PROCEEDINGS OF SCIENCE

On the coherent inelastic binary and multiparticle processes in the ultrarelativistic hadron–nucleus, photon–nucleus and nucleus–nucleus interactions

Valery Lyuboshitz*
Joint Institute for Nuclear Research (Dubna, Russia)
E-mail: Valery.Lyuboshitz@jinr.ru
Vladimir Lyuboshitz
Joint Institute for Nuclear Research (Dubna, Russia)

The coherent inelastic processes of the type $a \to b$, which may take place in the interaction of hadrons and γ quanta with nuclei at very high energies (the nucleus remains the same), are theoretically investigated. For taking into account the influence of the nucleus matter, the optical model, based on the concept of refraction index, is used . Analytical formulas for the effective cross section $\sigma_{\rm coh}(a \to b)$ are obtained, taking into account that at ultrarelativistic energies the main contribution into $\sigma_{\rm coh}(a \to b)$ is provided by very small transferred momenta in the vicinity of the minimum longitudinal momentum transferred to the nucleus. It is shown that the cross section $\sigma_{\rm coh}(a \to b)$ may be expressed through the "forward" amplitudes of inelastic scattering $f_{a+N\to b+N}(0)$ and elastic scattering $f_{a+N\to a+N}(0)$, $f_{b+N\to b+N}(0)$ on a separate nucleon, and it depends on the ratios L_a/R and L_b/R, where L_a, L_b are the respective mean free paths in the nucleus matter for the particles a, b and R is the nuclear radius.

The above formalism may be generalized also for the case of coherent inelastic multiparticle processes on a nucleus of the type $a \to \{b_1, b_2, b_3 \dots b_i\}$ and for the case of coherent processes at collisions of two ultrarelativistic nuclei.

XXIst International Europhysics Conference on High Energy Physics
July 21-27, 2011
Grenoble, Rhône-Alpes France

*Speaker.

PoS(EPS-HEP2011)476

1. Momentum transfer at ultrarelativistic energies and coherent reactions on nuclei

It is known that at ultrarelativistic energies the minimal longitudinal momentum, transferred to a nucleus, tends to zero, and in connection with this the role of coherent processes increases.

Let $f_{a+N\to b+N}(\mathbf{q})$ be the average amplitude of an inelastic process $a+N\to b+N$ on a separate nucleon in the rest frame of the nucleus (laboratory frame). Here $\mathbf{q}=\mathbf{k}_b-\mathbf{k}_a$ is the momentum transferred to the nucleon, $\mathbf{k}_a$ and $\mathbf{k}_b$ are the momenta of the particles a and b, respectively. In the framework of the impulse approximation [1], taking into account the interference phase shifts at the inelastic scattering of the particle a on a system of nucleons, the expression for the effective cross-section of the coherent inelastic process $a\to b$ on a nucleus can be presented in the form:

$$\sigma_{\rm coh}(a\to b)=\int|f_{a+N\to b+N}(\mathbf{q})|^2P(\mathbf{q})d\Omega_b. \tag{1.1}$$

where $d\Omega_b$ is the element of the solid angle of flight of the particle b in the laboratory frame, and the magnitude $P(\mathbf{q})$ has the meaning of the probability of the event that at the collision with the particle a all the nucleons will remain in the nucleus and the quantum state of the nucleus will not change:

$$P(\mathbf{q})=|\int n(\rho,z)\exp(-i\mathbf{q}_\perp\rho)\exp(-iq_{||}z)d^2\rho dz|^2. \tag{1.2}$$

Here $n(\mathbf{r})$ is the nucleon density normalized by the total number of nucleons in the nucleus ($\int n(\mathbf{r})d^3\mathbf{r}=A$); the axis z is parallel to the initial momentum $\mathbf{k}_a$, $\mathbf{q}_\perp$ and $q_{||}$ are the transverse and longitudinal components of the transferred momentum, respectively.

It is easy to see that the momenta $|\mathbf{q}|\lesssim 1/R$, transferred to a nucleon (R is the radius of a nucleus), give the main contribution to the effective cross-section of the coherent inelastic process $a\to b$ on the nucleus. In doing so, the recoil energy of the nucleus can be neglected, the effective flight angles for the ultrarelativistic particle b are very small: $\theta\lesssim 1/kR\ll 1$, where $k=E_b\approx E_a$. Then it is possible to assume in Eqs. (1.1) and (1.2) that the transverse and longitudinal transferred momenta are the following:

$$|\mathbf{q}_\perp|=k\theta,\qquad q_{||}=q_{\rm min}=\frac{m_a^2-m_b^2}{2k}, \tag{1.3}$$

where m_a and m_b are the masses of the particles a and b, respectively. Here $q_{\rm min}$ is the *minimal* transferred momentum corresponding to the "forward" direction.

