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The Chiral Magnetic Effect and symmetry breaking in SU(3) quenched theory

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We study some properties of the non-Abelian vacuum induced by strong external magnetic field. For this purpose we use the quenched SU(3) lattice gauge theory with tadpole-improved Lüscher-Weisz action and chirally invariant lattice Dirac operator. Within this approach the following results have been obtained: The chiral symmetry breaking is enhanced by the magnetic field. The corresponding chiral condensate depends on the field as a function of power $\nu = 1.6 \pm 0.2$. There is a paramagnetic polarization of the vacuum. The corresponding susceptibility and other magnetic properties have been calculated and compared with theoretical estimations. Finally, there are non-zero local fluctuations of the both chirality and electromagnetic current, which grow with the magnetic field strength. These fluctuations can be recognized as an evidence of the Chiral Magnetic Effect (CME), which is observed by the STAR Collaboration in heavy ion collisions at RHIC.

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1. Introduction

The modern experiments provide a possibility to discover new physical effects caused by presence of the strong (hadronic scale) magnetic field. At the Relativistic Heavy Ion Collider (RHIC) at the first moments ($\tau \sim 1$ fm/c) of noncentral collision the very strong ($B \sim 10^{15}$T, $\sqrt{eB} \sim 300$ MeV) magnetic fields appear[1, 4]. Such strong magnetic fields can be also created in ALICE experiment at LHC, at the Facility for Antiproton and Ion Research (FAIR) at GSI and in the experiment NICA in Dubna. The additional motivation for such discovery could also come from the physics of the early Universe, where the strong fields ($B \sim 10^{16}$T, $\sqrt{eB} \sim 1$ GeV) could have been produced after the electroweak phase transition[6]. It seems interesting to find some properties of strongly interacting matter, which can be relevant for these experiments or theoretical models. Due to the nonperturbative nature of the effects we perform the calculations on the lattice. We use quenched approximation for the simplicity and show that for some class of problems it provides values of the right order or even very close to the present estimations. It makes sense to suggest this approximation for some further predictions.

This work has been done in spirit of the previous SU(2) lattice studies[7, 8, 9, 10] and here we analyze SU(3) gauge theory to make some quantitative predictions, which one can directly compare with QCD. Let us make a short list of effects caused by the magnetic field and studied in the paper.

The strong magnetic can enhance the chiral symmetry breaking. There are various models (see Sec.3) which predict the growing of the chiral condensate as an order parameter. In all the models dependence on B typically ranges from linear to quadratic behaviour, so we are interested in SU(3) lattice predictions to check how close is the quenched approximation to this models.

The second effect is the chiral magnetization of the QCD vacuum. This effect has a paramagnetic nature because the external magnetic field leads to orient quarks' spins along the field. Investigation of the vacuum magnetization is essential for the properties of the nucleon magnetic moments[20] and other nonperturbative effects of hadrons[22]. We calculate the magnetic susceptibility and other quantities in Sec.4. In addition, the quarks develop an electric dipole moment along the field due to local fluctuations of topological charge[9]. We study this effect in Sec.5.

Finally, the fluctuations of topological charge can be a source of the assymetry between numbers of quarks with different chiralities created in heavy-ion collisions. The so called "event-by-event P- and CP-violation"[1] can be explained by this assymetry and observed at RHIC. So, our aim is also to see any evidences of this effect in SU(3) lattice simulations, nevertheless they can repeat qualitatively the SU(2) results[10].

2. Technical details

As a framework we use quenched $SU(3)$ lattice gauge theory with tadpole-improved Lüscher-Weisz action [11]. To generate statistically independent gauge field configurations we make use of the Cabibbo-Marinari heat bath algorithm. The lattice has been chosen of the size 14^4, with lattice spacing $a = 0.105\,fm$, which means that we consider a zero-temperature situation. All observables we present in the work have a similar structure: $\langle\bar{\Psi}\mathcal{O}\Psi\rangle$ for VEV of a single quantity or $\langle\bar{\Psi}\mathcal{O}_1\Psi\ \bar{\Psi}\mathcal{O}_2\Psi\rangle$ for dispersions or correlators. Here $\mathcal{O}$, $\mathcal{O}_1$, $\mathcal{O}_2$ are some operators in spinor and color space. These expectation values can be expressed through the sum over M low-lying

