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The η' meson with staggered fermions

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We report results from a high-statistics calculation of pseudoscalar flavor singlet masses with Asqtad staggered fermions and discuss the implications for the validity of the "fourth-root trick" used in the staggered formulation. We calculate the masses of the η and η' mesons and the associated mixing angles. We find no evidence of pathology introduced by the "fourth-root trick".

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1. Introduction

Correctly and accurately calculating the spectrum of flavor singlet mesons is an important test of lattice QCD methods. A full non-perturbative $N_f = 2+1$ flavor calculation can provide an understanding of how the topological effects in the fermion sea raise the mass of the η and η' mesons above that of the pion.

The pseudoscalar singlet sector is of particular interest to the question of the validity of the fourth-root trick employed in staggered fermion simulations. This trick reduces the native four degenerate flavors of staggered sea fermions to one, allowing the simulation of an arbitrary number of flavors. Although the staggered formulation has generated an impressive set of high-precision calculations with results in agreement with experiment, doubts have been expressed about the theoretical robustness of the fourth-root trick. In particular it has been suggested that the $\eta - \eta'$ sector would be where one would look for the failure of the formulation to be evident [5, 6]. Some of these concerns have been addresses theoretically, see e.g. [7], and [8].

Determinations of singlet quantities require the costly calculation of disconnected diagrams. With N_f flavors the pseudoscalar singlet propagator contains N_f connected terms:

$$\langle \sum_i \overbrace{\bar{q}_i(x')(\gamma_5 \otimes \mathbf{1})\, \underbrace{q_i(x') \sum_j \bar{q}_j}(x)(\gamma_5 \otimes \mathbf{1}) q_j}(x) \rangle \tag{1.1}$$

and N_f^2 disconnected terms:

$$\langle \sum_i \overbrace{\bar{q}_i(x')(\gamma_5 \otimes \mathbf{1}) q_i}(x') \sum_j \overbrace{\bar{q}_j(x)(\gamma_5 \otimes \mathbf{1}) q_j}(x) \rangle. \tag{1.2}$$

Here we use the operator $(\gamma_5 \otimes \mathbf{1})$ to indicate that the state has γ_5 Dirac structure and singlet staggered taste structure.

We can see the how the disconnected diagrams raise the mass of the singlet over the mass of the octet meson by examining a simple chiral expansion of the singlet propagator in the flavor-symmetric case. Using μ^2 to represent the effective coupling between two pion propagators, we can write:

$$\begin{aligned}
\tilde{G}_{\eta'}(p) &= \frac{1}{p^2+m_\pi^2} && (1.3)\\
&- \frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2} && (1.4)\\
&+ \frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2} && (1.5)\\
&- \frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2}\mu^2\frac{1}{p^2+m_\pi^2} && \\
&+ \ldots && (1.6)\\
&= \frac{1}{p^2+m_\pi^2+\mu^2}, && (1.7)
\end{aligned}$$

where the mass squared has shifted by μ^2.

Ensemble	N_f	$10/g^2$	$L^3 \times T$	$am_{l/s}$	m_π	N_{configs}	N_{traj}
			$a = 0.12$fm				
A	0	8.00	$20^3 \times 64$	0.050	0.757(1)	6154	
B	2+1	6.75	$24^3 \times 64$	0.006/0.03	0.280(1)	4453	26718
			$a = 0.09$fm				
C	2+1	7.095	$32^3 \times 64$	0.00775/0.031	0.357(3)	2811	16866

Table 1: Ensembles used in the simulations.

It is important to note that this expansion is valid only for full QCD, with sea quarks working properly. Line 1.3 is the connected contribution and the pion propagator. In a quenched world, the expansion would stop at the end of line 1.4, with the single μ^2 coupling the two valence quark loops that make up the quenched disconnected propagator.

We can examine the behavior or mis-behavior of sea quarks by taking a ratio of the disconnected contribution to the connected contribution. We expect that the connected propagator and the full singlet propagator at large time separations decays like $G_\pi(t) \sim e^{-m_\pi t}$ and $G_{\eta'}(t) \sim e^{-m_{\eta'} t}$, respectively, with $m_{\eta'} = m_\pi + \mu^2$. Therefore in full QCD we expect at large times

$$R(t) = \frac{N_f^2 D(t)}{N_f C(t)} = \frac{N_f C(t) - G_{\eta'}(t)}{N_f C(t)} = 1 - A \exp\left[-(m_\eta - m_\pi)t\right]. \tag{1.8}$$

Whereas in quenched QCD we expect

$$R(t) = \frac{N_f^2 D(t)}{N_f C(t)} = A' + B' t. \tag{1.9}$$

One can imagine performing some operation on the fermion matrix which would introduce some other pathology, say change the number of flavors of the sea quarks or the mass of the sea quarks with respect to the valence quarks. In these cases we might expect to be able measure a deviation of the behavior of the disconnected-to-connected ratio from the form of 1.8. The question is: does the fourth-root-trick cause such a measureable pathology?

