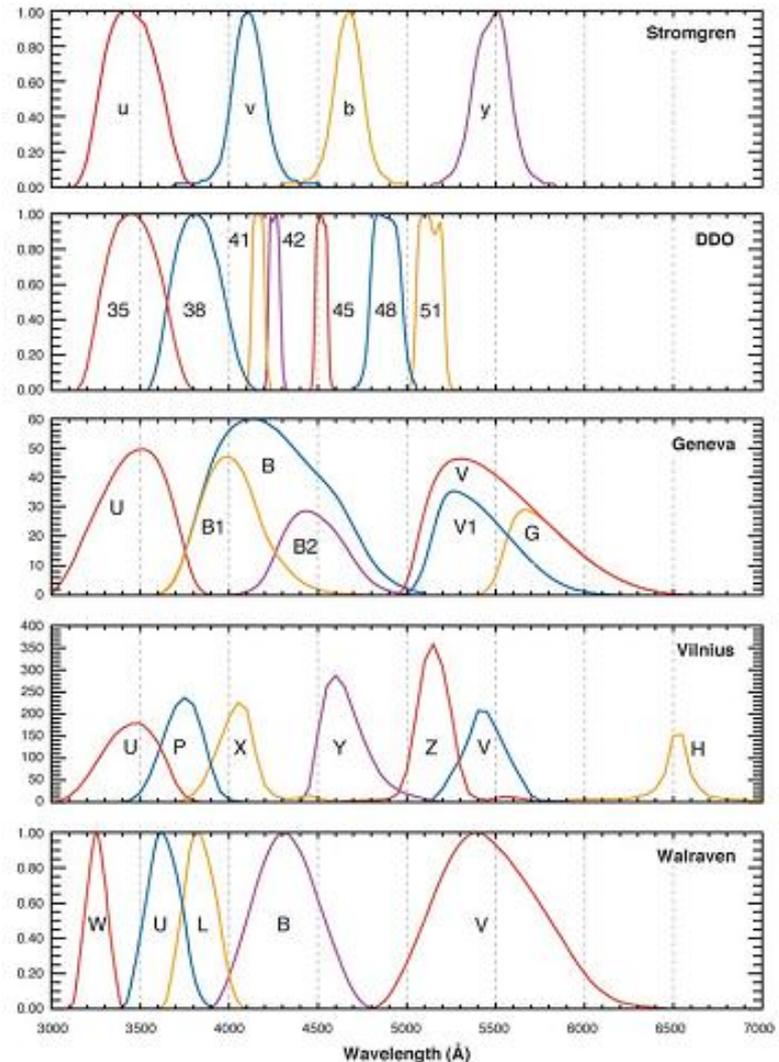
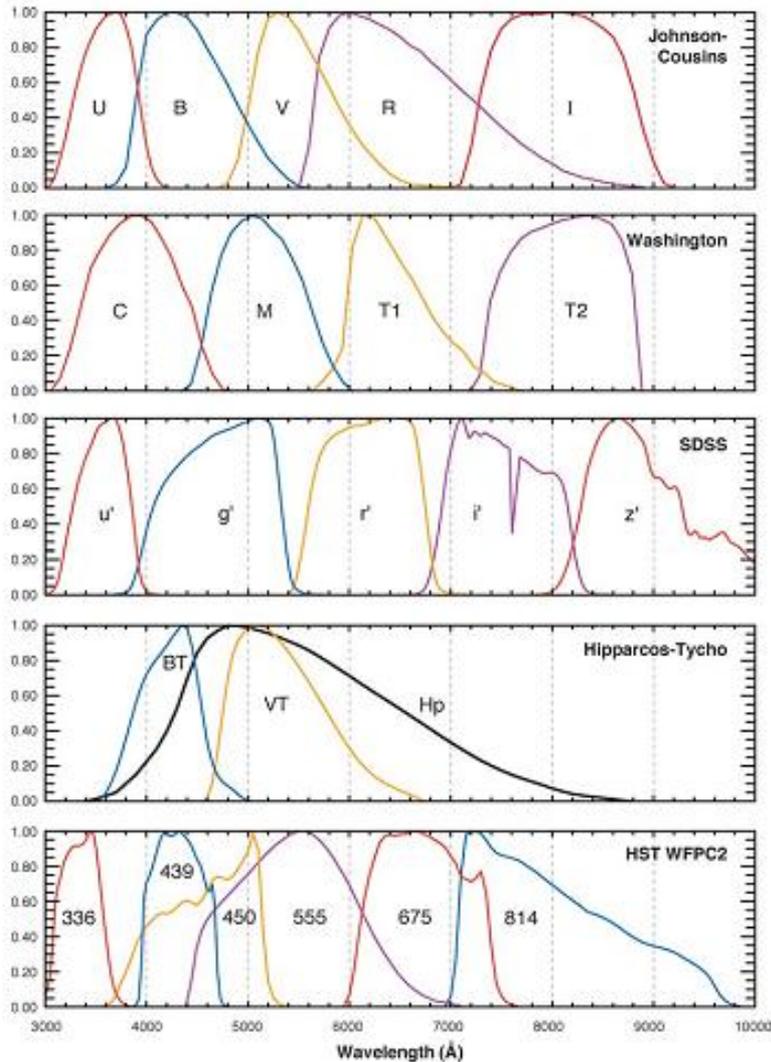
However, photometry is

- ❑ limited in which parameters can be derived,
- ❑ subject to extra parameters (reddening!)
- ❑ subject to parameters that cannot be determined well ( $\xi$ ,  $[\alpha/\text{Fe}]$ ).