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but non-zero eigenvalues $i\lambda_k$ of the chirally invariant Dirac operator D (Neuberger's overlap Dirac operator[12]):

$$\langle\bar{\Psi}\mathscr{O}\Psi\rangle = \sum_{|k|<M} \frac{\psi_k^\dagger \mathscr{O} \psi_k}{i\lambda_k + m} \tag{2.1}$$

and

$$\langle\bar{\Psi}\mathscr{O}_1\Psi\ \bar{\Psi}\mathscr{O}_2\Psi\rangle = \sum_{k,p} \frac{\langle k|\mathscr{O}_1|k\rangle\langle p|\mathscr{O}_2|p\rangle - \langle p|\mathscr{O}_1|k\rangle\langle k|\mathscr{O}_2|p\rangle}{(i\lambda_k+m)(i\lambda_p+m)}, \tag{2.2}$$

where all spinor and color indices are contracted and we omit them for simplicity. The λ_k are defined by the equation

$$D\psi_k = i\lambda_k\psi_k, \tag{2.3}$$

where ψ_k are the corresponding eigenfunctions and the uniform magnetic field $F_{12} = B_3 \equiv B$ is introduced as described in[7]. To perform calculations in the chiral limit one calculates the expression (2.1) or (2.2) for some non-zero m and average it over all configurations of the gauge fields. Then one repeats the procedure for other quark masses m and extrapolate the VEV to $m \to 0$ limit.

3. Chiral condensate

In this section we present some results for the chiral condensate

$$\Sigma \equiv -\langle 0|\bar{\Psi}\Psi|0\rangle, \tag{3.1}$$

as a function of the magnetic field B. This quantity is quite useful as an order parameter for the chiral symmetry breaking. The general tendency for Σ to grow with B was already obtained in various models: in the chiral perturbation theory [13, 14] ($\Sigma \propto B$ for weak fields, $\Sigma \propto B^{3/2}$ for strong fields), in the Nambu-Jona-Lasinio model [15] ($\Sigma \propto B^2$), in a confining deformation of the holographic Karch-Katz model [16] ($\Sigma \propto B^2$), in D3/D7 holographic system [17] ($\Sigma \propto B^{3/2}$ for low temperatures, $\Sigma \propto B$ for high temperatures) and in SU(2) lattice calculations [7]($\Sigma \propto B$). Here our aim is to see how the chiral condensate behaves in the $SU(3)$ quenched model.

For this purpose we use the Banks-Casher formula [18], which relates the condensate (3.1) with the density $\rho(\lambda)$ of near-zero eigenvalues of the Dirac operator:

$$\Sigma = \lim_{\lambda\to 0} \frac{\pi\rho(\lambda)}{V}, \tag{3.2}$$

where V is the four-volume of the Euclidean space-time. The result is shown on Fig.1(a).

In order to extract a power of the dependence we make a fit by the following function:

$$\Sigma^{fit}(B) = \Sigma_0 \left[1 + \left(\frac{eB}{\Lambda_B^2}\right)^\nu\right], \tag{3.3}$$

where $\Sigma_0 \equiv \Sigma(0)$. The obtained fitting parameters are

$$\Sigma_0 = [(228\pm 3)MeV]^3, \qquad \Lambda_B = (1.31\pm 0.04)\,GeV, \qquad \nu = 1.57\pm 0.23 \tag{3.4}$$

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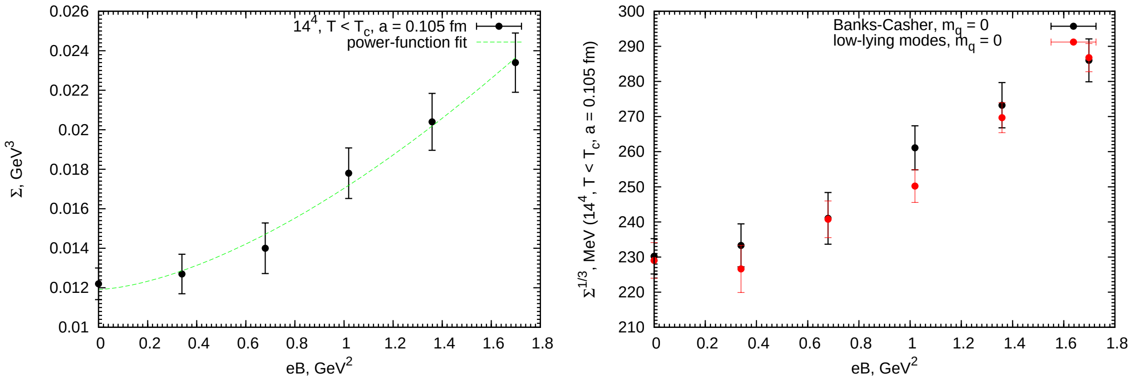


(a) (b)

