

## Neutrino Physics: a theoretical review

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I briefly review the status of neutrino physics from a theoretical point of view. After giving updated results from a global fit to neutrino oscillation data I discuss the most important open questions in neutrino phenomenology. I highlight the puzzling observations that neutrinos are so much lighter than all other fermions and that flavour mixing is very different in the lepton and quark sectors. I present a few approaches on how to extend the Standard Model in order to give mass to neutrinos including high- and low-scale seesaw mechanisms. The discussion is highly biased and does not aim of being complete.

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## 1. Introduction

Experimental results accumulated over the last 12 years or so provide convincing evidence for neutrino oscillations [1]. This implies that at least two out of the three neutrinos in the Standard Model (SM) have to have non-zero mass. In the presence of a mass term for neutrinos, one expects that the neutrino fields participating in the charged current (CC) interaction, will be superpositions of the fields with definite mass. Hence, there will be mixing in the lepton sector, in the same way as CKM mixing in the quark sector. Let us consider, to be specific, a Majorana mass term for neutrinos, together with the mass term for charged leptons:

$$\mathcal{L}_M = -\frac{1}{2} \sum_{i=1}^3 \bar{\nu}_{iL}^T C^{-1} \nu_{iL} m_i^{\nu} - \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha R} \ell_{\alpha L} m_{\alpha}^{\ell} + \text{h.c.} \quad (1.1)$$

Then in general the CC interaction is not diagonal in the basis of the neutrino mass eigenfields  $\nu_1, \nu_2, \nu_3$ , used in eq. 1.1:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W^{\rho} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^3 \bar{\nu}_{iL} U_{\alpha i}^* \gamma_{\rho} \ell_{\alpha L} + \text{h.c.}, \quad (1.2)$$

where  $(U_{\alpha i}) \equiv U_{\text{PMNS}}$  is the Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix [2]. Note that the ‘‘effective’’ mass term in equation 1.1 violates gauge invariance. Therefore the SM has to be extended in order to give mass to neutrinos in a consistent way. In particular, it is necessary to add new degrees of freedom to the SM, such as right-handed neutrinos or new scalar representations.

The conventional parameterization [3] for the lepton mixing matrix is  $U_{\text{PMNS}} = V^{\text{Dirac}} D^{\text{Maj}}$ , with

$$V^{\text{Dirac}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1.3)$$

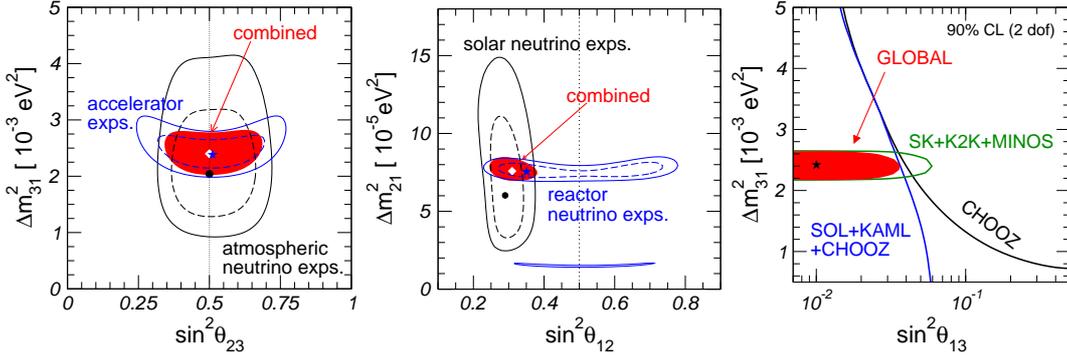
where  $s_{ij} \equiv \sin \theta_{ij}$  and  $c_{ij} \equiv \cos \theta_{ij}$ . The three mixing angles are given by

$$\tan \theta_{23} = \frac{|U_{\mu 3}|}{|U_{\tau 3}|} \quad \sin \theta_{13} = |U_{e 3}| \quad \tan \theta_{12} = \frac{|U_{e 2}|}{|U_{e 1}|}. \quad (1.4)$$

There is one Dirac phase  $\delta$  which leads to CP violation in neutrino oscillations. Furthermore, if neutrinos are Majorana particles there are two physical Majorana phases in  $D^{\text{Maj}} \equiv \text{diag}(1, e^{i\alpha/2}, e^{i\beta/2})$ , which have no effect in neutrino oscillations, but appear in lepton-number violating processes such as neutrino-less double beta decay.