2. Lattice simulation

Specifically for this project we have generated two long ensembles of dynamical staggered gauge configurations. These are listed as ensembles B and C in Table 1. We refer to these as the "coarse" and "fine" ensembles, respectively. We used the ASQTAD improved staggered fermion action, with the standard fourth root trick. We used tadpole improved Symanzik gauge action. The two long dynamical ensembles were generated on the UKQCD's QCDOC installation.

For comparison we also generated a 6154-configuration quenched ensemble, labeled A in Table 1. This is an extension of a 408-configuration generated by the MILC collaboration. This extension was generated on a small cluster in Liverpool.

We calculated the connected components of the using standard point sources. Due to the inherent noisiness of disconnected correlators we use a stochastic source method.

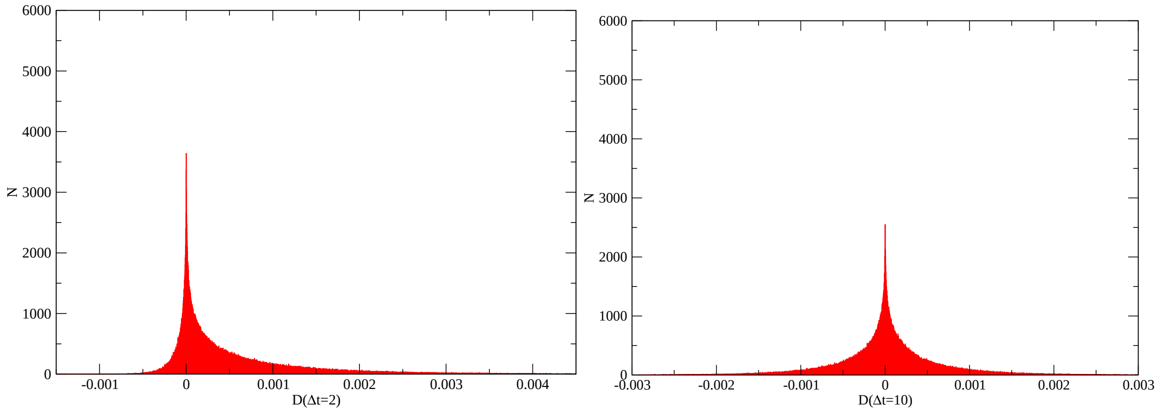


Figure 1: Histograms of disconnected correlator measurements on 393856 measurements (6154 configs $\times$ 64 starting time slices) on quenched configurations for $\Delta t = 2$ and $\Delta t = 10$. Note how symmetric the distribution has become for moderate time separations.

In practice, for the disconnected correlator we use a variance-reduction trick due to Venkataraman and Kilcup [4]. which is applicable to staggered operators with an even number of links separating the quark and antiquark. The disconnected correlator is the product of two valence loop operators:

$$D(\Delta t) = \langle \mathscr{O}_{\gamma_5\otimes\mathbf{1}}(t)\mathscr{O}_{\gamma_5\otimes\mathbf{1}}(0)\rangle \tag{2.1}$$

In [1] we showed that since individual measurements of $\mathscr{O}_{\gamma_5\otimes\mathbf{1}}(t)$ fall in a Gaussian distribution, their product, the individual measurements of the disconnected correlator $D(t)$, will fall in a long-tailed distribution shaped like a modified Bessel function of the second kind. Such a distribution always has the peak at zero, and the mean is strongly dependent on the asymmetry of the long tails. That is, outliers many standard deviations from the mean are relatively common. Figure 1 shows such distributions for $\Delta t = 2$ and $\Delta t = 10$. This is the essence of the difficulty in measuring disconnected diagrams.

We measure disconnected and connected correlators with light and strange valence quarks with both local and fuzzed sources.

3. Results

3.1 D/C Ratio

To test the behavior of the sea quarks we constructed the ratio of the disconnected contribution to the connected contribution of the propagator as described in Section 1. For the $N_f = 2+1$ flavor ensembles, the ratio we construct is a generalization of Eqn. 1.8:

$$R(t) = \frac{4D_{qq}(t) + 4D_{qs}(t) + D_{ss}(t)}{2C_{qq}(t) + C_{ss}(t)}. \tag{3.1}$$

The dynamical data are consistent with the $1 - Ae^{-\delta m t}$ form of Eq. 1.8, saturating near unity. They are easily distinguished from the linear curve followed by the quenched data. It is worth reiterating that we were unable to resolve this behavior in earlier works when we were using ensembles of ~ 400 configurations.

