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Consistent Higher-Order Corrections to $\tilde{t}_i \to \tilde{b}_j H^+$ in the Complex MSSM

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We review an analysis of a consistent renormalization of the top and bottom quark/squark sector of the MSSM with complex parameters (cMSSM). Various renormalization schemes are defined, analyzed analytically and tested numerically in the decays $\tilde{t}_2 \to \tilde{b}_i H^+/W^+$ $(i = 1,2)$. No scheme is found that produces numerically acceptable results over all the cMSSM parameter space, where problems occur mostly already for real parameters. Some numerical examples for $\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$ in our preferred scheme, "m_b, A_b $\overline{\mathrm{DR}}$" are shown.

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1. Introduction

One of the main tasks of the LHC is to search for Supersymmetry (SUSY) [1]. The Minimal Supersymmetric Standard Model (MSSM) predicts two scalar partners for all Standard Model (SM) fermions as well as fermionic partners to all SM bosons. Of particular interest are the scalar partners of the heavy SM quarks, the scalar top quarks, $\tilde{t}_i$ ($i = 1,2$) and scalar bottom quarks $\tilde{b}_j$ ($j = 1,2$) due to their large Yukawa couplings. Depending on the SUSY mass patterns, possibly important decay modes of the scalar tops are,

$$\tilde{t}_i \to \tilde{b}_j H^+ \quad (i,j = 1,2)\ , \tag{1.1}$$

$$\tilde{t}_i \to \tilde{b}_j W^+ \quad (i,j = 1,2)\ , \tag{1.2}$$

where H^+ denotes the (positively) charged MSSM Higgs boson. These processes can constitute a large part of the total stop decay width, and, in case of decays to a Higgs boson, they can serve as a source of charged Higgs bosons in cascade decays at the LHC.

For a precise prediction of the partial decay widths corresponding to Eq. (1.1) and Eq. (1.2), at least the one-loop level contributions have to be taken into account. This in turn requires a renormalization of the relevant sectors, especially a simultaneous renormalization of the top and bottom quark/squark sector. Due to the $SU(2)_L$ invariance of the left-handed scalar top and bottom quarks, these two sectors cannot be treated independently. Within the framework of the MSSM with complex parameters (cMSSM) we review the analysis of various bottom quark/squark sector renormalization schemes [2], while for the top quark/squark sector a commonly used on-shell renormalization scheme is applied throughout all the investigations. An extensive list of earlier analyses and corresponding references can be found in Ref. [2]. The evaluation of the partial decay widths of the scalar top quarks are being implemented into the Fortran code `FeynHiggs` [3–6].

2. The bottom/sbottom sector and its renormalization

2.1 The generic structure

The bilinear part of the Lagrangian with top and bottom squark fields, $\tilde{t}$ and $\tilde{b}$,

$$\mathscr{L}_{\tilde{t}/\tilde{b}\ \text{mass}} = -\left(\tilde{t}_L^\dagger, \tilde{t}_R^\dagger\right) \mathbf{M}_{\tilde{t}} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} - \left(\tilde{b}_L^\dagger, \tilde{b}_R^\dagger\right) \mathbf{M}_{\tilde{b}} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}\ , \tag{2.1}$$

contains the stop and sbottom mass matrices $\mathbf{M}_{\tilde{t}}$ and $\mathbf{M}_{\tilde{b}}$, given by

$$\mathbf{M}_{\tilde{q}} = \begin{pmatrix} M_{\tilde{Q}_L}^2 + m_q^2 + M_Z^2 c_{2\beta} (T_q^3 - Q_q s_\mathrm{w}^2) & m_q X_q^* \\ m_q X_q & M_{\tilde{q}_R}^2 + m_q^2 + M_Z^2 c_{2\beta} Q_q s_\mathrm{w}^2 \end{pmatrix} \tag{2.2}$$