## 2. Present status of neutrino oscillations

Neutrino oscillation is a quantum mechanical phenomenon similar to interferometry, which leads to the transition between different neutrino flavours, i.e., a neutrino originating from a CC interaction together with a charged lepton  $\ell_{\alpha}$  can produce a charged lepton  $\ell_{\beta}$  in a CC reaction in a detector separated by some distance  $L$  from the production point. This effect occurs if the lepton mixing matrix is non-trivial, if neutrino masses are non-degenerate, and if the experimental



**Figure 1:** Determination of oscillation parameters from the interplay of global data [5].

conditions are such that the phase  $\Delta m_{ij}^2 L / 2E_\nu \gtrsim 1$ , where  $\Delta m_{ij}^2$  is the mass-squared difference between two neutrino mass states and  $E_\nu$  is the neutrino energy. A recent discussion of quantum mechanical aspects of neutrino oscillations can be found in Ref. [4].

The determination of the three-flavour neutrino oscillation parameters from the global data is illustrated in Fig. 1. From solar and atmospheric neutrino oscillations we know that two out of the three mixing angles are large, and in the case of  $\theta_{23}$  even consistent with maximal mixing:<sup>1</sup>

$$\sin^2 \theta_{12} = 0.318_{-0.016}^{+0.019}, \quad \sin^2 \theta_{23} = 0.51_{-0.07}^{+0.08}, \quad (2.1)$$

The third mixing angle,  $\theta_{13}$ , is constrained to be small from the combined global data, with an important constraint from the CHOOZ reactor experiment:

$$\sin^2 \theta_{13} = 0.013_{-0.010}^{+0.013}, \quad \sin^2 \theta_{13} \leq 0.031 \text{ (0.047)} \quad \text{at 90\% CL (3}\sigma\text{)}. \quad (2.2)$$

Hence there is a slight hint for a non-zero value of  $\theta_{13}$  (at about  $1.5\sigma$ , depending somewhat on assumptions concerning the solar model) which, however, is not statistically significant. It is interesting to compare the current upper bound on the smallest element in the PMNS matrix,  $|U_{e3}| = \sin \theta_{13} \leq 0.217$  at  $3\sigma$ , to the largest off-diagonal element of the CKM matrix,  $|V_{us}| = 0.2252 \pm 0.0009$  [3], which are of comparable size.

Neutrino oscillations are sensitive to the mass-squared differences of neutrinos,  $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ , where  $m_i$  is the mass of the neutrino mass eigenstate  $\nu_i$ . Hence, no information can be obtained on the absolute scale of neutrino masses from oscillations.  $\Delta m^2$ 's are determined best by the energy spectrum in the KamLAND reactor experiment ( $\Delta m_{21}^2$ ) and the MINOS long-baseline accelerator experiment ( $\Delta m_{31}^2$ ), see fig. 1:

$$\Delta m_{21}^2 = 7.59_{-0.18}^{+0.23} \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = \left\{ \begin{array}{c} 2.47 \\ -2.36 \end{array} \right\} \pm 0.10 \times 10^{-3} \text{ eV}^2. \quad (2.3)$$

By convention  $m_1 < m_2$  and hence  $\Delta m_{21}^2 > 0$ . Thanks to the matter effect inside the sun we know that  $\theta_{12} < 45^\circ$ , which together with the fact that  $\theta_{13}$  is small implies that the mass state  $\nu_1$  contains dominantly the electron neutrino. On the other hand current data does not allow to determine the

<sup>1</sup>Values for oscillation parameters given here are based on updated results from [5] (errors at  $1\sigma$ ).

sign of  $\Delta m_{31}^2$ . Therefore either  $\nu_1$  or  $\nu_3$  could be the lightest neutrino, which is referred to as normal (NH) or inverted (IH) hierarchy, respectively. These two possibilities correspond to the two values for  $\Delta m_{31}^2$  given in eq. 2.3. Note that the absolute values of  $\Delta m_{31}^2$  are slightly different for the two mass orderings. Generic three-flavour effects such as CP violation are suppressed by the small quantities  $\Delta m_{21}^2/|\Delta m_{31}^2| \approx 0.03$  and  $\theta_{13}$ .

