

## Two-loop mixed QCD-EW virtual corrections to the Drell-Yan production of Z and W bosons

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The Drell-Yan production of Z and W bosons is a very important process for physics studies at hadron colliders. At the moment, the theoretical prediction includes up to the NNLO QCD corrections, together with the resummation of logarithmic terms originating from soft gluon emission. In these proceedings, we present the analytic calculation of the matrix elements for the mixed  $\alpha\alpha_S$  two-loop virtual corrections, for the production of an on-shell Z or W.

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## 1. Introduction

The Drell-Yan production of  $Z$  and  $W$  bosons [1],  $pp(\bar{p}) \rightarrow Z + X \rightarrow l^+l^- + X$  and  $pp(\bar{p}) \rightarrow W + X \rightarrow lv + X$ , is a fundamental process for an accurate test of the SM at hadron colliders, since it has a big cross section, a clean experimental signature, and it is very sensitive to the properties of the gauge bosons. In particular, the Drell-Yan production of  $W$ s is important for an accurate determination (via transverse mass and  $p_T$  distributions) of the  $W$  mass,  $m_W$ , which together with the top mass plays an important role in constraining the Higgs mass via radiative corrections. The  $W$  mass is supposed to be measured in the near future at Tevatron with high accuracy ( $\Delta m_W \sim 15 \text{ MeV}$ ) and at the LHC even more precisely ( $\Delta m_W \sim 10 \text{ MeV}$ ). This level of accuracy requires an equally accurate theoretical control on the kinematic distributions. The Drell-Yan production of  $Z$  bosons allows for a precise measurement of another important parameter of the SM,  $\sin^2 \theta_W$ . An accurate theoretical prediction for the forward-backward asymmetry in the  $Z \rightarrow l^+l^-$  decay channel can allow the LHC to match the performance of LEP. Furthermore, Drell-Yan is a background process for many important reactions as, for instance, the production of  $t\bar{t}$  pairs or the production of new vector resonances,  $Z'$  and  $W'$ , present in many extensions of the SM. Finally, the Drell-Yan mechanism can be used for detector calibration and determination of the collider luminosity at the LHC. Because of all these reasons, an accurate and reliable theoretical prediction for the cross section and the distributions of the Drell-Yan production mechanism, that means control on the higher-order perturbative corrections, is demanded for physics studies at hadron colliders.

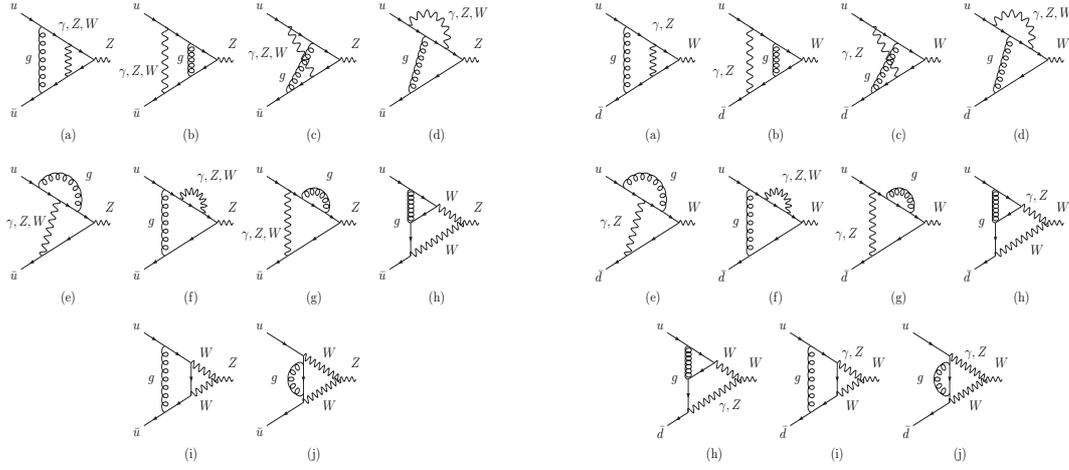
The NLO QCD corrections to the total cross section are known since many years. They were calculated in [2], finding a sizable increase of the cross section with respect to the LO. The NNLO QCD corrections [3] stabilize the convergence of the perturbative series. The electroweak NLO corrections are known for the  $W$  [4] and  $Z$  [5] production cross sections, both in narrow-width approximation and exact calculation. The total cross section is supplemented by the resummation of the logarithmically enhanced terms due to the soft gluon emission up to NNLL accuracy [6].

More exclusive observables are known in the literature. The  $Z$  and  $W$  production at non-zero transverse momentum  $p_T$  is known at the NLO in QCD [7] and in the full SM [8]. Recently, the two-loop QCD helicity amplitudes for the production of a  $Z$  or a  $W$  with a photon have also been calculated [9]. For small  $p_T$  ( $p_T \ll m_W, m_Z$ ) the convergence of the fixed-order calculation is spoiled by the large logarithmic terms  $\alpha_s^n \log^m(m_W^2/p_T^2)$  that have to be resummed [10].

Finally, the rapidity distribution of a vector boson is known at the NNLO in QCD [11].

Although the NLO corrections are available in a fully differential description (they are included in NLO Monte Carlo event generators like MC@NLO [12] and POWHEG [13]), they are not accurate enough for the performances of the new generation of colliders (Tevatron run II and LHC). The NNLO results mentioned above, however, are widely inclusive and they cannot provide realistic descriptions, that necessarily have to include experimental cuts. Therefore, a fully differential description of the Drell-Yan process at the NNLO is needed. Very recently, this goal was achieved in [14, 15, 16], where the decay products of the vector boson, the spin correlations and the finite-width effects are taken into account.