(c) F. Bresolin

# Photometric standard systems

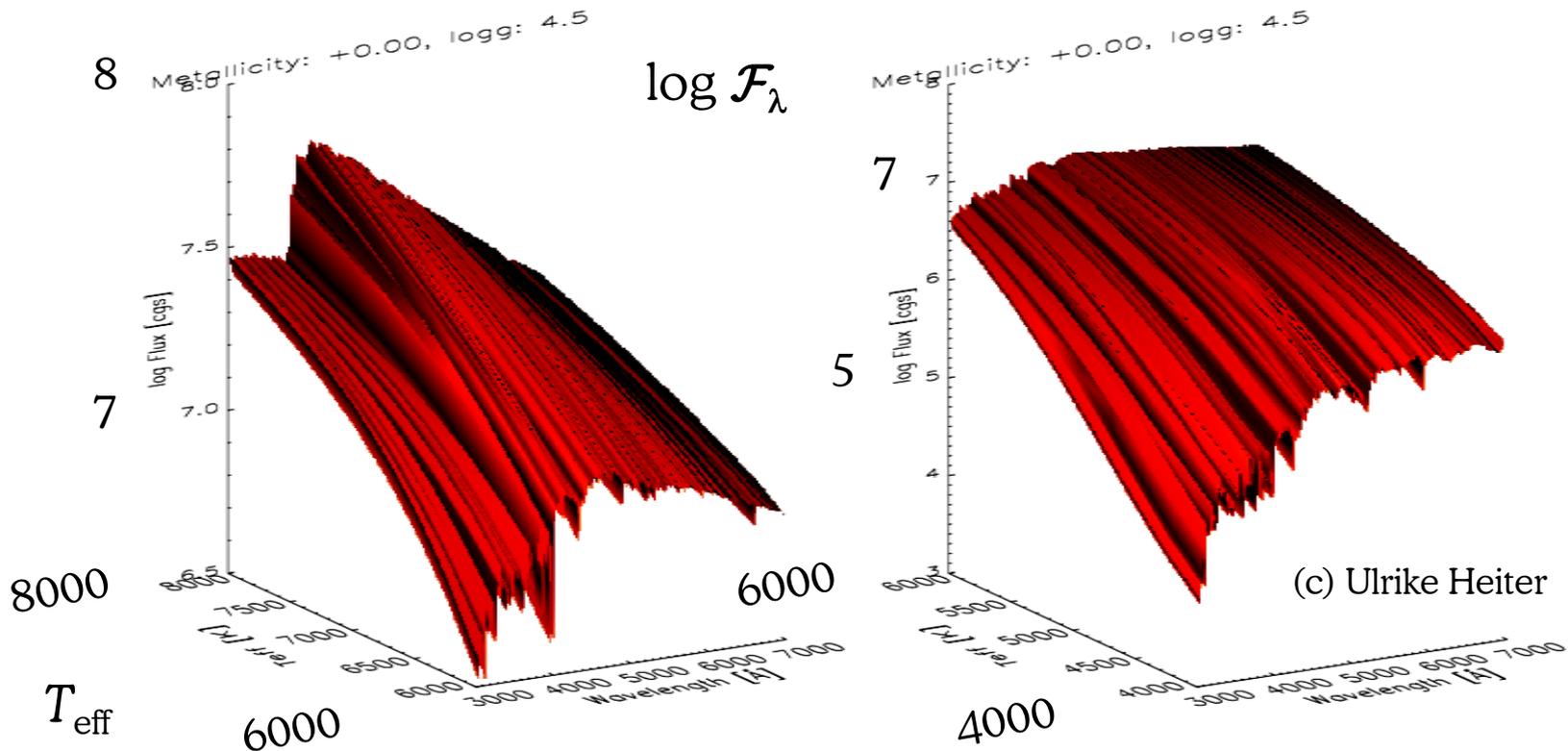


**Warning:** there is often more than one filter set for one system!

# Photometry: $T_{\text{eff}}$ dependence

$T_{\text{eff}}$  variations dominate the flux variations of cool stars.

In the BB approximation to stellar fluxes, it suffices to measure the flux at two points to uniquely determine  $T$ . In reality,  $[m/\text{Fe}]$  and reddening complicate the derivation of photometric stellar parameters.

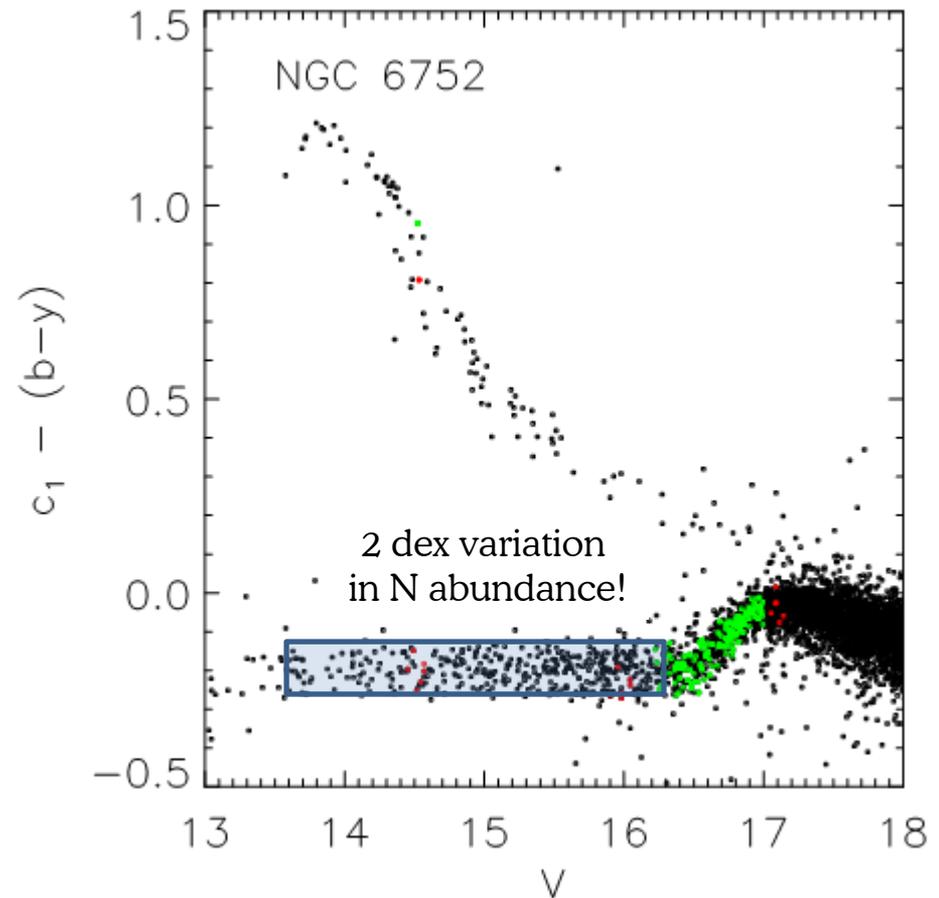


# Photometry: metallicities

After  $T_{\text{eff}}$ , the *global* metallicity has the largest influence on stellar fluxes (with the potentially disastrous exception of **reddening!**).

But the **precision** with which metallicities can be determined **is limited** (of order 0.3 dex). In addition, it is difficult to determine metallicities for stars with  $[\text{Fe}/\text{H}] < -2$ , as classical indicators like  $\delta(U - B)$  lose sensitivity.

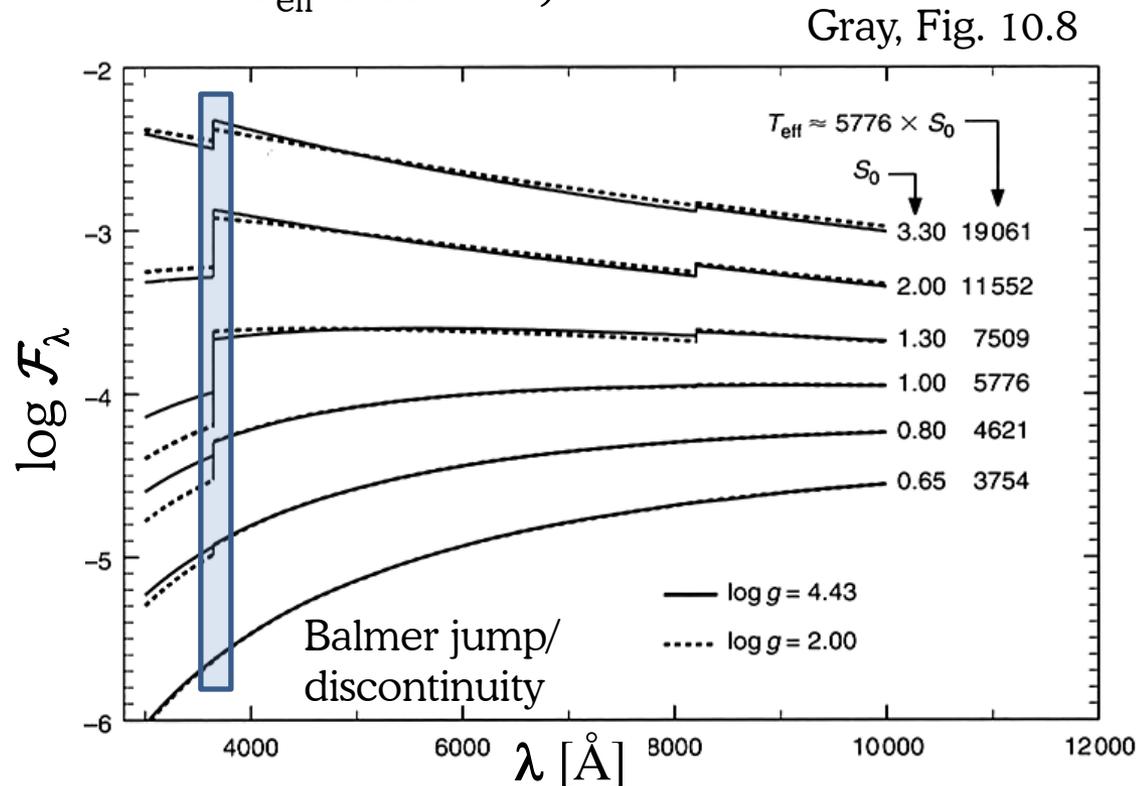
On the other hand, there are narrow-band indices which allow one to measure abundance variations (e.g. via molecular bands).