### 3. The next steps

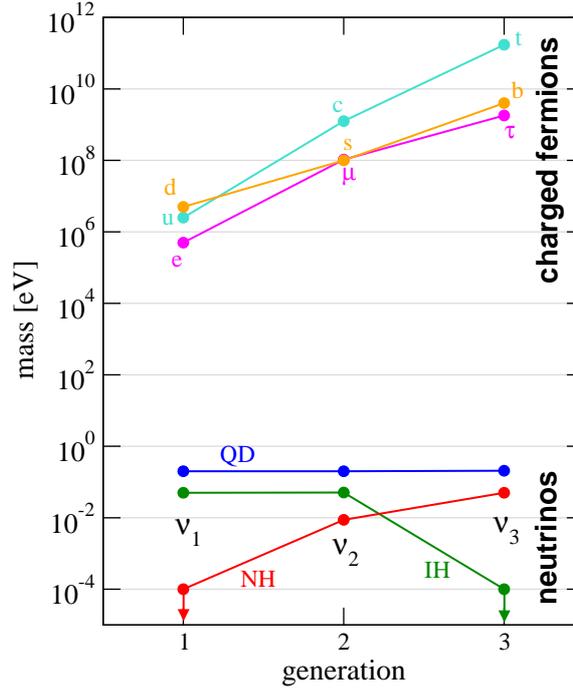
The next important goal in neutrino oscillations is the determination of the mixing angle  $\theta_{13}$ . There are 5 experiments under way with  $\theta_{13}$  being their primary target, the three reactor experiments Daya Bay, Double Chooz, and RENO, looking at  $\bar{\nu}_e$  disappearance at distances of order 1 km, and the two accelerator experiments T2K and NOvA, searching for  $\nu_\mu \rightarrow \nu_e$  transitions due to  $\Delta m_{31}^2$ . These experiments will explore the range  $\sin^2 \theta_{13} \gtrsim 0.0025$  in the next couple of years, about one order of magnitude smaller than the present bound, see [6, 7]. Once a finite value of  $\theta_{13}$  is established one may aim for the ultimate goals of neutrino oscillation experiments, namely the search for CP violation in neutrino oscillations and the determination of the neutrino mass hierarchy. This will require a next generation long-baseline neutrino experiment such as an upgraded super-beam, a beta beam, or a neutrino factory, see [6].

Neutrino oscillations are not sensitive to the absolute value of neutrino mass. Currently the strongest constraint comes from cosmology, bounding the sum of neutrino masses to be less than about 0.5 eV. The upcoming KATRIN experiment searches for kinematical effects of neutrino mass on the endpoint of the electron spectrum in Tritium beta decay and aims at a sensitivity of about 0.2 eV.

Important information on the nature of neutrinos will come from neutrino-less double beta decay experiments [8]. These experiments search for the transition of a nucleus  $(A, Z)$  to  $(A, Z+2)$  accompanied by the emission of two electrons (but no neutrino). This is a lepton number violating process (it violates lepton number by two units), an observation of which would be a ground breaking discovery. This process can be mediated by neutrino Majorana masses or some other lepton number violating mechanism, such as R-parity violating supersymmetry, right-handed  $W_R$  bosons, or doubly-charged scalars. Although these latter mechanisms may be unrelated to neutrino masses themselves, one can prove the following statement [9, 10]. If neutrino-less double beta decay is observed it is not possible to forbid a Majorana mass term for neutrinos by any symmetry. Therefore, in a “natural theory” (without cancellations of loop induced terms) a Majorana mass for neutrinos will be generated at some level, if neutrino-less double beta decay is observed.

### 4. Why are neutrino masses so small?

Since oscillation data tell us that neutrinos cannot be massless the question arises, why neutrinos are so much lighter than any other fermion in the Standard Model. Fig. 2 illustrates the fermion masses in the Standard Model. Reading this figure vertically, one finds that neutrinos have to be at least 6 orders of magnitude lighter than the charged fermions of the same generation. When looking at the figure horizontally, it appears that—especially for IH or quasi-degenerate (QD) neutrinos—the mass pattern between generations of neutrinos may be very different than for charged fermions.



**Figure 2:** Fermion masses in the Standard Model.

In the SM neutrinos are massless since (i) the absence of right-handed neutrinos prevents a Dirac mass term for neutrinos, and (ii) given the field content and the gauge structure of the SM lepton number ( $L$ ) is conserved (or  $B - L$  at the quantum level), which prevents the appearance of a Majorana mass. If right-handed neutrinos are added to the SM neutrinos could be Dirac particles. However, in the presence of right-handed neutrinos (which are true singlets under the SM gauge group) lepton number is no-longer automatically conserved and one has to impose this as an additional constraint on the theory. Therefore, naively it appears to be more natural that neutrinos have Majorana mass. Indeed, maybe the special properties of neutrinos mentioned above could be related to the fact that they are Majorana particles, which is not possible for charged fermions.

