

Medium Induced Collinear Radiation via Soft Collinear Effective Theory (SCET)

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The propagation of hard partons through the strongly interacting matter created in high energy heavy-ion collisions involves widely separated scales. The methods of Effective Field Theories (EFT) can provide a factorized description at lowest nontrivial order, and a formalism where the corrections to this factorization are calculable systematically order by order in the small ratios between the different scales. We present our preliminary results on the medium induced collinear radiation by using the methods of Soft Collinear Effective Theory (SCET). The radiated gluon is collinear with the incoming hard parton and gets an arbitrary fraction of its energy.

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1. Introduction and SCET

The strongly interacting, high-density matter produced in ultra-relativistic heavy ion collisions has a lifetime far too short to be tested with external probes. Thus the study of its properties can be only be performed through its decay products and via internally generated probes. One of the most promising tools is the study of medium induced parton energy loss and parton fragmentation, which is a central part of the program of heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) [1] and at the Large Hadron Collider (LHC) [2]. In the high energy limit the parton loses energy dominantly by processes that are the QCD analogue of bremsstrahlung [3]. It is crucial to the calculation of this radiative energy loss process that the incident hard parton, the outgoing parton, and the radiated gluons are all continually being jostled by the medium in which they find themselves: they are *all* subject to transverse momentum broadening.

The calculation of parton energy loss and transverse momentum broadening involves widely separated scales, $Q \gg T$, where Q is the energy of the hard parton and T is the soft scale characteristic of the medium. We can ultimately hope for a factorized description, with physics at each of these scales cleanly separated at lowest nontrivial order in a combined expansion in the small ratio between these scales and in the QCD coupling α evaluated at scales which become large in the high parton energy limit. And, most importantly, we can aspire to having a formalism in which corrections to this factorization are calculable systematically, order by order in these expansions. We continue our effort in this direction: we formulate the calculation of the medium induced collinear radiation in the language of Soft Collinear Effective Theory (SCET) [4], which has rendered the calculation of many other processes involving soft and collinear degrees of freedom systematically improvable. Related works applying SCET to jet propagation in dense media can be found in reference [5].

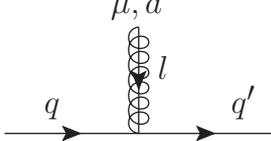
We applied SCET in reference [6] to treat transverse momentum broadening only in the absence of radiation. In particular the “jet quenching parameter” \hat{q} , defined as the mean transverse momentum picked up by the hard parton per unit distance travelled, was found to be given by

$$\hat{q} \equiv \frac{\langle k_{\perp}^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2 k_{\perp}}{(2\pi)^2} k_{\perp}^2 \int d^2 x_{\perp} e^{-ik_{\perp} \cdot x_{\perp}} \mathcal{W}_{\mathcal{R}}(x_{\perp}), \quad (1.1)$$

where \mathcal{R} is the $SU(N)$ representation to which the collinear particle belongs and $\mathcal{W}_{\mathcal{R}}(x_{\perp})$ is the expectation value of two light-like Wilson lines separated in the transverse plane by the vector x_{\perp} . Note that $\mathcal{W}_{\mathcal{R}}(x_{\perp})$ depends only on the properties of the medium and it is independent of the energy of the hard parton, meaning that so is \hat{q} in the limit in which this hard parton energy is taken to infinity. Transverse momentum broadening without radiation thus does “measure” a field-theoretically well-defined property of the strongly coupled medium. The goal of the present work is to extend the transverse momentum broadening analysis, by allowing the hard parton to emit a collinear gluon.

The effective theory fields we need to describe the radiative process are: collinear quarks, collinear gluons and Glauber gluons. Two different interactions are relevant: the one between the collinear partons and the Glauber gluons (transverse broadening), and the one between the collinear partons and the collinear gluons (energy loss). The Feynman rules to account for momentum

broadening can be found in reference [5, 6], whereas for the collinear emission we have



$$= i g t_F^a \left[\bar{n}_\mu - \frac{1}{2} \frac{q_\perp + l_\perp}{n \cdot (q+l)} \frac{q_\perp}{n \cdot q} n_\mu + \gamma_\mu^\perp \frac{q_\perp}{2 n \cdot q} + \frac{q_\perp + l_\perp}{2 n \cdot (q+l)} \gamma_\mu^\perp \right] \not{n} \quad (1.2)$$

where the partons are collinear in the same direction, and the light cone vectors are defined as $\bar{n} \equiv \frac{1}{\sqrt{2}}(1, 0, 0, -1)$ and $n \equiv \frac{1}{\sqrt{2}}(1, 0, 0, 1)$.

2. Medium Induced Collinear Radiation

In the high parton energy limit, the parton loses energy dominantly by inelastic processes that are the QCD analogue of bremsstrahlung: the parton radiates gluons as it interacts with the medium. We consider a very energetic parton created in a hard process propagating through some form of QCD matter, which for definiteness we will take to be quark-gluon plasma (QGP) in equilibrium at temperature T , although our discussion would also apply to propagation through other forms of matter. We will assume throughout that Q is very much larger than the highest momentum scales that characterize the medium, which for the case of QGP means $Q \gg T$. We study the emission by this hard parton of a single gluon, which is assumed to be collinear in the same direction of the incoming parton.