# Photometry: gravity dependence

The only feature that has a sufficiently large gravity sensitivity to be exploited by photometry is the **Balmer jump at 3647 Å** (in hot stars it can be used as a sensitive  $T_{\text{eff}}$  indicator).

Colours like  $(U - B)$  or  $(u - y)$  measure the Balmer discontinuity, but the usefulness as a precise gravity indicator is hampered by the high line density in this spectral region (missing opacity problem), the difficulties with ground-based observations in the near-UV and a proper treatment of the overlapping Balmer lines.



The  $c_1$  index ( $\equiv (u - b) - (b - y)$ ) works well for metal-poor giants (Önehag *et al.* 2008).

# IRFM: a semi-fundamental $T_{\text{eff}}$ scale

Basic idea of the infrared-flux method:

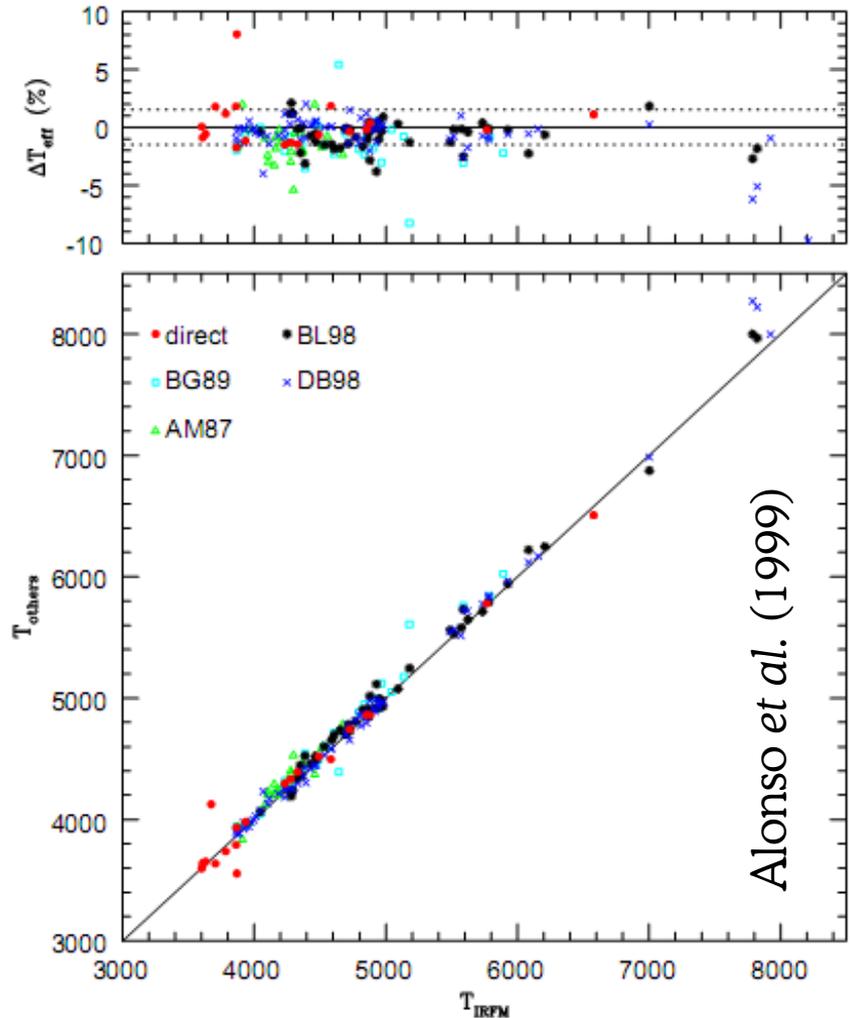
$$\frac{\mathcal{F}(\text{surface})}{\mathcal{F}_{\lambda_{\text{IR}}}(\text{Earth})} = \frac{\sigma T_{\text{eff}}^4}{\mathcal{F}_{\lambda_{\text{IR}}}(\text{model})}$$

$\mathcal{F}_{\lambda_{\text{IR}}}(\text{model})$  is said to be only weakly model dependent (but cf. Grupp 2004).

Once calibrated on stars with known diameters, any colour index can be calibrated on the IRFM.

**Direct** sample:  $\Delta T_{\text{eff}} = 0.06 \pm 1.25\%$

Comparing different IRFM calibrations (Blackwell *et al.*, Ramírez & Meléndez, Casagrande *et al.*), the **zero point** proves to be **uncertain by  $\pm 100$  K**, in particular for metal-poor stars.



$$\mathcal{F}_{\lambda_{\text{IR}}}(\text{Earth}) = q(\lambda_{\text{IR}}) \mathcal{F}_{\lambda_{\text{IR}}}^{\text{std}}(\text{Earth}) 10^{-0.4(m_{\text{IR}} - m_{\text{IR}}^{\text{std}})}$$

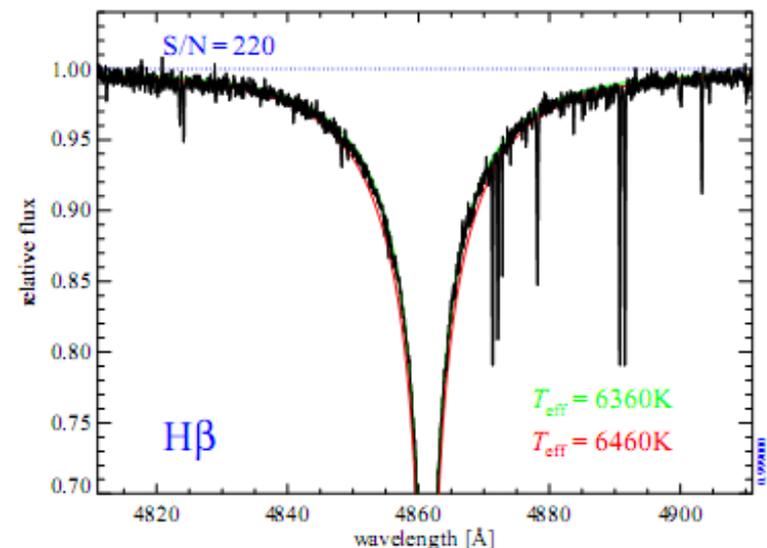
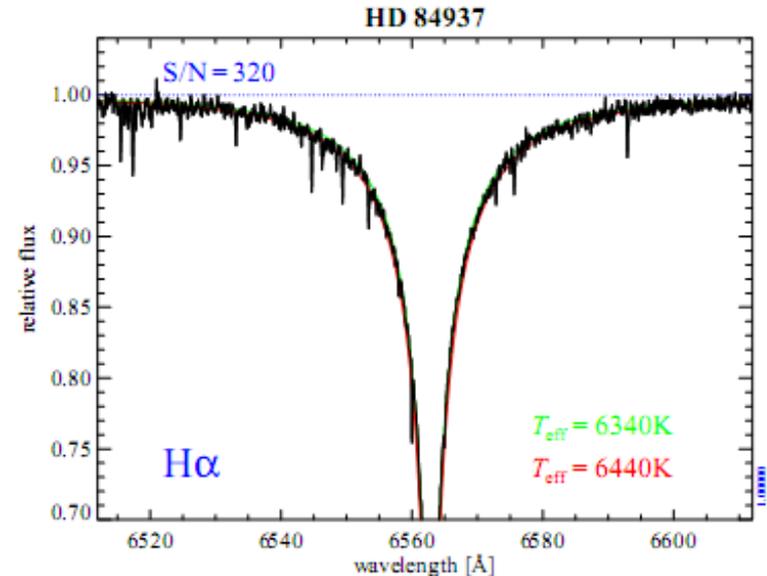
# Spectroscopic $T_{\text{eff}}$ indicators: H lines

Above 5000 K, the **wings of Balmer lines** are a sensitive  $T_{\text{eff}}$  indicator, **broadened by H + H collisions** (mainly  $H\alpha$ ) **and the linear Stark effect** ( $H + e^-$ ).