A sizable impact on the  $pp(\bar{p}) \rightarrow W \rightarrow lv$  distributions, and therefore on the determination of the  $W$  mass, comes from the QCD initial state radiation (ISR) with QED final state radiation (FSR) or from the real-virtual (factorizable) corrections. However, at the level of precision required



**Figure 1:** Left: the 40 Feynman diagrams contributing to the Drell-Yan production of an on-shell  $Z$  boson at  $\mathcal{O}(\alpha\alpha_S)$ . Right: the 44 Feynman diagrams contributing to the Drell-Yan production of an on-shell  $W$  boson at  $\mathcal{O}(\alpha\alpha_S)$ .

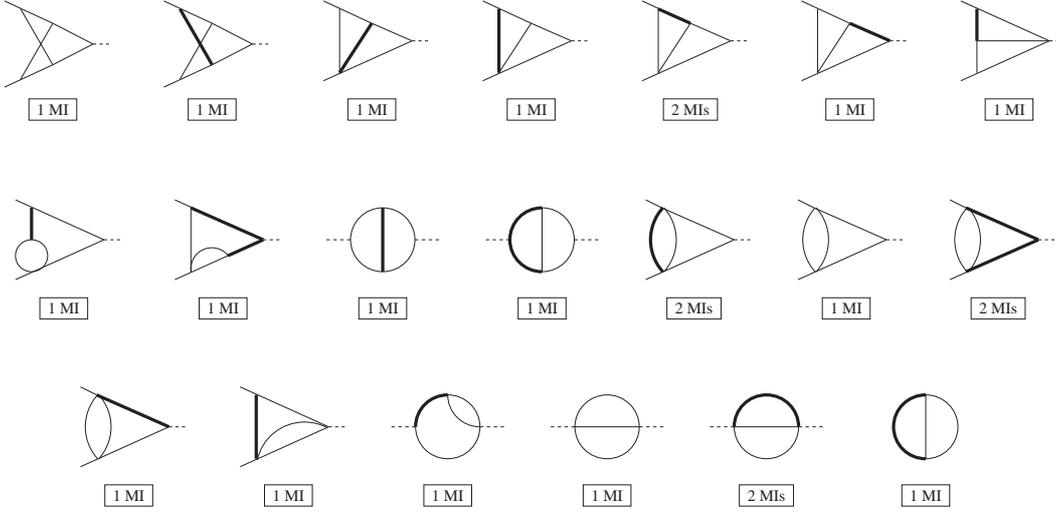
( $\Delta m_W \sim 10$  MeV), the complete set of mixed QCD-EW corrections may be important<sup>1</sup> and have to be considered. In [17], the mixed two-loop corrections to the form factors for the production of a  $Z$  boson were calculated analytically, expressing the result in terms of harmonic polylogarithms and related generalizations [18]. The QCD $\times$ QED corrections to the process were considered in [19].

In these proceedings, we report on the analytic calculation of the  $\mathcal{O}(\alpha\alpha_S)$  corrections to the form factors for the Drell-Yan production of a  $Z$  and a  $W$  boson [20].

## 2. The Calculation

We consider the partonic processes  $q\bar{q} \rightarrow Z + X \rightarrow l^+l^- + X$  and  $q\bar{q}' \rightarrow W + X \rightarrow l\nu + X$ . In the case only QCD corrections are taken into account, these affect exclusively the initial state. From a technical point of view, this means that NNLO QCD corrections involve at most two-loop vertices. If, now, we consider the  $\mathcal{O}(\alpha\alpha_S)$  corrections, the situation changes considerably. The final state can interfere with the initial state by the exchange of an electroweak gauge boson. This means that these corrections involve two-loop boxes with massive propagators, pentagons for the virtual-real radiation and  $2 \rightarrow 4$  diagrams for the double real radiation. The calculation is much more complicated. However, the bulk of the contribution due to these corrections comes from the region in which the  $W$  or the  $Z$  are nearly on-shell. Therefore, we can compute the corrections in narrow-width approximation, expanding formally the modulus square of the amplitude in the limit  $\Gamma_V/m_V \rightarrow 0$ , where  $\Gamma_V$ ,  $m_V$  are the vector boson width and mass, respectively. This limit makes in such a way that the corrections to the initial state are disconnected from those on the final state, and the process can be described as the product of the production of an on-shell vector boson and its subsequent leptonic decay. This gives the opportunity to simplify the calculation.

<sup>1</sup>For instance, they can improve considerably the stabilization of the renormalization/factorization scale dependence of the pure electroweak corrections.



**Figure 2:** Master Integrals for the Drell-Yan production of a  $Z$  and  $W$  bosons at  $\mathcal{O}(\alpha\alpha_s)$ . Thick lines represent a massive propagator, while thin lines represent mass-less propagators.

The Feynman diagrams involved in the calculation of the  $\mathcal{O}(\alpha\alpha_s)$  corrections to the production of a  $Z$  or  $W$ , considered in [20], are shown in Fig. 1. The calculation is performed using the Laporta algorithm [21] for the reduction of the dimensionally regularized scalar integrals to the set of Master Integrals (MIs), and the differential equations method [22] for their analytic evaluation. Many MIs (the full set is shown in Fig. 2) were already published in the literature [23], with the exception of two, that can be found in [20]. The diagrams in which both  $Z$  and  $W$  propagators are present at the same time, are calculated expanding the  $Z$  propagator in power series of the small parameter  $\xi = \Delta M^2/m_Z^2 = 1 - m_W^2/m_Z^2$ , in such a way that the topologies that have to be reduced contain massive propagators with the same mass. The results are, therefore, expressed in terms of Harmonic Polylogarithms (and their generalizations) of a single variable.

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