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$|V_{ub}|$ and $|V_{cb}|$ from semileptonic B decays, and $B \to \tau\nu$

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We are entering an era where many of the important measurements in B physics will be made at hadron colliders. However, the determination of the transition strengths of weak b-quark decays to up or charm quarks is likely to remain within the realm of e^+e^- colliders. In this talk I review the experimental techniques used in studying semileptonic decays and in the determination of $|V_{ub}|$ and $|V_{cb}|$. The use of both inclusive and exclusive semileptonic decays provides a check on our understanding, as they are subject to different uncertainties both theoretically and experimentally. Unfortunately, a persistent $\sim 2\sigma$ discrepancy between the inclusive and exclusive determinations of both $|V_{ub}|$ and $|V_{cb}|$ suggests that more work is needed before we can claim to have a robust understanding of these fundamental parameters. I also present the current status of measurements of the leptonic decay $B \to \tau\nu$.

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1. Introduction

Our best tool for the determination of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ is the study of semileptonic B decays. These decays involve only a single hadronic current, which allows the inherent uncertainties due to QCD to be limited using an appropriate combination of theoretical tools and experimental measurements. Since $|V_{cb}|$ is roughly ten times as large as $|V_{ub}|$, decays involving charm in the final state dominate the overall semileptonic B decay rate. The CKM-suppressed $b \to u$ decays are harder to isolate experimentally, and suffer from higher backgrounds and more restrictive kinematic acceptances.

The description of semileptonic decays can be approached in two complementary ways. Exclusive decays, in which the final state hadron X in $\bar{B} \to X\ell\bar{\nu}$ is explicitly reconstructed, are described using Lorenz-invariant form factors that depend on the squared momentum q^2 transferred to the lepton pair and, for decays to final state hadrons with non-zero spin, on additional angular variables. The form factor normalization must be calculated using non-perturbative methods such as lattice QCD or light-cone sum rules (LCSR). At present, the decay modes $\bar{B} \to D^{(*)}\ell\bar{\nu}$ and $\bar{B} \to \pi\ell\bar{\nu}$ are the most relevant in determinations of $|V_{ub}|$ and $|V_{cb}|$. In contrast, inclusive decays, in which the nature of the final state hadron is not specified (i.e., where we sum over all final states X), are described using an operator product expansion (OPE) in α_S and Λ/m_b, where Λ is a scale that parametrizes non-perturbative effects. This allows for a systematic expansion in terms of parametrically small quantities, since $\alpha_S \sim 0.22$ at the relevant energy scale and since $\Lambda/m_b \sim 0.15$ and the first non-zero power correction occurs at order $(\Lambda/m_b)^2$. The OPE can be applied to both $b \to c$ and $b \to u$ transitions, but convergence is compromised in the restricted regions of phase space appropriate to certain measurements of inclusive $b \to u\ell\bar{\nu}$ decays.

Semileptonic decays are best measured at present in the B factories, where a $B\bar{B}$ pair is produced nearly at rest in the $\Upsilon(4S)$ frame and where $B\bar{B}$ production accounts for approximately 1/4 of the $e^+e^- \to$ hadrons cross-section. The decay products of the B mesons overlap, and the neutrino from the semileptonic B decay goes undetected. As a result, hadrons cannot be unambiguously associated with a semileptonic B decay unless the second B meson in the event is fully reconstructed. A range of experimental methods are in use:

Efficiency low⟶high			Purity high⟵low
	Fully inclusive lepton	any e or μ with sufficient momentum	
	Missing ν constraint	$|\cos\theta_{BY}| < 1$ for a single missing neutrino	
	Recoil methods	Semileptonic tag: assign particles to each B	
		Hadronic tag: determine B 4-vector	

The kinematic constraint ensuring consistency with a single neutrino is given by

$$\cos\theta_{BY} = \frac{2E_B E_Y - m_B^2 - m_Y^2}{2|\vec{p}_B||\vec{p}_Y|} \tag{1.1}$$

where Y refers to the hadronic system presumed to come from the semileptonic B decay. The cleanest measurements are done in events where the second B is fully reconstructed, but come at the cost of a low efficiency ($\sim 1\%$ for semileptonic tags, $\sim 0.3\%$ for hadronic tags). As a result, with current B factory statistics the more inclusive methods remain important.