with $X_q = A_q - \mu^* \kappa$ and $\kappa = \{\cot\beta, \tan\beta\}$ for $q = \{t, b\}$. $M_{\tilde{Q}_L}^2$ and $M_{\tilde{q}_R}^2$ are the soft SUSY-breaking mass parameters. m_q is the mass of the corresponding quark. Q_q and T_q^3 denote the charge and the isospin of q, and A_q is the trilinear soft SUSY-breaking parameter. The mass matrix can be diagonalized with the help of a unitary transformation $\mathbf{U}_{\tilde{q}}$,

$$\mathbf{D}_{\tilde{q}} = \mathbf{U}_{\tilde{q}} \mathbf{M}_{\tilde{q}} \mathbf{U}_{\tilde{q}}^\dagger = \begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix}\ , \qquad \mathbf{U}_{\tilde{q}} = \begin{pmatrix} U_{\tilde{q}_{11}} & U_{\tilde{q}_{12}} \\ U_{\tilde{q}_{21}} & U_{\tilde{q}_{22}} \end{pmatrix}\ . \tag{2.3}$$

The scalar quark masses, $m_{\tilde{q}_1}$ and $m_{\tilde{q}_2}$, will always be mass ordered, i.e. $m_{\tilde{q}_1} \leq m_{\tilde{q}_2}$:

$$m^2_{\tilde{q}_{1,2}} = \frac{1}{2}\left(M^2_{\tilde{Q}_L} + M^2_{\tilde{q}_R}\right) + m_q^2 + \frac{1}{2} T_q^3 M_Z^2 c_{2\beta} \mp \frac{1}{2}\sqrt{\left[M^2_{\tilde{Q}_L} - M^2_{\tilde{q}_R} + M_Z^2 c_{2\beta}(T_q^3 - 2Q_q s_\mathrm{w}^2)\right]^2 + 4m_q^2|X_q|^2} \,. \tag{2.4}$$

2.2 Renormalization of the bottom/sbottom sector

The field renormalization constants of the bottom/sbottom (as well as of the top/stop) sector are chosen according to an on-shell prescription [2].

The parameter renormalization can be performed as follows,

$$\mathbf{M}_{\tilde{q}} \to \mathbf{M}_{\tilde{q}} + \delta\mathbf{M}_{\tilde{q}} \tag{2.5}$$

which means that the parameters in the mass matrix $\mathbf{M}_{\tilde{q}}$ are replaced by the renormalized parameters and a counterterm. After the expansion $\delta\mathbf{M}_{\tilde{q}}$ contains the counterterm part,

$$\delta\mathbf{M}_{\tilde{q}_{11}} = \delta M^2_{\tilde{Q}_L} + 2m_q \delta m_q - M_Z^2 c_{2\beta}\, Q_q\, \delta s_\mathrm{w}^2 + (T_q^3 - Q_q s_\mathrm{w}^2)(c_{2\beta}\, \delta M_Z^2 + M_Z^2\, \delta c_{2\beta}) \,, \tag{2.6}$$

$$\delta\mathbf{M}_{\tilde{q}_{12}} = (A_q^* - \mu\kappa)\, \delta m_q + m_q(\delta A_q^* - \mu\, \delta\kappa - \kappa\, \delta\mu) \,, \tag{2.7}$$

$$\delta\mathbf{M}_{\tilde{q}_{21}} = \delta\mathbf{M}^*_{\tilde{q}_{12}} \,, \tag{2.8}$$

$$\delta\mathbf{M}_{\tilde{q}_{22}} = \delta M^2_{\tilde{q}_R} + 2m_q \delta m_q + M_Z^2 c_{2\beta}\, Q_q\, \delta s_\mathrm{w}^2 + Q_q s_\mathrm{w}^2 (c_{2\beta}\, \delta M_Z^2 + M_Z^2\, \delta c_{2\beta}) \,. \tag{2.9}$$