Assuming that there is some new physics at a high energy scale  $\Lambda$ , at low energy one expects this new physics to show up in the form of non-renormalizable operators of dimension  $d > 4$ , suppressed by powers of  $\Lambda^{4-d}$ . It has been noted by Weinberg in 1979 [11] that while there are many operators at dimension 6, there is only one gauge invariant dimension-5 operator in the Standard Model, and this operator leads to a Majorana neutrino mass after electroweak symmetry breaking:

$$Y^2 \frac{(L^T \tilde{\phi}^*)(\tilde{\phi}^\dagger L)}{\Lambda} \longrightarrow m_\nu \sim Y^2 \frac{v^2}{\Lambda}, \quad (4.1)$$

where  $L$  is a lepton doublet,  $\phi$  is the Higgs doublet,  $v = 246$  GeV is the VEV of the Higgs, and  $Y$  is a matrix of Yukawa couplings. Flavour indices are implicit. If  $\Lambda \gg v$  one obtains a suppression of neutrino masses relative to the electroweak scale which in this context is usually called “seesaw mechanism”. At tree-level there are three possible UV completions of this operator by introducing different  $SU(2)$  representations of mediator particles, namely singlet fermions (right-handed

neutrinos), triplet scalars, or triplet fermions. These possibilities are referred to as type-I [12], type-II [13], and type-III [14] seesaw, respectively. Of course the immediate question is, what is the scale of the new physics  $\Lambda$ ?

#### 4.1 Neutrino masses from the GUT scale?

When we take Yukawa couplings of order 1 in eq. 4.1 one finds that neutrino masses  $m_\nu \lesssim 0.1$  eV are induced by  $\Lambda \sim 10^{14}$  GeV. Unfortunately this scale is impossible to probe directly in experiments, however, it is somewhat close to the scale of Grand Unification,  $\Lambda_{\text{GUT}} \sim 10^{16}$  GeV, which might suggest a GUT origin of neutrino masses.

In particular, it is tempting that all Standard Model fermions of one generation plus a right-handed neutrino fit into the spinorial 16 dimensional representation of SO(10). Furthermore, typically in SO(10) theories scalar representations containing a triplet under SU(2) are needed. Therefore, SO(10) theories are an attractive framework to implement type-I and/or type-II seesaw mechanism.

One predictive example is the so-called minimal renormalizable supersymmetric SO(10) GUT, where all fermion masses are generated by the VEVs of 10- and 126-dimensional Higgses, see e.g. [15, 16, 17, 18]. This leads to a constrained system of sum-rules between quark and lepton mass matrices and gives specific predictions for neutrino mass and mixing parameters. It turns out, however, that in the minimal version of this model the fact that the seesaw scale has to be somewhat lower than the GUT scale, leads to the appearance of pseudo-Goldstone bosons which spoil the unification of gauge coupling constants [18]. This suggests that modifications of this minimal setup are necessary. Certainly there exist many alternative setups based on SO(10), for example allowing for non-renormalizable operators [19], or modified assignment of fermion representations [20].

Another attractive feature of the high-scale seesaw mechanism is Leptogenesis [21]. A net-lepton number is created at a high temperature in the early universe due to CP violating out-of-equilibrium processes of some heavy leptons (for example the decay of the right-handed neutrinos, in the case of type-I seesaw). This lepton number is later transferred to baryons by  $B-L$  conserving sphaleron processes, and accounting in that way for the small excess of matter over anti-matter. The typical Leptogenesis scale is about  $10^{10}$  GeV, although it might also be possible to lower this scale down to the TeV range in certain scenarios. See [22] for a review.

#### 4.2 Neutrino masses from the TeV scale?

For various reasons (unrelated to neutrino mass) we expect new physics to show up at the TeV scale, to be probed in the near future at LHC. As an alternative to the high-scale seesaw mechanism mentioned above, one can also invoke a neutrino mass mechanism at the TeV scale, which opens the fascinating possibility to test neutrino masses at collider experiments. However, for  $\Lambda \sim \text{TeV}$  the seesaw suppression is not sufficient and one needs some additional ingredients to make neutrino masses small, for example, make Yukawa couplings small, invoke cancellations between large terms, or use loop-factor suppressions by generating neutrino mass radiatively.

What are the prospects of testing the three seesaw types at LHC? In case of type-I seesaw the mediators are singlets, and their only interaction with SM particles is via Yukawa interactions.

If the masses of the singlets are at the TeV scale, there are two possibilities to get small enough neutrino masses: (i) make the Yukawa couplings small,  $Y \lesssim 10^{-6}$ , which, however, leads to a negligible production rate at colliders; and (ii) invoke cancellations between the contributions of the different heavy singlets. In this case the leading-order structure of the mass matrices leads to vanishing light neutrino masses, and non-vanishing masses are generated by small perturbations, which however, cannot be probed by LHC, see [23] for a recent discussion and references.