2. Exclusive semileptonic decays

2.1 CKM-favored transitions

The differential decay rates for semileptonic transitions are proportional to the square of a CKM matrix element times a form factor. For example, consider the decay $\bar{B} \to D\ell\bar{\nu}$:[1]

$$\frac{d\Gamma(\bar{B} \to D\ell\bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (\mathscr{G}(w))^2 \Phi(w); \quad w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D}; \quad 1 \leq w < 1.59; \tag{2.1}$$

where $\Phi(w)$ is a known phase-space factor, $\mathscr{G}(w)$ is a form factor and w is the product of four-velocities of the initial and final hadrons. The point $w = 1$ corresponds to maximum q^2, where the D meson is at rest in the B frame. The form factor slope at $w = 1$ is given by the parameter $\rho^2_{\mathscr{G}}$. In the limit of heavy quark symmetry, $\mathscr{G}(1) = 1$. Parameterizations of the form factor that respect unitarity and analyticity are given in terms of a power series in the variable $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$, where $0 \leq z < 0.065$.

The exclusive decay $\bar{B} \to D\ell\bar{\nu}$ has recently been measured with significantly improved precision in two BABAR analyses. An analysis[2] of decays recoiling against fully reconstructed B mesons gives good separation between $\bar{B} \to D\ell\bar{\nu}$ and $\bar{B} \to D^*\ell\bar{\nu}$ and results in $\mathscr{G}(1)|V_{cb}| = (42.3 \pm 1.9_{\text{stat}} \pm 1.0_{\text{sys}}) \times 10^{-3}$ and $\rho^2_{\mathscr{G}} = 1.20 \pm 0.09_{\text{stat}} \pm 0.04_{\text{sys}}$. A global fit[3] in $(p_\ell, p_D, \cos\theta_{BY})$-space to $D\ell$ pairs from the decays $\bar{B} \to D(X)\ell\bar{\nu}$ is largely uncorrelated with the tagged analysis and finds $\mathscr{G}(1)|V_{cb}| = (43.1 \pm 0.8_{\text{stat}} \pm 2.1_{\text{sys}}) \times 10^{-3}$ and $\rho^2_{\mathscr{G}} = 1.20 \pm 0.04_{\text{stat}} \pm 0.06_{\text{sys}}$. These results dominate the HFAG[4] averages $\mathscr{G}(1)|V_{cb}| = (42.3 \pm 0.7_{\text{stat}} \pm 1.3_{\text{sys}}) \times 10^{-3}$ and $\rho^2_{\mathscr{G}} = 1.18 \pm 0.04_{\text{stat}} \pm 0.04_{\text{sys}}$. Using the average $\mathscr{G}(1)|V_{cb}|$ along with an old unquenched Lattice QCD calculation[5] $\mathscr{G}(1) = 1.074(18)(16)$ and a QED correction[6] of 1.007 gives

$$|V_{cb}|_{D\ell\nu} = (39.1 \pm 1.4_{\text{exp}} \pm 0.9_{\text{th}}) \times 10^{-3}. \tag{2.2}$$

The experimental uncertainty can be further reduced with present B-factory data sets; BELLE has not published this channel since 2002 and the BABAR global fit analysis[3] used less than half the final data set. Calculations of the form factor normalization at values $w > 1$ may allow more precise determinations; they are currently available only in the quenched approximation[7]. Renewed efforts to provide state-of-the-art calculations of the form factor normalization are needed.

The $\bar{B} \to D^*\ell\bar{\nu}$ decay rate depends on four kinematic variables (w and 3 angles) and involves three non-trivial form factors, usually denoted A_1, A_2 and V. The form factor $\mathscr{F}(w)$ is usually parameterized[8] using an expansion of A_1 in terms of w (or z), with the w-dependence of the form factor ratios $R_1(w) \propto A_2(w)/A_1(w)$ and $R_2(w) \propto V(w)/A_1(w)$ taken from theory. The slope of the form factor $\mathscr{F}(w)$ at $w = 1$ is denoted by $\rho^2_{\mathscr{F}}$. A variety of methods have been used to measure these decays. Untagged measurements of $\bar{B}^0 \to D^{*+}\ell\bar{\nu}$ and $B^- \to D^{*0}\ell\bar{\nu}$ provide high statistics, allowing $R_1(1)$ and $R_2(1)$ to be measured, but require the reconstruction of a soft π^+ or π^0. The global fit and recoil analyses discussed previously also provide measurements of $\bar{B} \to D^*\ell\bar{\nu}$ and are sensitive to a different mix of systematic uncertainties.