Another possibility for the parameter renormalization is to start out with the physical parameters which corresponds to the replacement:

$$\mathbf{U}_{\tilde{q}}\, \mathbf{M}_{\tilde{q}}\, \mathbf{U}_{\tilde{q}}^\dagger \to \mathbf{U}_{\tilde{q}}\, \mathbf{M}_{\tilde{q}}\, \mathbf{U}_{\tilde{q}}^\dagger + \mathbf{U}_{\tilde{q}}\, \delta\mathbf{M}_{\tilde{q}}\, \mathbf{U}_{\tilde{q}}^\dagger = \begin{pmatrix} m^2_{\tilde{q}_1} & Y_q \\ Y_q^* & m^2_{\tilde{q}_2} \end{pmatrix} + \begin{pmatrix} \delta m^2_{\tilde{q}_1} & \delta Y_q \\ \delta Y_q^* & \delta m^2_{\tilde{q}_2} \end{pmatrix} \,, \tag{2.10}$$

where $\delta m^2_{\tilde{q}_1}$ and $\delta m^2_{\tilde{q}_2}$ are the counterterms of the squark masses squared. δY_q is the counterterm[1] to the squark mixing parameter Y_q (which vanishes at tree level, $Y_q = 0$, and corresponds to the off-diagonal entries in $\mathbf{D}_{\tilde{q}} = \mathbf{U}_{\tilde{q}}\, \mathbf{M}_{\tilde{q}}\, \mathbf{U}_{\tilde{q}}^\dagger$, see Eq. (2.3)). Using Eq. (2.10) one can express $\delta\mathbf{M}_{\tilde{q}}$ by the counterterms $\delta m^2_{\tilde{q}_1}$, $\delta m^2_{\tilde{q}_2}$ and δY_q. Especially for $\delta\mathbf{M}_{\tilde{q}_{12}}$ one yields

$$\delta\mathbf{M}_{\tilde{q}_{12}} = U^*_{\tilde{q}_{11}} U_{\tilde{q}_{12}} (\delta m^2_{\tilde{q}_1} - \delta m^2_{\tilde{q}_2}) + U^*_{\tilde{q}_{11}} U_{\tilde{q}_{22}}\, \delta Y_q + U_{\tilde{q}_{12}} U^*_{\tilde{q}_{21}}\, \delta Y_q^* \,. \tag{2.11}$$

For the top/stop sector we use an on-shell renormalization, see e.g. Refs. [2, 7, 8]. The various options to renormalize the bottom/sbottom sector are listed in Tab. 1.

2.3 Summary of the renormalization scheme analysis

A bottom quark/squark sector renormalization scheme always contains dependent counterterms which can be expressed by the independent ones. According to our six definitions, these dependent parameters can be δm_b, δA_b or δY_b. A problem can occur when the MSSM parameters are chosen such that the independent counterterms (nearly) drop out of the relation determining the

[1] The unitary matrix $\mathbf{U}_{\tilde{q}}$ can be expressed by a mixing angle $\theta_{\tilde{q}}$ and a corresponding phase $\varphi_{\tilde{q}}$. Then the counterterm δY_q can be related to the counterterms of the mixing angle and the phase (see Ref. [7]).

scheme	$m_{\tilde{b}_{1,2}}$	m_b	A_b	Y_b	name
analogous to the $t/\tilde{t}$ sector: "OS"	OS	OS		OS	RS1
"m_b, A_b $\overline{\text{DR}}$"	OS	$\overline{\text{DR}}$	$\overline{\text{DR}}$		RS2
"m_b, Y_b $\overline{\text{DR}}$"	OS	$\overline{\text{DR}}$		$\overline{\text{DR}}$	RS3
"m_b $\overline{\text{DR}}$, Y_b OS"	OS	$\overline{\text{DR}}$		OS	RS4
"A_b $\overline{\text{DR}}$, ReY_b OS"	OS		$\overline{\text{DR}}$	ReY_b: OS	RS5
"A_b vertex, ReY_b OS"	OS		vertex	ReY_b: OS	RS6