In cool stars, the  $\log g$  sensitivity is low (line and continuous opacity both depend on  $P_{\rho}$ ), as is the metallicity dependence. There is some dependence on the mixing-length parameter ( $H\beta$  and higher).

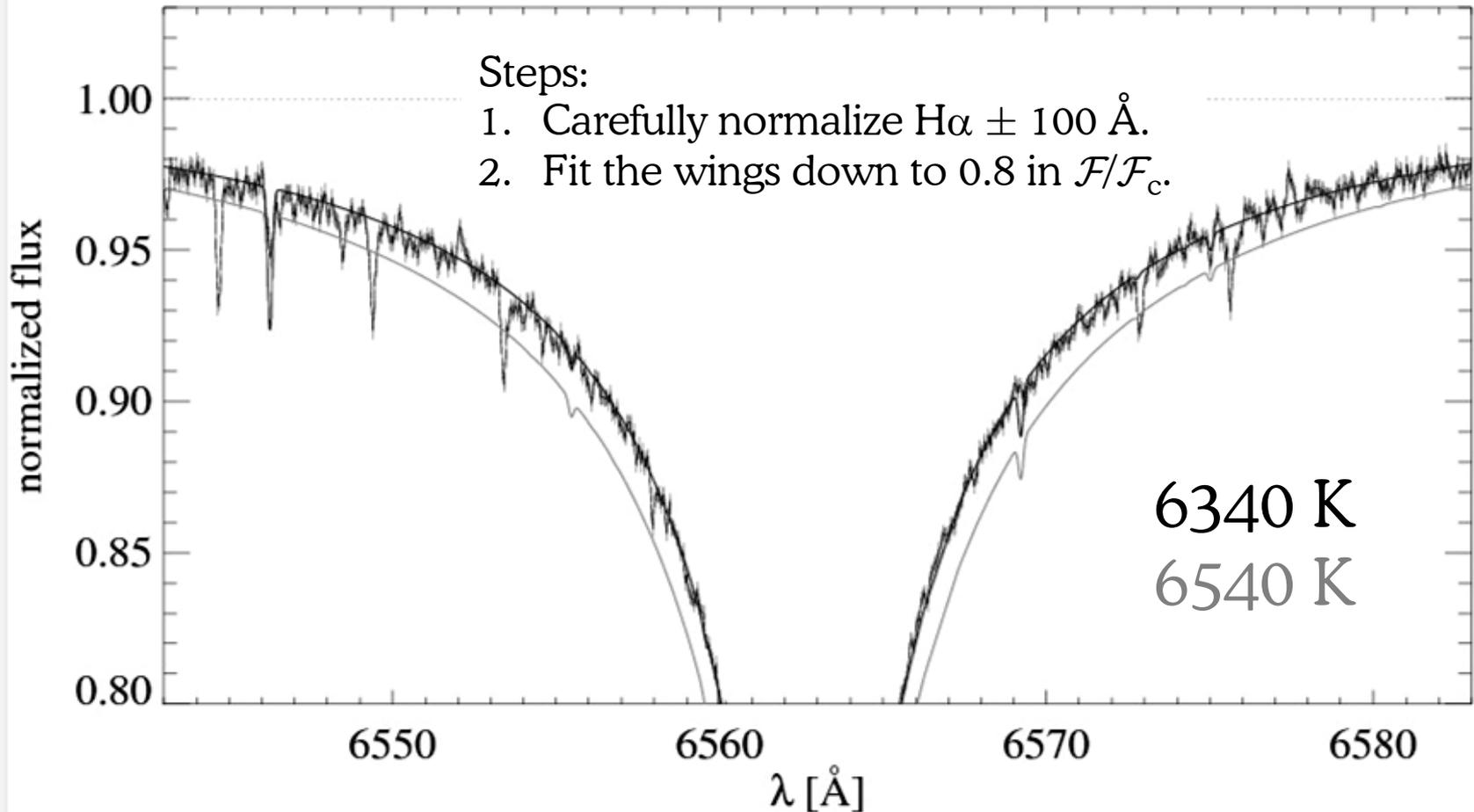
Main **challenge** (apart from the surprisingly complex broadening): **recovering the intrinsic line profiles** from (echelle) observations.

In hot stars, Balmer lines can constrain the surface gravity.



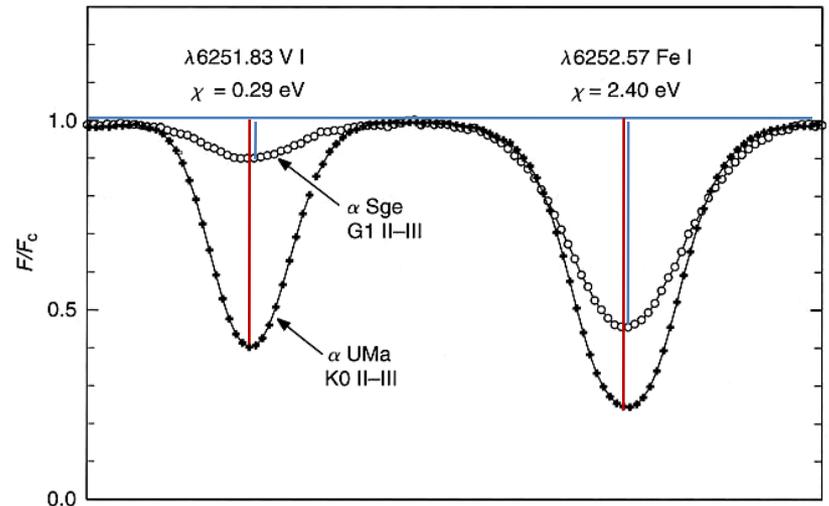
# H $\alpha$ as a function of $T_{\text{eff}}$

## HD84937 FOCES



# Line-depth ratios (LDRs)

Using the **ratio of two lines'** central depths (rather than  $W_\lambda$ ) can be a **remarkably sensitive temperature indicator** (precision as high as 5 K!), if the lines are chosen to have different sensitivities to  $T$ . Ideally, the LDR is close to 1 and the lines should not be too far apart.



Gray Fig.14.7

The main **challenge** lies in a **proper  $T_{\text{eff}}$  calibration** across a usefully large part of the HRD.

# Gravity sensitivity of ionized lines

Recall that ionized lines of an element that is mainly ionized have a  $P_e^{-1}$  sensitivity via the continuous opacity of  $H^-$ .