A recent publication by BELLE[9] provides improved results on $\mathscr{F}(1)|V_{cb}|$, $\rho^2_{\mathscr{F}}$, $R_1(1)$ and $R_2(1)$. The BELLE measurement is based on 123×10^3 $\bar{B} \to D^*\ell\bar{\nu}$ decays where the D^* is reconstructed in $D^0\pi^+$ with $D^0 \to K^-\pi^+$ (see Fig. 1). Their measurements of $R_1(1) = 1.401 \pm$

[1] More detailed discussion of this decay can be found in Ref.[1].

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$0.034_{\rm stat} \pm 0.018_{\rm sys}$ and $R_2(1) = 0.864 \pm 0.024_{\rm stat} \pm 0.008_{\rm sys}$ are compatible with previous results from BABAR[10] and CLEO[11]. However, the overall consistency of the measurements of $\mathscr{F}(1)|V_{cb}|$ and $\rho^2_{\mathscr{F}}$ is poor (CL$\simeq$ 0.02). A crude average of just the B factory measurements[2] also has a low confidence level, ~ 0.06, so the errors on the average are scaled by $\sqrt{\chi^2/\mathrm{ndf}} = 1.41$ to give $\mathscr{F}(1)|V_{cb}| = (35.5 \pm 1.0) \times 10^{-3}$ and $\rho^2_{\mathscr{F}} = 1.154 \pm 0.035$. Using $\mathscr{F}(1) = 0.908(5)(16)$ from a recent calculation[12] gives

$$|V_{cb}|_{D^*\ell\nu} = (38.8 \pm 1.1_{\rm exp} \pm 0.7_{\rm th}) \times 10^{-3}. \tag{2.3}$$

The results from $D\ell\bar{\nu}$ and $D^*\ell\bar{\nu}$ decays are compatible; they are essentially uncorrelated and their weighted average is

$$|V_{cb}|_{\rm excl} = (38.9 \pm 1.1) \times 10^{-3}. \tag{2.4}$$

The measured ratio $\mathscr{G}(1)/\mathscr{F}(1) = 1.19 \pm 0.05$ agrees with the theoretical value 1.18 ± 0.03.

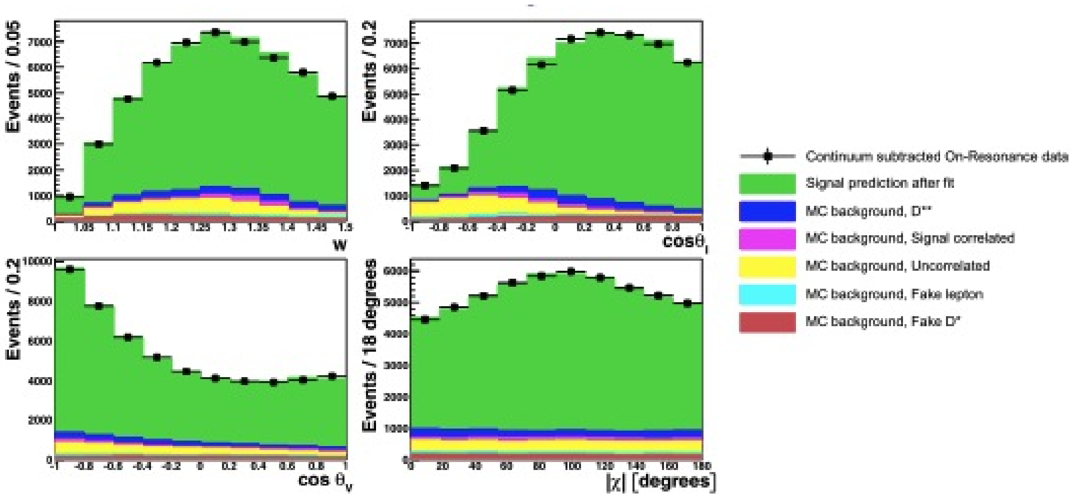

Figure 1: Measured distributions of $\bar{B} \to D^*\ell\bar{\nu}$ decay from BELLE[9].