Our goal is to compute the emitted gluon spectrum, differential in its light-cone energy P and its transverse momentum l_\perp . This radiative process takes place in a dense medium, and all the collinear partons interact repeatedly with Glauber gluons. And, as we have done for the momentum broadening case, the recoil of the medium is neglected, therefore the Glaubers can be considered as a background field. More specifically, we evaluate the spectrum for a given background field configuration and average over the thermal ensemble only at the end of the calculation.

We have two possible classes of diagrams: the case when the gluon is emitted in the vacuum, so there is no interaction between the medium and the final states, and the case when the gluon is emitted in the medium. The two contributions are shown in the figure 1, and the full amplitude for the process is given by the sum of the two contributions, namely $\mathcal{M} = \mathcal{M}_v + \mathcal{M}_m$. Therefore the total probability reads

$$|\mathcal{M}|^2 = |\mathcal{M}_v|^2 + |\mathcal{M}_m|^2 + 2 \text{Re}(\mathcal{M}_m \mathcal{M}_v^*) . \quad (2.1)$$

We present our results for the case of a propagating collinear quark. The analysis of the medium induced radiation for a propagating collinear gluon can be performed by using similar techniques. The first piece we evaluate is the amplitude squared for the vacuum emission $|\mathcal{M}_v|^2$. After resumming over an arbitrary number of interactions between the incoming collinear quark and the Glauber gluons from the medium we get the following contribution

$$|\mathcal{M}_v|^2 = 4 g^2 C_F \int d^2 x_\perp \exp[i l_\perp \cdot x_\perp] \mathcal{Y}[x_\perp, l_\perp, P] , \quad (2.2)$$

where C_F is the Casimir factor in the $SU(N)$ representation, and we define

$$\mathcal{Y}[x_\perp, l_\perp, P] = \mathcal{W}_F(x_\perp) \int \frac{d^2 k_\perp}{(2\pi)^2} \exp[i k_\perp \cdot x_\perp] \left(\frac{k_\perp^2}{Q-P} + \frac{l_\perp^2}{P} \right)^{-2} \times$$

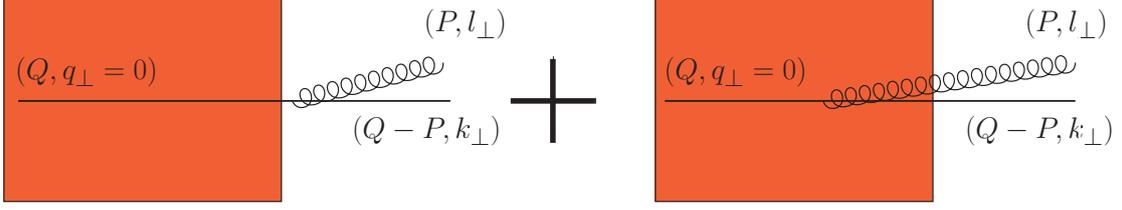


Figure 1: Two contributions to the medium induced gluon radiation: emission in the vacuum (left) and emission in the medium (right). The red box is the strongly coupled medium through which the hard parton passes.

$$\left[\frac{1}{2} \frac{k_{\perp}^2}{(Q-P)^2} + \frac{1}{2} \frac{(k_{\perp} + l_{\perp})^2}{Q^2} + \frac{l_{\perp}^2}{P^2} - \frac{l_{\perp} \cdot k_{\perp}}{P(Q-P)} - \frac{l_{\perp} \cdot (k_{\perp} + l_{\perp})}{QP} \right]. \quad (2.3)$$

We therefore discover that $|\mathcal{M}_v|^2$, the amplitude for emission in vacuum after the hard parton exits the medium, does depend on the properties of the medium — in fact through the same soft function $\mathcal{W}_F(x_{\perp})$ constructed from the expectation value of light-like Wilson lines that controls transverse momentum broadening and appears in (1.1). This is at first a surprise, since in treatments in the literature (for example in [7]) $|\mathcal{M}_v|^2$ is independent of the properties of the medium. However, in the kinematic limit in which the radiated gluon is much softer than the incoming hard parton, namely $P \ll Q$, we find that (2.3) becomes independent of the medium, as in the literature. In this limit

$$\mathcal{Y}[x_{\perp}, l_{\perp}, P] \simeq \mathcal{W}_F(x_{\perp}) \delta^2(x_{\perp}) \frac{1}{l_{\perp}^2} \quad \Rightarrow \quad |\mathcal{M}_v|^2 = \frac{4g^2 C_F}{l_{\perp}^2} \quad (\text{for } P \ll Q). \quad (2.4)$$

The medium emission squared $|\mathcal{M}_m|^2$ and the interference $2\text{Re}(\mathcal{M}_m \mathcal{M}_v^*)$ contributions are also computed, and we recover the correct limit [7] when the radiated gluon is much softer than the incoming quark. However the expressions for arbitrary values of P have some interesting different features. The final state collinear quark, after it radiates a collinear gluon, is not necessarily described by a light-like Wilson line as in [7], but more generally as a path integral. This takes into account the region of phase space where the radiated gluon takes away a significant amount of its energy. In addition there are transverse derivatives acting not only on the path integrals describing the motion of the radiated gluon, but also on the fundamental Wilson line and path integral, describing the incoming and outgoing quark, respectively. We will report the detailed expressions in a future publication.

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