In most cases the characteristic momentum transferred to the nucleus at the inelastic coherent scattering ($|\mathbf{q}|\sim 1/R$) is small as compared with the characteristic momentum transferred to the nucleon in the process $a+N\to b+N$. In connection with this, the amplitude $f_{a+N\to b+N}(\mathbf{q})$ in Eq. (1.1) can be replaced by its value $f_{a+N\to b+N}(0)$ corresponding to the flight of the particle b in the "forward" direction. Taking into account that at small angles θ the solid angle in Eq. (1.1) is $d\Omega=\sin\theta d\theta d\phi\approx d^2\mathbf{q}_\perp/k^2$ and using the properties of the two-dimensional δ-function, we obtain, as a result of the integration of the expression (1.1) over the transverse transferred momenta and over the nucleus volume, the following equation:

$$\sigma_{\rm coh}(a\to b)=\frac{4\pi^2}{k^2}|f_{a+N\to b+N}(0)|^2\int\left(|\int_{-\infty}^{\infty}n(\rho,z)\exp(-iq_{\rm min}z)\,dz|^2\right)d^2\rho, \tag{1.4}$$

where $q_{\min}$ is determined by Eq. (1.3).

For a spherical nucleus with the radius R and the constant density of nucleons $n_0 = 3A/4\pi R^3$, Eq. (1.4) gives at sufficiently high energies, when $|q_{\min}|R \ll 1$:

$$\sigma_{\rm coh}(a \to b) = \frac{8\pi^3}{k^2} n_0^2 |f_{a+N\to b+N}(0)|^2 R^4 = \frac{9\pi}{2k^2R^2} A^2 |f_{a+N\to b+N}(0)|^2. \qquad (1.5)$$

2. Effect of the nucleus matter on coherent processes

In the relations obtained above the multiple scattering of the initial and final particles on nucleons of the nucleus was neglected. This is possible when the mean lengths of free path of the particles a and b inside the nucleus are much greater than the nucleus radius R. Actually, the role of the nucleus matter may be essential,– especially in the case of medium and heavy nuclei. For the analysis of the effects of the nucleus matter we will apply the optical model of the nucleus at high energy based on the conception of the refraction index [1, 2].

Taking into account the refraction indexes of the particles a and b, the influence of the nucleus matter on the coherent inelastic processes implies the introduction of the additional complex phase shift into Eq. (1.4): the exponential factor $\exp(-iq_{\min}z)$ is replaced by $Q = \exp[-iq_{\min}z + i\delta(\rho, z)]$. In the case of the spherical nucleus with the constant density $n(\rho, z) = n_0$ inside the interval $0 \le |z| \le \sqrt{R^2-\rho^2}$ ($\rho = |\rho|$) and $n(\rho, z) = 0$ outside this interval, the additional phase inside the indicated interval is described by the equation:

$$\delta(\rho,z) = (\chi_a - \chi_b)\, z + (\chi_a + \chi_b)\sqrt{R^2-\rho^2}, \qquad 0 \le |z| \le \sqrt{R^2-\rho^2}, \qquad (2.1)$$

where

$$\chi_a = \frac{2\pi n_0}{k} f_{a+N\to a+N}(0), \quad \chi_b = \frac{2\pi n_0}{k} f_{b+N\to b+N}(0).$$

Here $f_{a+N\to a+N}(0)$ and $f_{b+N\to b+N}(0)$ are the average amplitudes of elastic scattering of the particles a and b on a nucleon at the zero angle in the laboratory frame. The relations (2.1) are valid at $|\chi_a|/k \ll 1$, $|\chi_b|/k \ll 1$.

After the replacement $q_{\min}z \to q_{\min}z - \delta(\rho, z)$ in Eq. (1.4) and the integration over z we obtain the following expression for the cross-section of the coherent reaction $a \to b$:

$$\sigma_{\rm coh}(a \to b) = \frac{8\pi^3}{k^2} n_0^2 \frac{|f_{a+N\to b+N}(0)|^2}{|q_{\min} + \chi_a - \chi_b|^2} \times$$

$$\times \int_0^R \left| \exp[-2i\,(q_{\min} - \chi_a)\sqrt{R^2-\rho^2}] - \exp[2i\chi_b\sqrt{R^2-\rho^2}]\right|^2 \rho\, d\rho. \qquad (2.2)$$

3. Dependence of cross-sections of inelastic coherent processes on the nucleus radius

Taking into account the optical theorem [3], $\mathrm{Im}\,\chi_a = n_0\sigma_{aN}/2$, $\mathrm{Im}\,\chi_b = n_0\sigma_{bN}/2$, where σ_{aN} and σ_{bN} are the total cross-sections of interaction of the particles a and b with nucleons, averaged over the protons and neutrons of the nucleus.