2.2 CKM-suppressed transitions

The cleanest exclusive decay mode for the determination of $|V_{ub}|$, for both theory and experiment, is $\bar{B} \to \pi\ell\bar{\nu}$. While measurements that use the recoil method have significantly higher signal-to-background ratios, the untagged measurements are more precise, especially for differential measurements as a function of q^2. Recent measurements of the differential branching fraction for $\bar{B} \to \pi\ell\bar{\nu}$ have been made by BABAR[13, 14] and BELLE[15]. These analyses all fit for the normalization of $\bar{B} \to X_c\ell\bar{\nu}$ and $\bar{B} \to X_u\ell\bar{\nu}$ backgrounds, and determine signal yields in bins of q^2 by fitting distributions of signal and background in ΔE ($\equiv E_{\rm beam} - E_{\pi\ell\nu}$) and m_{ES} $\left(\equiv \sqrt{E^2_{\rm beam} - p^2_{\pi\ell\nu}}\right)$, where all quantities are in the $\Upsilon(4S)$ rest frame (see Fig. 2). The analyses give compatible branching fractions and q^2 spectra. Taking the BABAR averages from Ref.[14] and treating the BELLE measurement as uncorrelated with these, the averaged values are

$$\mathscr{B}(\bar{B}^0 \to \pi\ell\bar{\nu}) = (1.45 \pm 0.06) \times 10^{-4}, \tag{2.5}$$

[2] Using the same method as HFAG, but applied only to the BABAR and BELLE measurements.

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$$\Delta\mathscr{B}(q^2 > 16\,\mathrm{GeV}^2) = (0.379 \pm 0.024) \times 10^{-4}, \tag{2.6}$$
$$\Delta\mathscr{B}(q^2 < 16\,\mathrm{GeV}^2) = (1.096 \pm 0.045) \times 10^{-4}, \tag{2.7}$$

providing good precision in all regions of q^2. The measured q^2 spectra can be combined with theoretical points (from LQCD or LCSR) in a fit to determine $|V_{ub}|$ with reduced uncertainties:

$$\begin{array}{ll} |V_{ub}| = (2.95 \pm 0.31) \times 10^{-3} & (\mathrm{BABAR} + \mathrm{FNAL/MILC})[13] \\ |V_{ub}| = (3.43 \pm 0.33) \times 10^{-3} & (\mathrm{BELLE} + \mathrm{FNAL/MILC})[15] \\ |V_{ub}| = (3.25 \pm 0.12 \pm 0.28) \times 10^{-3} & (\mathrm{BABAR} + \mathrm{BELLE} + \mathrm{FNAL/MILC})[16] \end{array} \tag{2.8}$$

The last fit is used below as the $|V_{ub}|$ determination from exclusive decays.

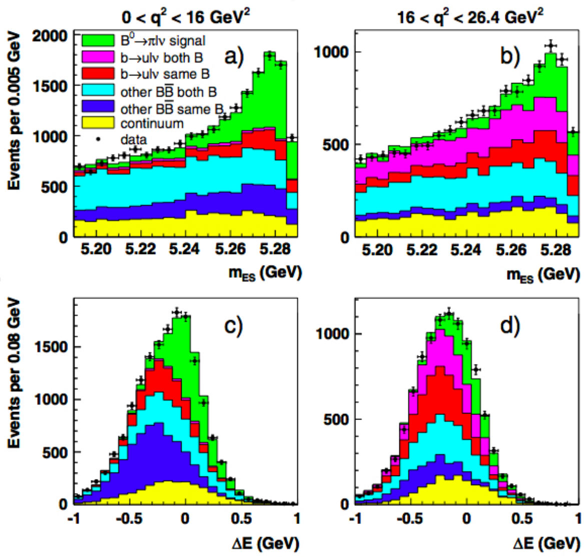

Figure 2: Measured distributions of $\bar{B} \to \pi\ell\bar{\nu}$ decay from BABAR[14].