Figure 1: Chiral condensate

It would we also interesting to compare quantitatively the condensate obtained by the Banks-Casher formula and one calculated by the expression (2.1) with $\mathscr{O} = \mathbb{1}$. The result is shown on Fig.1(b). The value of the condensate in absence of the magnetic field equals $\Sigma(0) = (230 \pm 5)\, MeV$ which is not so far away from the value, which can be estimated by the Gell-Mann-Oakes-Renner formula[19]:

$$\Sigma(0) = \frac{F_\pi^2 m_\pi^2}{2(m_u + m_d)} \simeq (240 \pm 10 MeV)^3 \tag{3.5}$$

4. Chiral magnetization and susceptibility

In this section we calculate the quantity

$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi} \Psi \rangle q F_{\alpha\beta}, \tag{4.1}$$

where $\sigma_{\alpha\beta} \equiv \frac{1}{2i}\left[\gamma_\alpha, \gamma_\beta\right]$ and $\chi(F)$ is some coefficient of proportionality (susceptibility), which depens on the field strength.

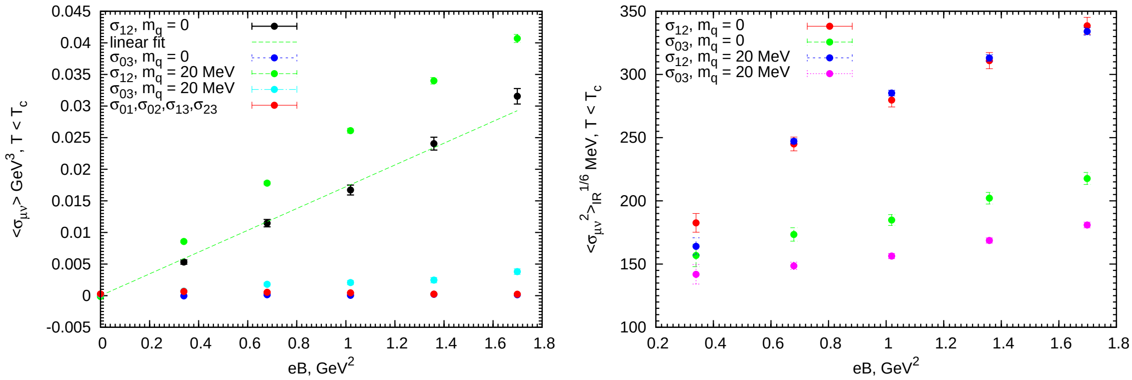
This quantity was introduced in[20] and can be used to measure spin polarization of the quarks in external magnetic field. The magnetization can be described by the dimensionless quantity $\mu = \chi \cdot qB$ so that

$$\langle \bar{\Psi} \sigma_{12} \Psi \rangle = \mu \langle \bar{\Psi} \Psi \rangle \tag{4.2}$$

The expectation value (4.1) can be calculated on the lattice by (2.1) with $\mathscr{O} = \sigma_{\alpha\beta}$. The result is shown on Fig.2(a) (here for comparison we also plot series for some finite quark mass). We can see, that the 12-component grows linearly with the field, which agrees with[20]. This allows us to find the chiral susceptibility $\chi(0) \equiv \chi_0$ through the slope of the dependence. After making a linear approximation $\langle \bar{\Psi} \sigma_{12} \Psi \rangle = \Omega^{fit} eB$, where[1]

$$\Omega^{fit} \equiv -\frac{1}{3} \chi_0^{fit} \Sigma_0 \tag{4.3}$$

[1] in our simulation we calculate the magnetization of the d-quark condensate, thus $q = \left|-\frac{e}{3}\right|$



(a) (b)

Figure 2: Expectation values of $\bar{\Psi}\sigma_{\alpha\beta}\Psi$ and its square

we obtain $\Omega^{fit} = (172.3 \pm 0.5) MeV$ and extract χ_0^{fit} (see below.)

The corresponding theoretical value can be expressed as[21]:

$$\chi_0^{th} = -\frac{c_\chi N_c}{8\pi^2 F_\pi^2}, \tag{4.4}$$

where c_χ is a dimensionless parameter, which according to the pion dominace idea[21] can be chosen as $c_\chi = 2$. $F_\pi = 130.7 MeV$ is the pion decay constant for $N_c = 3$. Comparing this value with our result we find a good agreement:

$$\chi_0^{th} \simeq -4.46 GeV^{-2}, \qquad \chi_0^{fit} \simeq -4.24 GeV^{-2} \tag{4.5}$$