Integrating the hydrostatic equation, we find

$$P_g \propto g^{2/3}$$

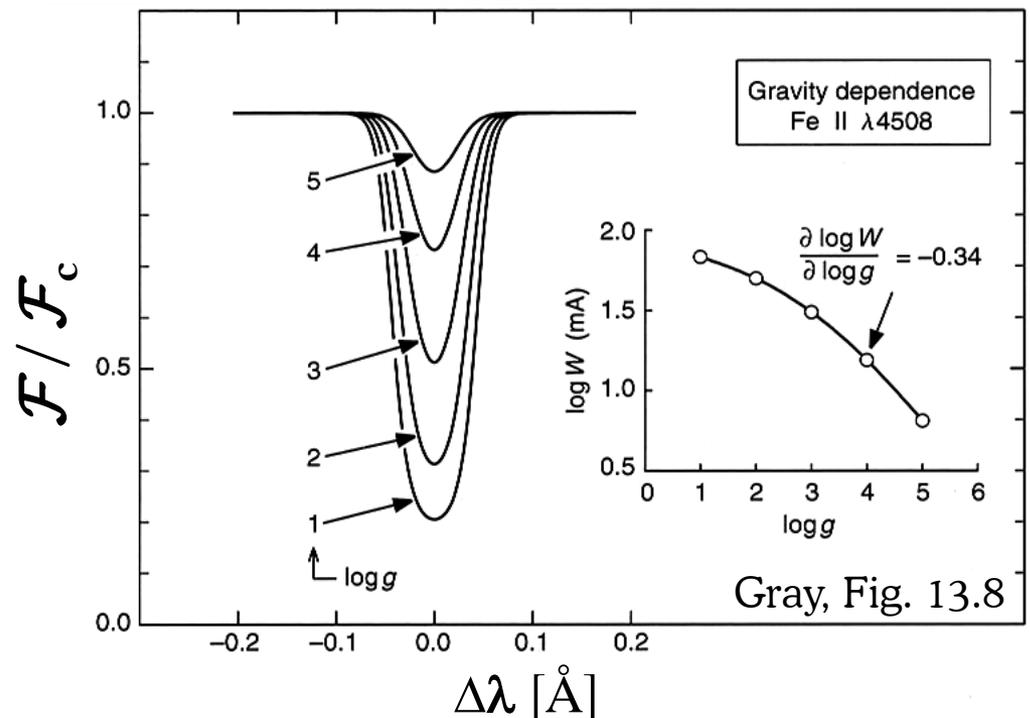
and together with  $P_e \propto \text{sqrt}(P_g)$  we expect

$$I_\nu / \kappa_\nu \propto g^{-1/3}.$$

This is borne out by actual calculations.

## Hydrostatic equilibrium

$$dP/d\tau_\nu = g / \kappa_\nu$$



# Practicalities of ionization equilibria

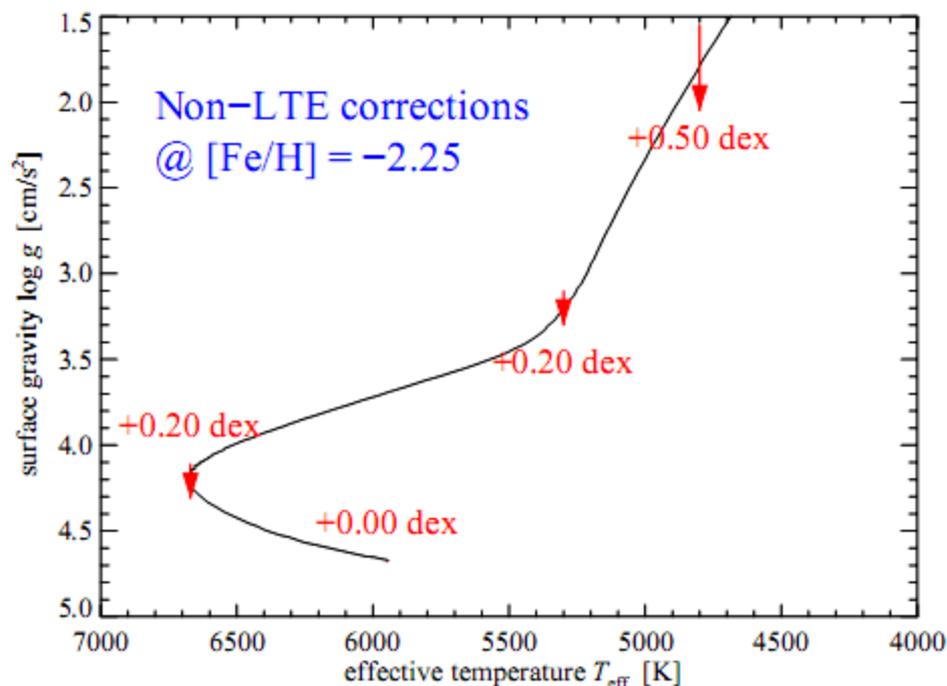
A change of **0.1 dex in  $\log \epsilon$**   
translates to a change of **0.3 dex  
in  $\log g$** .

Consequences:

A line-to-line scatter of 0.1 dex  
means that  $\log g$  is known to  
within 0.3 dex.

Relatively small changes in  $\log \epsilon$ ,  
e.g. because of a change in  $T_{\text{eff}}$   
or NLTE effects, can lead to  
factor-of-two changes in the  
surface gravity.

**Astrometry can help** to establish  
the correct surface-gravity scale.



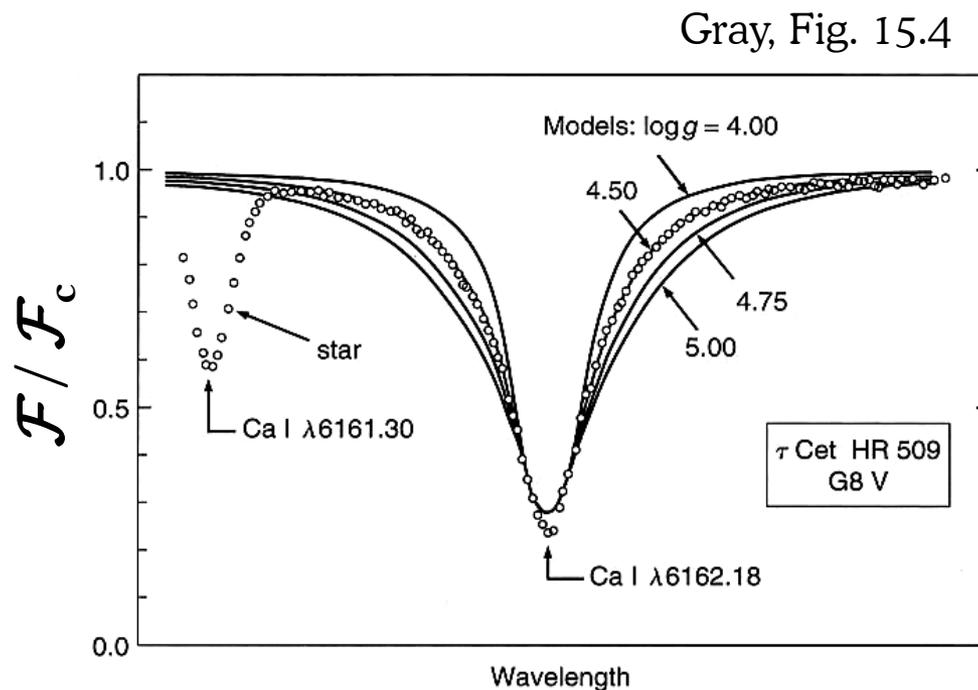
Korn (2004),  
Carnegie Observatories Centenary (2003)  
<http://www.ociw.edu/ociw/symposia/series/symposium4/proceedings.html>  
Mashonkina *et al.* (2010), *A&A submitted*

# The strong line method

Damped (neutral) lines show a strong gravity sensitivity, because

$$I_{\nu} \propto \gamma_6 \propto P_g \propto g^{2/3}.$$

Like with ionization equilibria,  $\log \varepsilon$  needs to be known. This is to be obtained from weak lines of the same ionization stage, preferably originating from the same lower state (to minimize differential NLTE effects).



Examples: Ca I 6162 (see above), Fe I 4383, Mg I 5183, Ca I 4226. Below  $[\text{Fe}/\text{H}] \approx -2$ , there are no optical lines strong enough to serve as a surface-gravity indicator.