3. Inclusive semileptonic decays

3.1 Inclusive CKM-favored decays

The description of inclusive semileptonic decays relies on the heavy quark expansion, an OPE in inverse powers of the b-quark mass. The first term in this series is the familiar free-quark decay, and the m_b^{-1} term vanishes, leaving a well-behaved series. Schematically, for $b \to c\ell\bar{\nu}$ decays,

$$\Gamma(b \to c\ell\bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2(1+A_{ew})A^{pert}(r,\mu)\left[z_0(r) + z_2(r)\left(\frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}\right) + z_3(r)\left(\frac{\rho_D^3}{m_b^3}, \frac{\rho_{LS}^3}{m_b^3}\right) + \ldots\right] \tag{3.1}$$

where $r = m_c/m_b$ and the z_j are known functions. The calculation must be done within a consistent renormalization scheme for the quark masses. The terms appearing at orders m_b^{-2} and m_b^{-3} are non-perturbative quantities that also depend on the renormalization scale μ. In addition to power corrections, there are perturbative electroweak (A_{ew}) and QCD (A^{pert}) corrections. Similar expansions can be written for moments of decay distributions, e.g. for powers of the electron energy or hadronic mass squared, in each case as a function of a minimum cut on the lepton momentum. These moments depend on the same set of non-perturbative quantities and allow them to be determined in an overconstrained fit. Many experimental collaborations have measured spectral moments[17, 18, 19]: CLEO, BABAR, BELLE, CDF and DELPHI. The measured moments at different lepton momentum cuts can be highly correlated, making it mandatory to provide full correlation matrices for both statistical and systematic uncertainties. Results from a global fit to these moments[4] in the kinetic mass scheme[20] are given in Table 1.

Table 1: Results of global fit to $b \to c\ell\bar{\nu}$ moments.

Input	$\lvert V_{cb}\rvert \times 10^3$	$m_b^{\rm kin}$(GeV)	μ_π^2(GeV^2)	χ^2/ndf
all moments	$41.85 \pm 0.42 \pm 0.59$	4.591 ± 0.031	0.454 ± 0.038	$29.7/(66-7)$
only $b \to c\ell\bar{\nu}$	$41.68 \pm 0.44 \pm 0.58$	4.646 ± 0.047	0.439 ± 0.042	$24.2/(55-7)$

The first uncertainty on $|V_{cb}|$ is from the fit, while the second is from the calculated decay rate. The unreasonably low χ^2/ndf values suggest uncertainties or correlations are not fully understood. A similar fit using the 1S scheme[21] gives $|V_{cb}| = (41.87 \pm 0.25 \pm 0.08) \times 10^{-3}$, and a similarly small χ^2/ndf. We use $|V_{cb}|_{\rm incl} = (41.85 \pm 0.42 \pm 0.59) \times 10^{-3}$ below.

3.2 Inclusive CKM-suppressed decays

The isolation of $b \to u\ell\bar{\nu}$ transitions is challenging experimentally due to the much larger ($\times$50) rate for $b \to c\ell\bar{\nu}$ decays. Kinematic separation can be achieved either by directly using the hadronic mass, which must be at least as large as m_{D^0} for $b \to c\ell\bar{\nu}$ decays, or through the impact of the quark mass difference on other variables, like q^2 or the lepton momentum. The only way to determine which particles are associated with the semileptonic decay is to fully reconstruct the second B meson in the event at a large cost in efficiency. As a result, more inclusive analyses that measure just the lepton momentum or q^2 continue to provide important information on $|V_{ub}|$.

Partial branching fraction measurements in several regions of phase space are given in Table 2. Some regions encompass only 15% of the total rate, while others include up to 90%. A caution in using some the most inclusive measurements: they are sensitive to the modeling of the $b \to u\ell\bar{\nu}$ signal, and the correlations between the corresponding uncertainty and the uncertainties in the theoretical calculations are not well understood.

Table 2: Partial branching fraction measurements ($\times 10^5$). The total $b \to u\ell\bar{\nu}$ branching fraction is ≈ 220. The first error is statistical, the second, systematic.