Table 1: Summary of the six renormalization schemes for the $b/\tilde{b}$ sector investigated in Ref. [2]. Blank entries indicate dependent quantities. ReY_b denotes that only the real part of Y_b is renormalized on-shell, while the imaginary part is a dependent parameter.

dependent counterterms. This can lead to (unphysically) large counterterm contributions in such a case. As it was shown in Ref. [2] it is possible already in very generic SUSY scenarios to find a set of MSSM parameters which show this behaviour for each of the chosen renormalization schemes. Consequently, it appears to be difficult *by construction* to define a renormalization scheme for the bottom quark/squark sector (once the top quark/squark sector has been defined) that behaves well for the full MSSM parameter space. One possible exception could be a pure $\overline{\text{DR}}$ scheme, which, however, is not well suited for processes with external top squarks and/or bottom squarks.

The analytical and numerical analysis performed in Ref. [2] identfied RS2 as "preferred scheme". This schemes showed the "relatively most stable" behavior, problems only occur for maximal sbottom mixing, $|U_{\tilde{b}_{11}}| = |U_{\tilde{b}_{12}}|$, where a divergence in δY_b appears. On the other hand, other schemes with δm_b or δA_b as dependent counterterms generally exhibit problems in larger parts of the parameter MSSM space and may induce large effects, since m_b (or the bottom Yukawa coupling) and A_b enter prominently into the various couplings of the Higgs bosons to other particles.

3. Numerical Example

In this section we show some example results for $\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$ [2]. This decay mode can serve potentially as a source of charged MSSM Higgs bosons in SUSY cascade decays. The parameters are chosen according to the two scenarios S1 and S2 as defined in Tab. 2.

In Fig. 1 we show the partial decay width $\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$ as a function of $\tan\beta$ (upper left), as a function of A_b (upper right), as a function of μ (lower left) and as a function of φ_{A_b} (lower right plot). "tree" denotes the tree-level value and "full" is the decay width including all one-loop corrections (including hard QED and QCD radiation, see Ref. [2] for details)[2]. For S1 the grey region is excluded and for S2 the dark grey region is excluded. The spikes and dips visible in the lower left plot are due to various particle thresholds, while the first dip in S1 is due to $|U_{\tilde{b}_{11}}| \approx |U_{\tilde{b}_{12}}|$.

[2]Corrections from imaginary parts of external leg self-energy contributions [10] are not included.

Scen.	$M_{H^\pm}$	$m_{\tilde{t}_2}$	μ	A_t	A_b	M_1	M_2	M_3
S1	150	600	200	900	400	200	300	800
S2	180	900	300	1800	1600	150	200	400

Table 2: MSSM parameters for the initial numerical investigation; all parameters are in GeV. We always set $m_b^{\overline{\rm MS}}(m_b) = 4.2$ GeV. In our analysis we use $M_{\tilde{Q}_L}(\tilde{t}) = M_{\tilde{t}_R} = M_{\tilde{b}_R} =: M_{\rm SUSY}$, where $M_{\rm SUSY}$ is chosen such that the above value of $m_{\tilde{t}_2}$ is realized. The parameters entering the scalar lepton sector and/or the first two generations do not play a relevant role in our analysis. The values for A_t and A_b are chosen such that charge- or color-breaking minima are avoided.

