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$N_f = 3$ chiral dynamics in the light of recent 2+1 lattice simulations

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Due to the intermediate position of the strange quark in the hierarchy of quark masses, a numerical competition may arise in chiral series between leading and next-to-leading order terms in three-flavour Chiral Perturbation Theory (χPT). A way to study this question is to use a modified version of χPT called Resummed Chiral Perturbation Theory, a reordering of the series expansion which does not assume the smallness of next-to-leading order terms compared to the leading order. Working within this framework, we fit recent lattice results on masses of pseudoscalar mesons, decay constants and $K\ell_3$ form factors from several simulations using 2+1 dynamical quarks. We show that it provides a good fit of the recent results of two lattice collaborations (PACS-CS and RBC-UKQCD), and we observe that the numerical competition between leading and next-to-leading orders indeed occurs. Numerical values for the usual order parameters of $N_f = 3$ chiral symmetry breaking are extracted, along with other quantities like the ratio of decay constants F_K/F_π, the light quark masses and the $K\ell_3$ vector form factor at zero momentum transfer $f_+(0)$.

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Investigating the non-perturbative dynamics of the strong interaction at low energies can be done within two complementary approaches: Lattice QCD, and Effective Field Theory methods, i.e. Chiral Perturbation Theory (χPT). Indeed, numerical simulations probe the quark-mass dependence of observables, while χPT provides a tool to extrapolate lattice data down to the physical masses. Some lattice collaborations reported difficulties when fitting their data with $2+1$ dynamical quarks with $N_f = 3$ chiral expansions [1, 2, 3]. This could be interpreted as the fact that the $N_f = 3$ chiral series suffer from problems of convergence, for their Leading-Order term (LO) is no longer dominant and compete with higher order (HO) ones, at least with the Next-to-Leading Order (NLO). One ends up with a weak convergence situation where LO and NLO numerically compete, instead of the usually expected situation where LO is dominant. Such instabilities in chiral series could come from significant vacuum fluctuations of $s\bar{s}$ pairs in the $N_f = 3$ theory [4, 5] due to the very special role played by the strange quark whose mass is of the order of $\mathscr{O}(\Lambda_{QCD})$.

Resummed Chiral Perturbation Theory (ReχPT) has been proposed to allow for such a numerical competition [5]. In this framework no specific hypothesis is made upon the relative size of LO and NLO. First, a subset of observables is chosen for which a "good" overall convergence is assumed, i.e. the sum of LO and NLO terms is large compared to the remaining HO terms of the series (in relation with the correlators described by χPT): for example, the squared pion and kaon decay constants, F_π^2, F_K^2 ($\langle A^\mu A^\nu \rangle$), the squared masses, $F_\pi^2 M_\pi^2$, $F_K^2 M_K^2$ ($\langle \partial_\mu A^\mu \partial_\nu A^\nu \rangle$) and the pion electromagnetic form factor, $F_\pi F_V^\pi$ ($\langle A^\nu j_\mu A^\sigma \rangle$). Their chiral expansion is performed in terms of χPT Lagrangian parameters up to NLO, *keeping track* of the HO which forms the (small) remainders of the series. Finally, three fundamental LO quantities, involving two main order parameters of chiral symmetry breaking, are kept free:

$$X(3) = \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2}, \quad Z(3) = \frac{F(3)^2}{F_\pi^2}, \quad r = \frac{m_s}{m}. \qquad \left(m = \frac{m_u + m_d}{2} \right) \tag{1}$$

$\Sigma(3)$ and $F(3)$ are respectively the quark condensate and the pseudoscalar decay in the $N_f = 3$ chiral limit ($m_u, m_d, m_s \to 0$). $X(3)$ and $Z(3)$ assess the saturation of $F_\pi^2 M_\pi^2$ and F_π^2 by their LO, thus probing the numerical LO saturation of these observables. Finally, the parameters of the Lagrangian which appear at NLO, the so-called low energy constants L_i, are expressed as functions of $X(3)$, $Z(3)$, r, some conveniently chosen observables and small HO remainders. The relations thus obtained can be inserted into the chiral expansions of other observables, such as the $K_{\ell 3}$ form factors $F_\pi F_K f_{+,-}$ ($\langle A^\nu V_\mu A^\sigma \rangle$) which are important quantities to test the Standard Model and probe New Physics in the light-quark sector. The new necessary remainders have to be introduced appropriately. The important point is *not to trade* leading-order terms for physical ones systematically (for exemple $2mB_0$ is not traded for M_π^2), as is usually done.