# Spectroscopy of the Solar neighbourhood

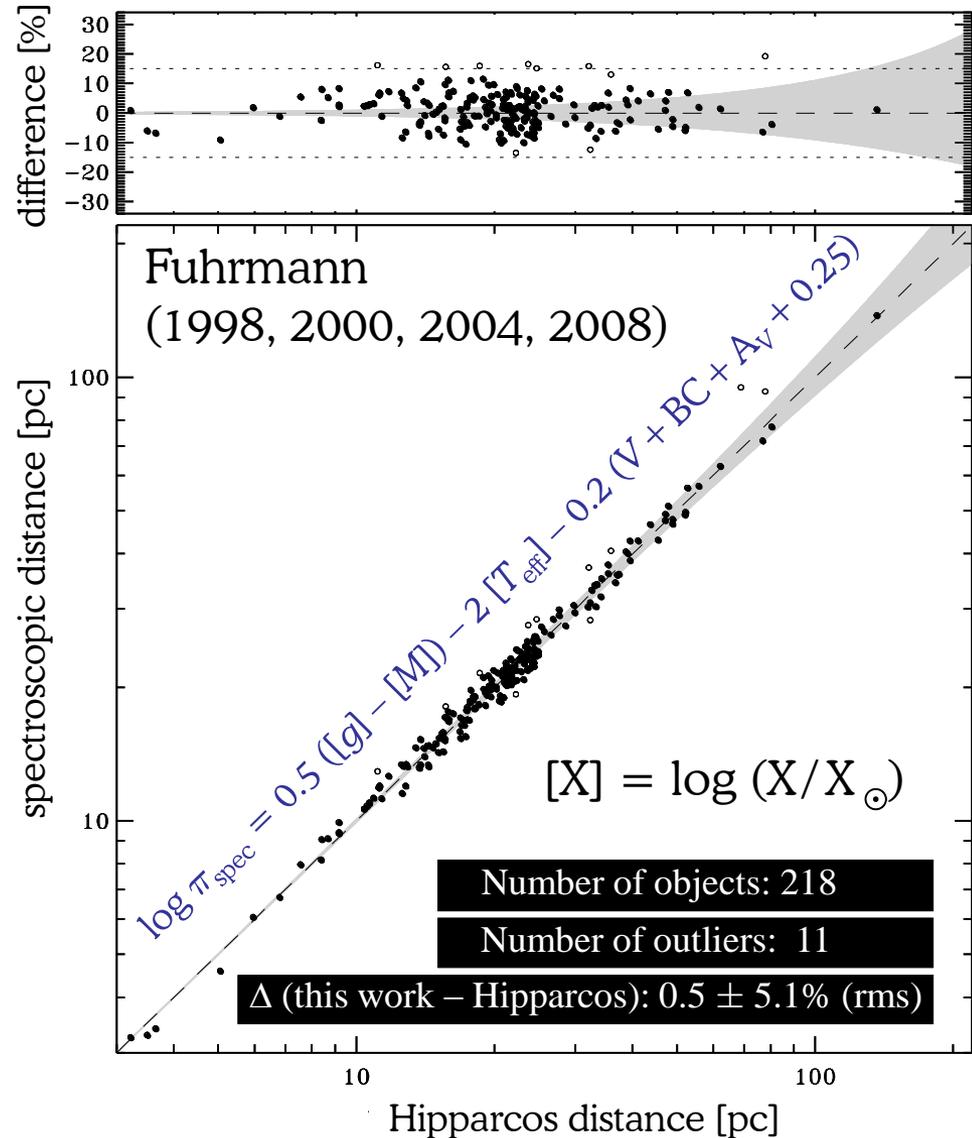
## Aim:

Derive precise stellar parameters and chemical abundances of FGK stars within  $d = 25$  pc.

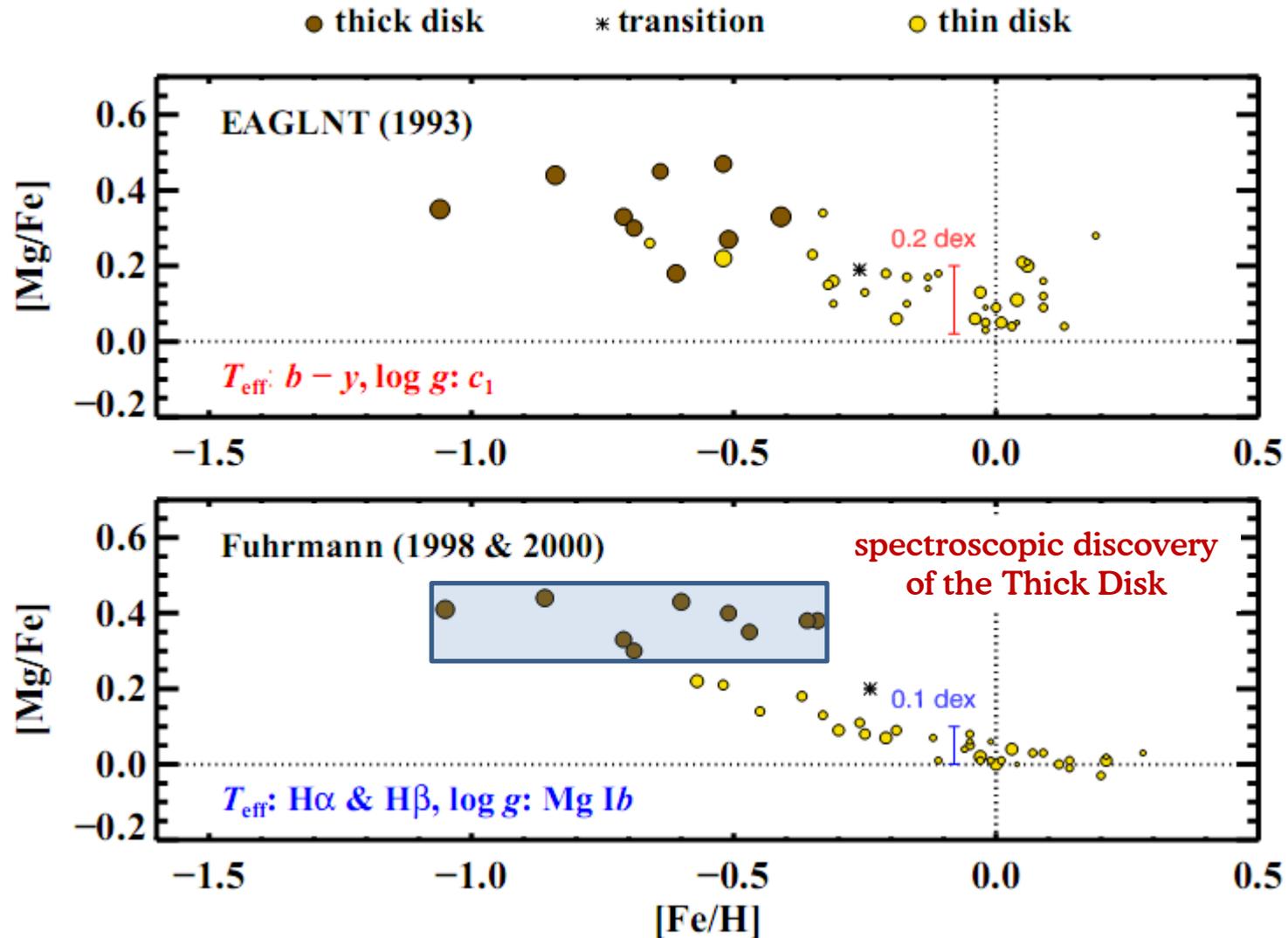
## Example:

The strong-line method as a surface-gravity indicator for not too metal-poor, not-too-evolved stars coupled with  $T_{\text{eff}}$  values from Balmer lines.