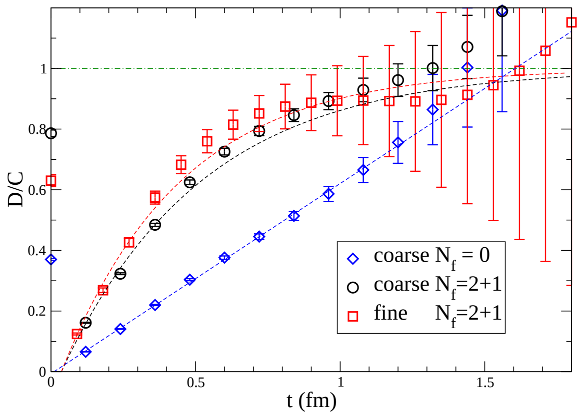


Figure 2: The ratio of the disconnected to connected contributions to pseudoscalar singlet propagators. The circles and squares are the coarse and fine $N_f = 2+1$ flavor ensembles respectively and fall on a curve consistent with Eqn. 1.8. The diamond symbols are the quenched data and fall on a linear curve consistent with Eqn. 1.9.

3.2 Spectroscopy & mixing angles

We perform factorizing fits of a 4×4 correlator matrix with elements corresponding to source and sink flavor (q or s) and fuzzing (F or L for fuzzed of local).

$$G = \begin{bmatrix} C_{qqLL} - 2D_{qqLL} & C_{qq} - 2D_{qqLF} & -\sqrt{2}D_{qsLL} & -\sqrt{2}D_{qsLF} \\ C_{qqFL} - 2D_{qqFL} & C_{qq} - 2D_{qqFF} & -\sqrt{2}D_{qsFL} & -\sqrt{2}D_{qsFF} \\ -\sqrt{2}D_{sqLL} & -\sqrt{2}D_{sqLF} & C_{ssLL} - D_{ssLL} & C_{ssLF} - D_{ssLF} \\ -\sqrt{2}D_{sqFL} & -\sqrt{2}D_{sqFF} & C_{ssFL} - D_{ssFL} & C_{ssFF} - D_{ssFF} \end{bmatrix} \tag{3.2}$$

We fit to a common ladder of mass states:

$$G_{ij}(t) = \sum_{k=0}^{N_{\text{exp}}} \frac{a_{i,k}a_{j,k}}{2E_i} e^{-m_k t}. \tag{3.3}$$

We identify the ground state and first excited state with the η and η' mesons. A second excited state likely corresponds to the $\eta(1295)$. We find :

Ensemble	aE_0	aE_1	aE_2
B	0.410(3)	0.529(12)	1.14(13)
C	0.296(3)	0.462(15)	0.822(68)

Unfortunately, computer time was not sufficient to run the finer dynamical ensemble (B) at the same light quark mass as the coarse lattice, preventing us from taking a meaningful continuum limit. We therefore quote these results with statistical errors only. In Figure 3 we summarize our results as a function of both a^2 and m_π^2. In Figure 3 we compare our results with those of RBC/UKQCD collaboration [2] and HSC collaboration[3], as a function of squared pion mass.

The η-η' mixing can be described using either the $SU(3)$ basis or the quark-flavor basis. We use the latter in which