A way out of this problem is to give the seesaw messengers additional interactions, allowing them to be produced efficiently at colliders. For instance, if the type-I seesaw is embedded into a left-right symmetric model at the TeV scale, the right-handed neutrinos can be produced via the  $W_R$  interaction [24]. In the cases of the type-II and type-III seesaws, the mediators are triplets under the SM SU(2) gauge symmetry. Therefore, they can be produced efficiently via gauge interactions if they are in the kinematical reach of LHC. In the type-II scenario the observation of same-sign dilepton events would open the phantastic possibility to directly probe the flavour structure of the neutrino mass matrix, including Majorana phases, see e.g. [25, 26, 27, 28] for recent studies. TeV scale Type-III seesaw has been discussed recently for example in [29, 30]. A review of seesaw tests at LHC can be found in Ref. [31].

I mention very briefly two other examples for TeV scale neutrino masses. (i) In R-parity violating supersymmetry neutrino masses are induced due to tiny lepton-number violating couplings. In these models the lightest supersymmetric particle is not stable. It decays via the couplings which are also responsible for neutrino mass, which allows interesting correlations of collider signals and the lepton mixing matrix, e.g. [32]. (ii) Neutrino masses can be generated radiatively, which provides additional suppression factors. For example in the Zee-Babu model [33] a singly and a doubly charged scalar are added to the SM, which allows neutrino masses to be induced at the two-loop level. If the scalars are in the kinematical reach of LHC this model is highly testable by correlating LHC signatures, charged lepton-flavour violation, and neutrino mixing parameters [34, 35, 36].

## 5. Lepton mixing and the problem of flavour

The results of neutrino oscillation experiments discussed in sec. 2 imply that the PMNS lepton mixing matrix looks very different than the CKM quark mixing matrix:

$$U_{\text{PMNS}} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \varepsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}, \quad U_{\text{CKM}} = \begin{pmatrix} 1 & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon \\ \varepsilon & \varepsilon & 1 \end{pmatrix}, \quad (5.1)$$

where  $\varepsilon \ll \varepsilon \ll 1$ . Indeed, the PMNS matrix is consistent with resulting from just random numbers in flavour space, so-called ‘‘anarchy’’ [37]. This hypothesis can be excluded in case  $\theta_{13}$  turns out to be much smaller than its present bound.

On the other hand, the numbers we observe for the mixing angles could also indicate a very special flavour structure. The so-called tri-bimaximal ansatz [38] for the mixing matrix assumes  $\sin^2 \theta_{12} = 1/3$ ,  $\sin^2 \theta_{23} = 1/2$ , and  $\theta_{13} = 0$ , which is consistent with present data within about one

sigma and results in

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (5.2)$$

This mixing matrix implies that the second mass state  $\nu_2$  is equally mixed with all flavours, whereas the third mass state  $\nu_3$  is equally mixed with  $\nu_\mu$  and  $\nu_\tau$  and has no  $\nu_e$  component. A mixing matrix with such special values could indicate an underlying flavour symmetry. There is a large literature on using discrete symmetries such as for example a  $\mu$ - $\tau$  exchange symmetry, or  $S_2, S_3, A_4, D_4, D_5, D_8, \Delta(27)$  groups to explain (some of) the observed values of the lepton mixing angles, see [39] for a review and references. In particular, the  $A_4$  permutation group is found to be very useful to predict tri-bimaximal mixing. It has three one-dimensional and one three-dimensional irreducible representation which can be assigned in various ways to the lepton representations; an overview of various  $A_4$  models can be found in [40].

In general it turns out that writing down a full model gets often quite involved. A “strong” breaking of the symmetry is required in the charged lepton sector, and typically the flavour symmetry has to be augmented by some mechanism of vacuum alignment. Furthermore, it seems difficult to extend the flavour symmetry to the quark sector, in particular in the context of unified theories.

## 6. Conclusions

Neutrinos have surprised us with a number of interesting properties. At present there is no clear understanding about how neutrino masses are generated and why the mixing pattern in the lepton sector is so different from the one in the quark sector. A wealth of experiments will provide important information on the issue in the upcoming years. These include neutrino oscillation experiments and experiments searching for lepton number violation as well as charged lepton flavour violation. Results of these experiments will have to be put in the context of the physics at the TeV energy scale to be discovered at the LHC. In this sense neutrino properties may provide another puzzle piece towards a more complete picture of the physics beyond the Standard Model.

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