## Benchmark: Hipparcos



# Stellar populations around the Sun

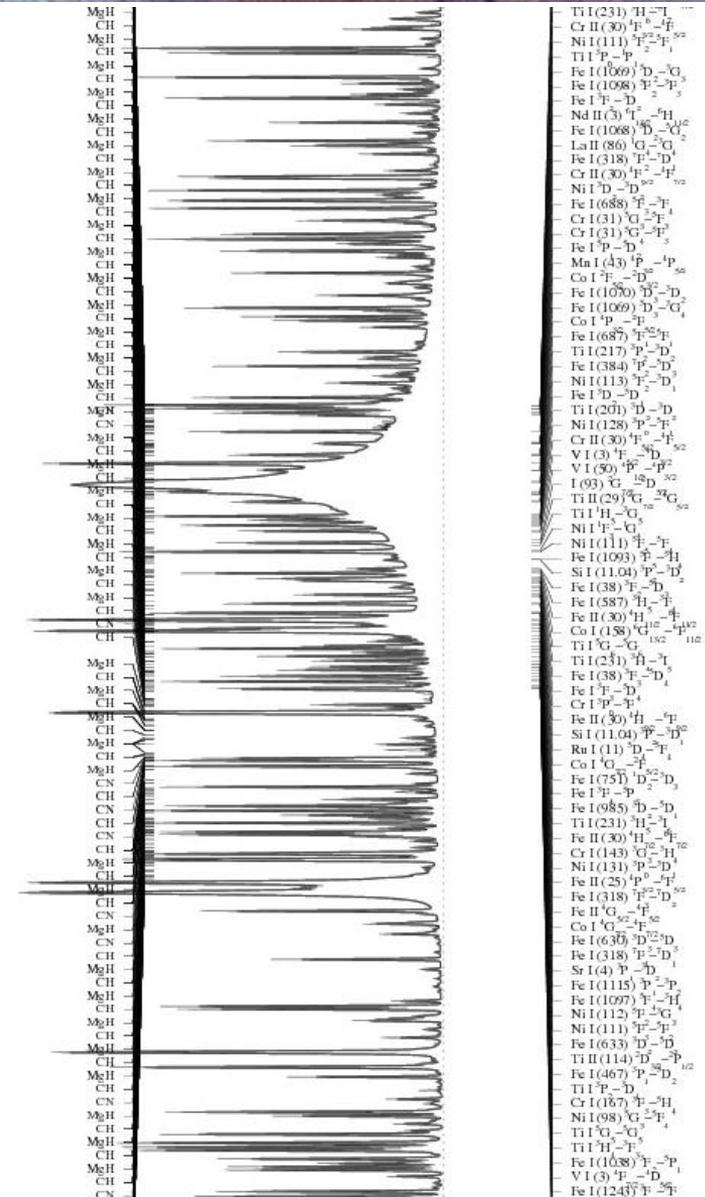


# Abundances from H to U

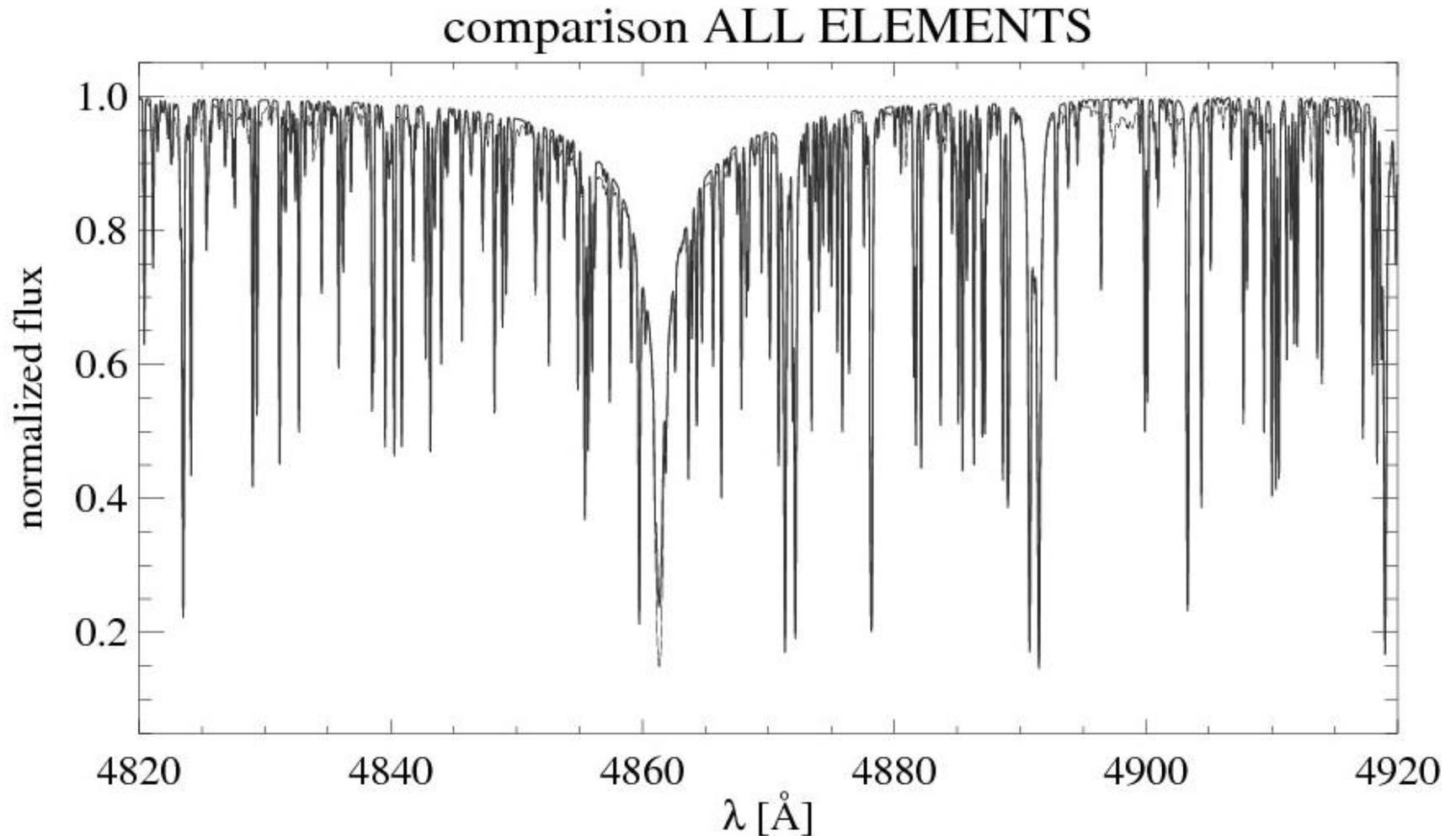
Once you have good stellar parameters, it is relatively easy to determine chemical abundances for your favourite element(s).

## Caveats

- ❑ some elements are not visible, e.g. noble gases in cool stars
- ❑ lines may lack or have inaccurate atomic data
- ❑ lines can be blended leading to overestimated abundances
- ❑ lines can be subject to effect you are unaware of, e.g. 3D and NLTE effects, hfs, isotopic and Zeeman splitting
- ❑ ...



# Quantitative spectroscopy: the Sun



# Spectroscopy: pros *vs.* cons

Spectroscopy is

- ✓ a way of determining a great number of stellar parameters,
- ✓ the key technique for obtaining detailed chemical abundances,
- ✓ (usually) reddening-free.

However, hi-res spectroscopy is

- ❑ comparatively costly at the telescope,
- ❑ currently limited to 18<sup>m</sup> in *V*,
- ❑ more difficult to master than photometry.



...especially when they accept photometry as a source of valuable information.

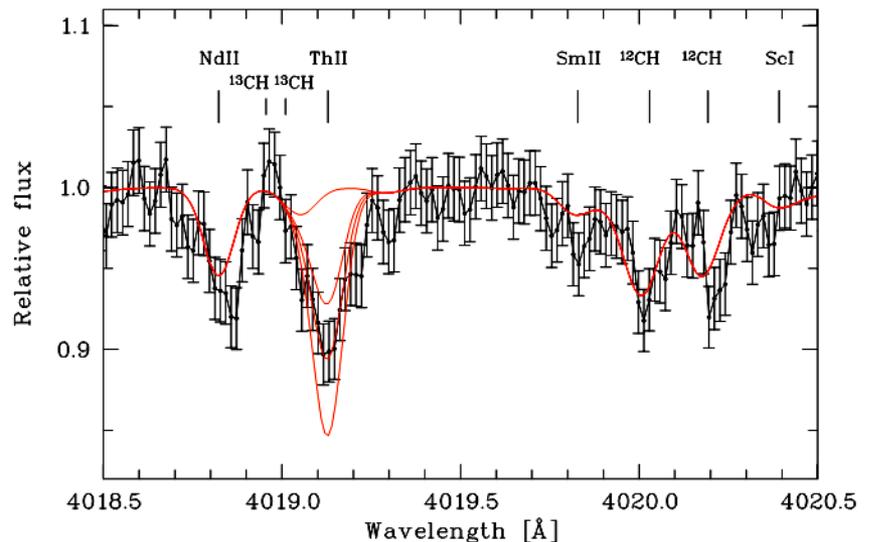
# An *r*-process example

Hayek *et al.* (2009) analysed two metal-poor stars with significant enhancement of *r*-process elements ( $[r/Fe] > 1.0$ ).