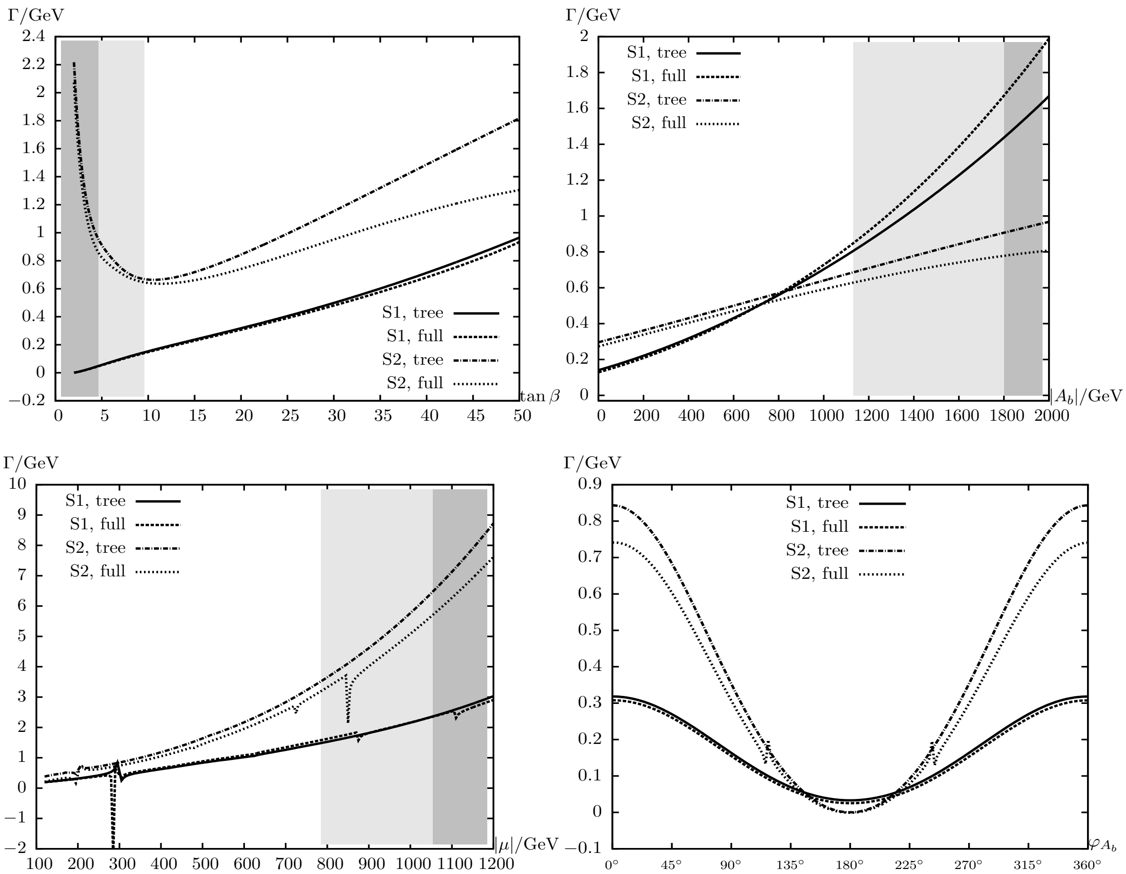

Figure 1: $\Gamma(\tilde{t}_2 \to \tilde{b}_1 H^+)$. Tree-level and full one-loop corrected partial decay widths for the renormalization scheme RS2. The parameters are chosen according to the scenarios S1 and S2. For S1 the grey region is excluded and for S2 the dark grey region is excluded. Upper left plot: $\tan\beta$ varied. Upper right plot: $\tan\beta = 20$ and $|A_b|$ varied. Lower left plot: $\tan\beta = 20$ and $|\mu|$ varied. Lower right plot: $\tan\beta = 20$ and φ_{A_b} varied.

The two spikes in the lower right plot are also due to $|U_{\tilde{b}_{11}}| \approx |U_{\tilde{b}_{12}}|$, which leads to a divergence in RS2, which, however, is confined to very narrow intervals. The loop corrections, as can be observed in all four plots, are relatively modest, staying below $\sim 25\%$ for all parameters. The fact of relatively small one-loop corrections shows that no unphysically large contributions via large counterterms are introduced, a characteristic of a suitable renormalization scheme.

The real quantity of interest at the LHC is the $\mathrm{BR}(\tilde{t}_2 \to \tilde{b}_1 H^+)$. This, however, requires the evaluation of *all* decay modes (at the same level of accuracy). The corresponding results will be presented elsewhere [11].

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