$$|\eta_q\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right)$$

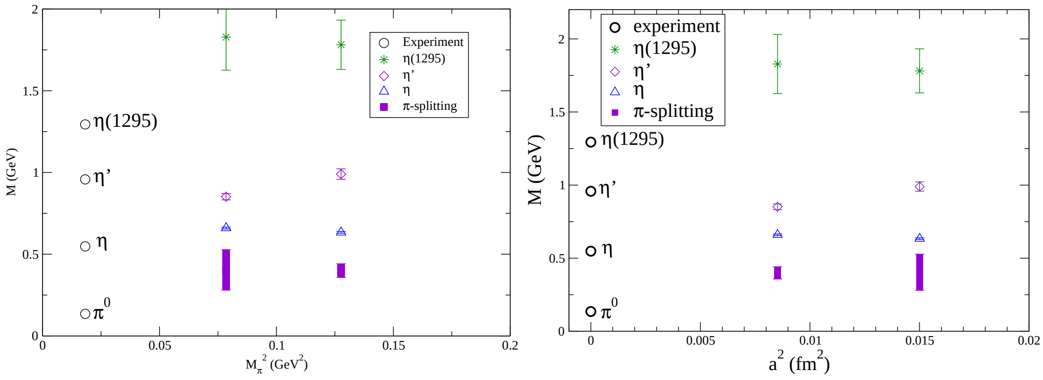
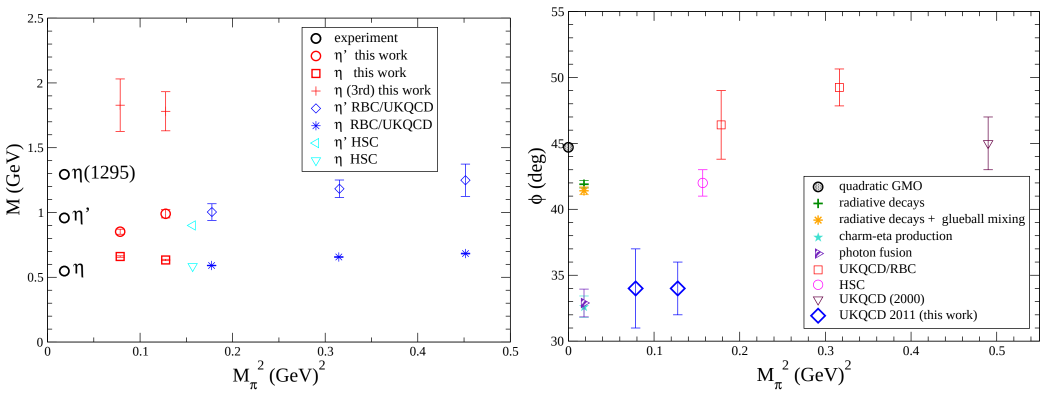


Figure 3: Spectrum results for $N_f = 2+1$ dynamical pseudoscalar singlets, showing the ground state (triangle) and first two excited states (diamond burst) which we identify as the η, η' and $\eta(1295)$. We plot against Goldstone pion mass squared (left) and lattice spacing squares (right). For comparison we display the staggered pion splitting (bars).



a. b.

Figure 4: Comparison of spectrum results for the η, η' and $\eta(1295)$ (a), and mixing angle results (b). We convert the RBC/UKQCD angle from the $SU(3)$ basis to the quark flavor basis.

$$|\eta_s\rangle = |s\bar{s}\rangle \tag{3.4}$$

and

$$\begin{bmatrix} |\eta\rangle \\ |\eta'\rangle \end{bmatrix} = \begin{bmatrix} \cos\phi_P & -\sin\phi_P \\ \sin\phi_P & \cos\phi_P \end{bmatrix} \begin{bmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{bmatrix} . \tag{3.5}$$

With $SU(3)$ flavor-breaking we are the mixing is in principle described by *two* mixing angles [9]. In the flavor basis the two mixing angles can be related to the fit amplitudes and decay constants:

$$\begin{bmatrix} a_{q\eta} & a_{s\eta} \\ a_{q\eta'} & a_{s\eta'} \end{bmatrix} = \begin{bmatrix} f_q\cos\phi_q & -f_s\sin\phi_s \\ f_q\sin\phi_q & f_s\cos\phi_s \end{bmatrix} , \tag{3.6}$$

where the a's are the amplitudes in Eqn. (3.3) resulting from the fits.There are arguments [10, 11] suggesting that $\phi_s \sim \phi_q$. We solve for the angles separately using $\tan\phi_q = \frac{a_{q\eta'}}{a_{q\eta}}$ and $\tan\phi_s = -\frac{a_{s\eta'}}{a_{s\eta}}$. We also perform fits of our fit amplitudes to Eqn 3.6 with the constraint $\phi_q = \phi_s$. We present

β	$\phi_q^{\rm est}$	$\phi_s^{\rm est}$	$\phi^{\rm fit}$	χ^2/dof
6.75	25(4)	36(2)	34(3)	$8.2/3$
7.095	40(5)	34(2)	34(2)	$3.7/3$

Table 2: η-η' mixing angles in degrees defined and determined as described in the text.

the results in Table 2. We plot the result for the single-angle in Fig. 4b along with results from the RBC/UKQCD collaboration [2] and the Hadron Spectrum Collaboration [3]HSC and experimental numbers from the summary in [12].

4. Conclusions

We have calculated the pseudoscalar singlet spectrum and the $\eta - \eta'$ mixing angles with $N_f = 2+1$ flavors of dynamical staggered fermions. We get results in reasonable agreement with that from other formulations and experiment. We see no "smoking-gun" evidence of any pathologies caused by the "fourth-root trick" in the staggered-formulation.

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