CLEO[22]	BaBar[23]	Belle[24]	BaBar[25]	BaBar[26]	BaBar[26]	BaBar[26]	BaBar[26]	Belle[27]
$E_e > 2.1$	E_e-q^2	$E_e > 1.9$	$E_e > 2.0$	$m_X < 1.55$	m_X-q^2	P_+	$E_e > 1$	$E_e > 1$
$33 \pm 2 \pm 7$	$44 \pm 4 \pm 4$	$85 \pm 4 \pm 15$	$57 \pm 4 \pm 7$	$108 \pm 8 \pm 6$	$68 \pm 6 \pm 4$	$99 \pm 9 \pm 8$	$180 \pm 13 \pm 15$	$196 \pm 17 \pm 16$

There are several theoretical calculations available[28, 29, 30, 31] for the determination of $|V_{ub}|$ from the measured rates. They differ in how they calculate rates in restricted regions of phase space where the OPE breaks down. The $|V_{ub}|$ values determined for each of the rates in Table 2 agree within the estimated independent theoretical uncertainty of 3-5%. A common source of uncertainty in each case comes from the b quark mass. Weak annihilation, evaluated using different ansatze in the calculations, also affects the determination of $|V_{ub}|$.

BABAR semileptonic tag[33]	$180 \pm 80 \pm 10$	BABAR hadronic tag[34]	$180\,^{+57}_{-54} \pm 26$
BELLE semileptonic tag[35]	$154\,^{+38}_{-37}\,^{+29}_{-31}$	BELLE hadronic tag[36]	$179\,^{+56}_{-49}\,^{+46}_{-51}$

Table 3: Measurements of the branching fraction for $B \to \tau\nu$, in units of 10^{-6}.

For each calculation an average $|V_{ub}|$ is determined from the set of rates in Table 2. Instead of choosing one particular calculation, we quote the arithmetic average of the values and errors from this set of four $|V_{ub}|$ determinations to find

$$|V_{ub}|_{\text{incl}} = (4.25 \pm 0.15_{\text{exp}} \pm 0.20_{\text{th}}) \times 10^{-3}. \tag{3.2}$$

One caution: a calculation[32] of NNLO corrections results in an 8% increase of $|V_{ub}|$ for the prediction of Ref. [29]; this is not reflected in the average given here.

4. Summary of $|V_{ub}|$ and $|V_{cb}|$

The inclusive (Table 1) and exclusive (Eq. 2.4) determinations of $|V_{cb}|$ differ by 2.2σ. The average, after scaling the uncertainty by a factor $\sqrt{\chi^2/\text{ndf}} = 2.2$, is

$$|V_{cb}| = (41.0 \pm 1.3) \times 10^{-3}. \tag{4.1}$$

The difference between the inclusive and exclusive results currently limits our ability to provide a precise determination of $|V_{cb}|$.

An average of $|V_{ub}|$ determinations from $B \to \pi\ell\bar{\nu}$ (Eq. 2.8) and from inclusive $b \to u\ell\bar{\nu}$ decays (Eq. 3.2) has a P-value of 2.2%. After scaling the uncertainty by $\sqrt{\chi^2/\text{ndf}} = 2.3$,

$$|V_{ub}| = (3.97 \pm 0.45) \times 10^{-3}. \tag{4.2}$$

The continued tension between the inclusive and exclusive determinations of both $|V_{ub}|$ and $|V_{cb}|$ remains a puzzle. The tension is reduced slightly in the ratios $|V_{ub}/V_{cb}|_{\text{incl}} = 0.101 \pm 0.006$ and $|V_{ub}/V_{cb}|_{\text{excl}} = 0.084 \pm 0.008$. Improvements with existing data are still possible for $|V_{cb}|$ from $\bar{B} \to D\ell\bar{\nu}$ and on $|V_{ub}|$ from inclusive decays. Further progress also relies on improved calculations of form factor normalizations and radiative corrections in the heavy quark expansion.

5. $B \to \tau\nu$

The purely leptonic decay $B \to \tau\nu$ has a rate proportional to $f_B|V_{ub}|^2$. It is challenging to measure, since the presence of at least two neutrinos in the final state precludes the use of kinematic constraints. The recoil method can be used to identify which particles belong to the second B meson in the event, after which vetos on additional charged particles or neutral energy beyond that expected from the τ decay are employed. Both hadronic and semileptonic reconstruction of the second B meson can be used; the former is cleaner, but the latter provides a higher efficiency. Recent measurements are given in Table 3; the associated HFAG[4] average is $\mathcal{B}(B \to \tau\nu) = (164 \pm 34) \times 10^{-6}$. These measurements require a good understanding of vetos on calorimeter activity, and have sources of background that peak in the signal region; they are extremely challenging.

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