Let us consider the situation when the total cross-section of the interaction of the initial particle a with nucleons is small, so that $|\chi_a|R \ll 1$, but $|\chi_b|R \gg 1$. In so doing, $L_a \gg R$, $L_b \ll R$, where

$L_a = (n_0 \sigma_{aN})^{-1}$ and $L_b = (n_0 \sigma_{bN})^{-1}$ are the mean lengths of free path of the particles a and b, respectively, inside the nucleus. In particular, we can deal with the coherent production of vector mesons ρ^0, ω, ϕ at the interaction of very high energy photons with heavy nuclei.

It follows from Eq. (2.2), neglecting the terms, depending on the masses m_a and m_b and tending to zero at very high energies, that:

$$\sigma_{\rm coh}(a \to b) = \pi R^2 \left| \frac{f_{a+N\to b+N}(0)}{f_{b+N\to b+N}(0)} \right|^2 . \tag{3.1}$$

Let us emphasize that, according to Eq. (3.1), the effective cross-section of the coherent process $a \to b$ on a nucleus at very high energies has the same dependence on the number of nucleons (proportional to $A^{2/3}$) as the cross-section of scattering of the final particle b on the "black" nucleus, despite the smallness of the cross-section of interaction of the initial particle a with a separate nucleon (in connection with this, see [4]).

For the coherent process $\gamma \to \rho^0$ on the lead nucleus ($R = 1.1 \cdot 10^{-13} A^{1/3}$ cm ≈ 6.5 Fm, $L_\rho \sim 1.5$ Fm, $|f_{\gamma+N\to\rho+N}(0)/f_{\rho+N\to\rho+N}(0)|^2 \sim 10^{-3}$) the formula (3.1) is applicable at the energies of γ-quanta above several tens of GeV in the nucleus rest frame ($k \gg m_\rho^2 L_\rho \sim 4.5$ GeV). In doing so, $\sigma_{\rm coh}(\gamma + Pb \to \rho^0 + Pb) \sim 1.3$ mbn.

It is easy to verify that the expansion of exponents in the expression (2.2) into the power series under the conditions $|q_{\rm min}|R \ll 1$, $|\chi_a|R \sim R/L_a \ll 1$, $|\chi_b|R \sim R/L_b \ll 1$ leads to the relation (1.5), as one would expect. In this limit $\sigma_{\rm coh}(a \to b)$ is proportional to R^4 (or to $A^{4/3}$).

4. Summary

In the present work (see also, e.g., [5]) the coherent processes at the interaction of ultrarelativistic particles with atomic nuclei are investigated. The role of these processes essentially increases at very high energies due to the fact that the minimal momentum, transferred to a nucleon, tends to zero with increasing energy. For the purpose of the analysis of the influence of the nucleus matter on coherent reactions, the conception of the refraction index is used. The dependence of the effective cross-sections of the inelastic processes on the nuclear radius and the mean lengths of free path of the initial and final particles in the nucleus matter is analyzed.

Finally, let us remark that the above formalism may be easily generalized also for the case of coherent inelastic multiparticle processes on a nucleus of the type $a \to \{b_1, b_2, b_3 \ldots b_i\}$, and for the case of coherent processes at collisions of two ultrarelativistic nuclei.

References

[1]M. Goldberger, K. Watson, *Collision theory*, New York-London- Sydney, 1964, chapter 11.

[2]R. Jastrov, Phys. Rev. **82**, 261 (1951) .

[3]L.D. Landau, E.M. Lifshitz, *Quantum Mechanics. Nonrelativistic theory*, Moscow, Nauka, 1989; §125 [transl. Pergamon Press, New York, 1977].

[4]V.N. Gribov, Zh. Eksp. Teor. Fiz. **57**, 1307 (1969).

[5]V.L. Lyuboshitz and V.V. Lyuboshitz, Yad. Fiz. **70**, 1224 (2007) [Phys. At. Nucl. **70**, 1185 (2007)] .