In order to study the question of convergences of chiral series we performed fits of the π and K decay constants and masses, and the $K_{\ell 3}$ form factors [6]] from some recent $N_f = 2+1$ lattice data provided by PACS-CS [1] and RBC-UKQCD [2] collaborations (work to include the last results of RBC-UKQCD [3] for these quantities as well as a study of the topological susceptibility along the same lines is in progress). Results are shown in Tables 1 and 2. Note that we have chosen to keep the ratio F_K/F_π as a parameter of the fit to probe deviations from the value obtained from leptonic and semileptonic decays within the Standard Model. The χ^2/d.o.f is good for both data sets (and different from the two collaborations, who assumed LO dominance). In both cases, the LO does

Params.	PACS/CS (no $K\ell_3$)	RBC/UKQCD (with $K\ell_3$)
$Z(3)$	0.66 ± 0.09	0.46 ± 0.04
$X(3)$	0.59 ± 0.21	0.20 ± 0.14
r	26.5 ± 2.3	23.2 ± 1.5
F_K/F_π	1.237 ± 0.025	1.148 ± 0.015
χ^2/d.o.f.	0.9/3	4.4/8

Converg.	PACS-CS (no $K_{\ell 3}$)	RBC-UKQCD (with $K_{\ell 3}$)
F_π^2	0.66 + 0.22 + 0.12	0.45 + 0.69 - 0.14
F_K^2	0.44 + 0.48 + 0.08	0.34 + 0.76 - 0.10
$F_\pi^2 M_\pi^2$	0.60 + 0.30 + 0.10	0.20 + 0.95 - 0.15
$F_K^2 M_K^2$	0.42 + 0.50 + 0.08	0.14 + 0.97 - 0.11
$F_\pi F_K f_+(0)$	n.a.	0.40 + 0.75 - 0.15

Table 1: On the left: parameters of the fit. On the right: relative fraction of the LO+NLO+HO contributions for observables used in our fits. $f_+(0)$ is the $K_{\ell 3}$ vector form factor at zero momentum transfer.

LECs	PACS-CS	RBC-UKQCD
$L_4.10^3$	-0.1 ± 0.2	2.4 ± 2.0
$L_5.10^3$	1.8 ± 0.4	1.8 ± 1.6
$L_6.10^3$	0.1 ± 0.4	4.7 ± 7.1
$L_8.10^3$	0.8 ± 0.7	4.4 ± 7.1
$L_9.10^3$	none	4.4 ± 2.8

	PACS-CS	RBC-UKQCD
m_s(MeV)	70 ± 4	107
m(MeV)	2.6 ± 0.3	4.6 ± 0.3
F_0(MeV)	74.8 ± 4.9	62.2 ± 2.5
B_0(GeV)	3.34 ± 1.18	0.92 ± 0.67
$f_+(0)$	1.004 ± 0.149	0.985 ± 0.008

Table 2: On the left: values of $N_f = 3$ NLO low-energy constants at $\mu = M_\rho$, as determined from our fits. On the right: light-quark masses, $N_f = 3$ chiral order parameters ($F_0 \equiv F(3)$and $B_0 = \Sigma(3)/F_0^2$) and $K_{\ell 3}$ form factor $f_+(0)$. The discrepancy between the two collaborations for the strange quark mass is due to a different choice of renormalization procedure, and is independent of the ReχPT framework used here.

not saturate the series expansions, as reflected by the low values of F_0 and B_0. Values for the low energy constants can be compared with recent determinations from fits to NNLO expressions, hinting at a similar suppression of LO [7]. Not shown here are the values obtained for chiral order parameters in the $N_f = 2$ chiral limit ($m_u, m_d \to 0$ but physical m_s) which are in agreement with a saturation of two-flavour series by their LO. For more details, see ref. [6].

The ReχPT framework provides alternative expressions for chiral extrapolations of three-flavour quantities like F_K/F_π or $K_{\ell 3}$ form factors. More data are needed before reaching a complete understanding of $N_f = 3$ chiral dynamics. However our results show that the numerical competition between LO and NLO could be important and should be taken into account in chiral extrapolations.

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