The other interesting phenomenological quantity is the product of the chiral susceptibility χ and the condensate $\langle \bar{\Psi}\Psi \rangle$[22]. In our calculations it is equal to

$$-\chi_0^{fit} \langle \bar{\Psi}\Psi \rangle \simeq 52 MeV, \tag{4.6}$$

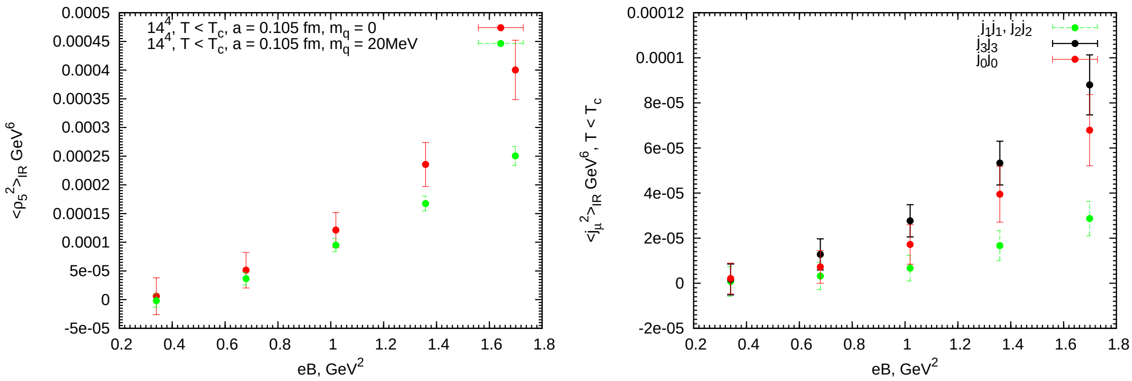
while from the QCD sum rules one can estimate this quantity as approximately 50 MeV [23], which is also close to our value.

5. Electric dipole moment

Other interesting effect of the magnetic field is that it induces a local electric dipole moment along the field[9]. This quantity corresponds to the $i0$-components of the (4.1):

$$d_i(x) \equiv \bar{\Psi}(x)\sigma_{i0}\Psi(x), \qquad i = \overline{1,3} \tag{5.1}$$

In the real CP-invariant vacuum the VEV of this quantity should be zero: $\langle d_i(x) \rangle = 0$, what we actually can see in our computations (Fig.2(a)). Let us show, that fluctuations of $d_i(x)$ can be sufficiently strong. For this purpose we measure VEV's (2.2) with $\mathcal{O}_1 = \mathcal{O}_2 = \sigma_{\alpha\beta}$. In the case of $i0$-components it corresponds to dispersions of $\vec{d}$. The result is shown on Fig.2(b), where we see, that longitudinal fluctuations of the local dipole moment grow with the field strenght, while



(a) (b)

Figure 3: Fluctuations of the chirality and electromagnetic current/charge

transverse fluctuations are absent. Here and after we use the “IR” subscript to emphasize, that we substract from the quantity its value at $B = 0$:

$$\langle Y \rangle_{IR}(B) = \frac{1}{V}\int\limits_V d^4x \langle Y(x)\rangle_B - \frac{1}{V}\int\limits_V d^4x \langle Y(x)\rangle_{B=0} \tag{5.2}$$

This procedure removes all UV-divergences and provides us with a quantity, which is insensitive to the UV-cutoff[10].

6. Some evidences of the Chiral Magnetic Effect

The nontrivial topological structure of QCD causes some unusual effects in the presence of strong magnetic field. One example of a such effect is the Chiral Magnetic Effect (CME), which generates an electric current along the field in the presence of the nontrivial gluonic background[1, 2]. This effect has been observed by the STAR collaboration at RHIC[3, 5] in heavy-ion collisions as a non-statistical asymmetry between numbers of positive and negative particles emmited on different sides of the reaction plane. This asymmetry can be interpreted as existing of a local imbalance between left- and right-handed quarks due to the quantum fluctuations of topological charge. An explanation of the effect and $SU(2)$ lattice studies can be found in[10, 24]. Here we implement the same procedure for the $SU(3)$ case and study the local chirality

$$\rho_5 = \overline{\Psi}(x)\gamma_5\Psi(x) \equiv \rho_L(x) - \rho_R(x) \tag{6.1}$$

and the electromagnetic current

$$j_\mu = \overline{\Psi}(x)\gamma_\mu\Psi(x). \tag{6.2}$$

The expectation value of the first one can be computed by (2.1) with $\mathscr{O} = \gamma_5$ and with $\mathscr{O} = \gamma_\mu$ for the second one. The both VEV’s are zero, as expected, but the corresponding fluctuations obtained from (2.2) are finite and grow with the field (see Fig.3).

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