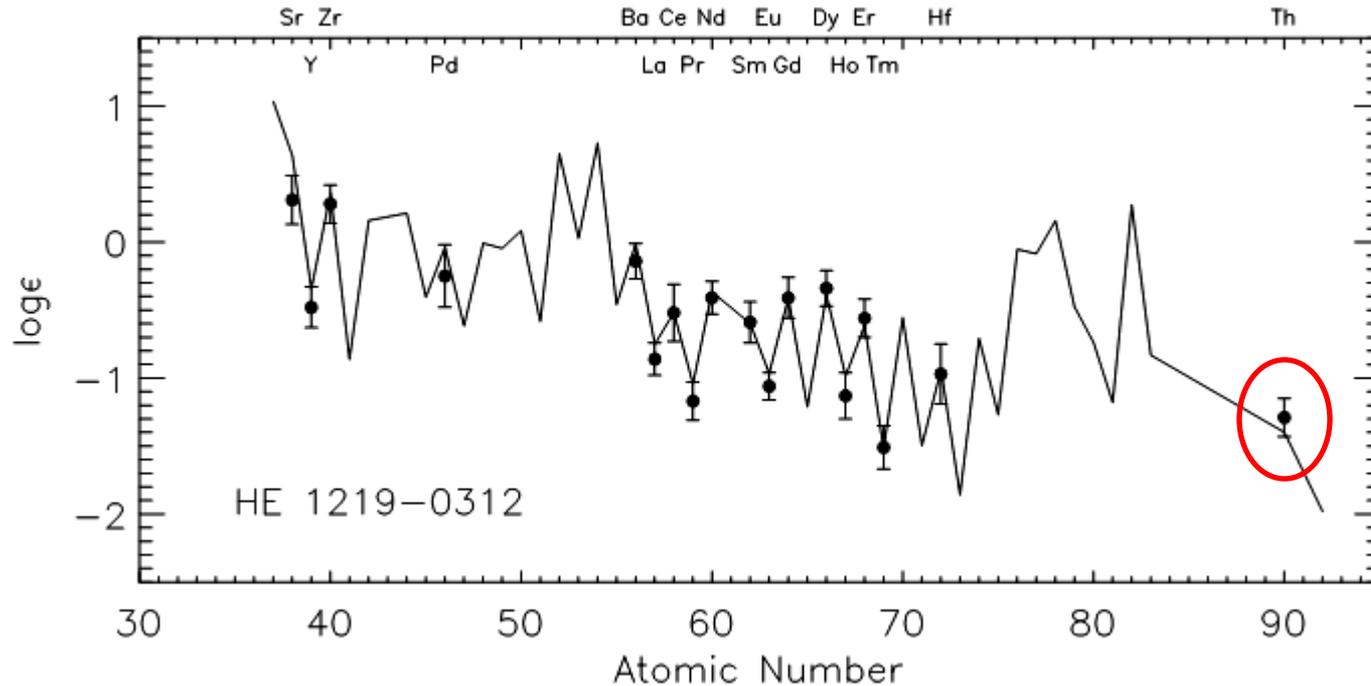
Stellar parameters were determined from high-resolution ( $R = 70,000$ ), high signal-to-noise-ratio ( $S/N \approx 100$ ) UVES spectra using interactive and automated techniques.

Abundances for up to 18 elements beyond the iron peak were derived for HE 1219–0312, including that of thorium.

HE 1219–0312		
Parameter	This work	HERES
$T_{\text{eff}}$ [K]	$5060 \pm 100$	$5140 \pm 100$
$\log g$ (cgs)	$2.3 \pm 0.2$	$2.4 \pm 0.4$
[Fe/H]	$-2.96 \pm 0.14$	$-2.80 \pm 0.12$
$\xi$ [ $\text{km s}^{-1}$ ]	$1.6 \pm 0.1$	$1.5 \pm 0.2$



# HE 1219-0312



The abundance pattern matches a scaled-solar  $r$ -process pattern well. But thorium is slightly more abundant than expected. This leads to a severe interpretation conflict, as the star seems to be rather young or even possess a negative age!

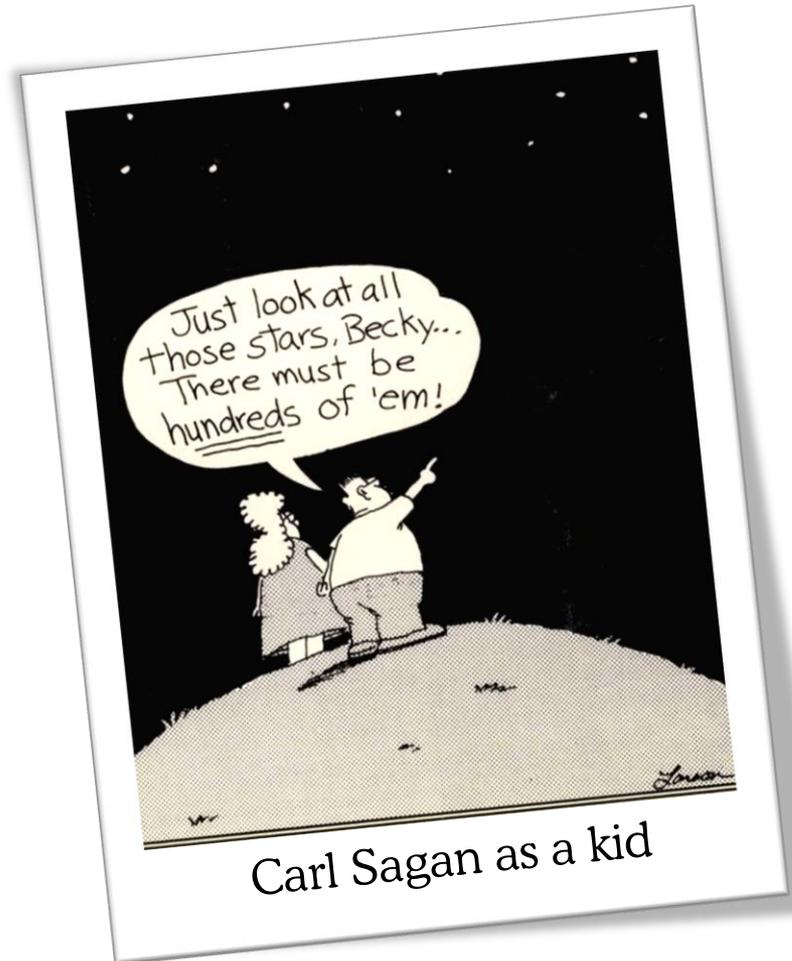
**What is wrong?**

# Summary

The determination of stellar parameters ( $T_{\text{eff}}$ ,  $\log g$ ,  $[\text{Fe}/\text{H}]$ ,  $[\alpha/\text{Fe}]$ , plus auxiliary parameters like  $\xi$  and  $\Xi$ ) is a crucial first step of stellar data analysis.

For some applications it suffices to use stellar parameters with limited *precision* (survey-type work), while for others high *accuracy* is mandatory (abundance fine analysis).

When chemical abundances are at focus, **spectroscopy is an indispensable technique to be mastered.**



Carl Sagan as a kid

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