

$K^0 - \bar{K}^0$ mixing in the Standard Model from $N_f = 2 + 1 + 1$ Twisted Mass Lattice QCD

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We present preliminary results at $\beta = 1.95$ ($a = 0.077$ fm) on the first unquenched $N_f = 2 + 1 + 1$ lattice computation of the B_K parameter which controls the neutral kaon oscillations in the Standard Model. Using $N_f = 2 + 1 + 1$ maximally twisted sea quarks and Osterwalder-Seiler valence quarks we achieve $O(a)$ improvement and a continuum-like renormalization pattern for the four-fermion operator. Our results are extrapolated/interpolated to the physical light/strange quark mass but not yet to the continuum limit. The computation of the relevant renormalization constants is performed non perturbatively in the RI'-MOM scheme using dedicated simulations with $N_f = 4$ degenerate sea quark flavours produced by the ETM collaboration.

We get $B_K^{RGI}(a = 0.077) = 0.747(18)$, which when compared to our previous unquenched $N_f = 2$ determination and most of the existing results, suggests a rather weak B_K^{RGI} dependence on the number of dynamical flavours. We are at the moment analysing lattice data at two additional β values which will allow us to perform an extrapolation to the continuum limit.

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1. Introduction

The mixing $K^0 - \bar{K}^0$ plays an important role in the understanding of the physics of CP-violation. In the Standard model weak effective Hamiltonian (with three light flavours) at the lowest order this oscillation is described only by the operator

$$O^{\Delta S=2} = \frac{1}{4} [\bar{s} \gamma_\mu (1 - \gamma_5) d] [\bar{s} \gamma^\mu (1 - \gamma_5) d] \quad (1.1)$$

The matrix element of the $\Delta S = 2$ effective hamiltonian can be factorized into a short distance contribution which can be computed perturbatively and a nonperturbative long distance contribution containing the strong interaction effects. These nonperturbative effects are described by the hadronic matrix elements of the renormalized four-fermion operator

$$\langle \bar{K}^0 | \mathcal{H}_{eff}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_W^2}{16\pi^2} \left[\sum_{l,m=u,c,t} C_1^{(l,m)}(\mu) V_{ls}^* V_{ld} V_{ms}^* V_{md} \right] \langle \bar{K}^0 | \hat{O}^{\Delta S=2}(\mu) | K^0 \rangle \quad (1.2)$$

The B_K parameter parametrizes the deviation of the hadronic element from the Vacuum Insertion Approximation (VIA)

$$\langle \bar{K}^0 | \hat{O}^{\Delta S=2}(\mu) | K^0 \rangle \equiv \langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle_{VIA} \hat{B}_K(\mu) = \frac{8}{3} f_K^2 M_K^2 \hat{B}_K(\mu) \quad (1.3)$$

Combining the lattice computation of the B_K parameter with the experimental value of ϵ_K one can constrain the values of the CKM matrix elements.

2. Lattice setup

In the gauge sector we use the Iwasaki action while the dynamical quarks have been regularized employing the twisted mass formalism at maximal twist [1] which provides automatic $O(a)$ improvement [2, 3]. The fermionic action for the light doublet in the sea is given by

$$S_l^{sea, Mtm} = \sum_x \bar{\chi}_l(x) [D_W[U] + m_{0,l} + i\mu_l \gamma_5 \tau_3] \chi_l(x) \quad (2.1)$$

where we follow the notation in [4]. In the heavy sector the sea quark action becomes

$$S_h^{sea, Mtm} = \sum_x \bar{\chi}_h(x) [D_W[U] + m_{0,h} + i\mu_h \gamma_5 \tau_1 + \mu_8 \tau_3] \chi_h(x) \quad (2.2)$$

The $O(a)$ improvement and a continuum-like renormalization pattern for the four-fermion operators can be achieved by introducing an Osterwalder-Seiler [5] valence quark action allowing for a replica of the down (d, d') and the strange (s, s') quarks [6]. The action for each OS valence flavour χ_f reads

$$S_f^{val, OS} = \sum_x \bar{\chi}_f(x) [D_W[U] + m_{0,f} + i\mu_f \gamma_5 r_f] \chi_f(x) \quad (2.3)$$

where the Wilson parameters should satisfy the relation $-r_s = r_d = r_{d'} = r_{s'}$.

In table 1 we give the details of the simulation of the bare B_K parameter. For the inversions in the valence sector we used the stochastic method with propagator sources located at random timeslices in order to increase the statistical information [7, 8].

	$a\mu^{sea}$	aM_{PS}^{ll} (MeV)	$L^3 \times T$	#confs
	0.0025	~ 270	$32^3 \times 64$	144
$\beta=1.95$	0.0035	~ 320	$32^3 \times 64$	144
($a \sim 0.077$ fm)	0.0055	~ 400	$32^3 \times 64$	144
$a\mu_\sigma = 0.135$ $a\mu_\delta = 0.17$	0.0075	~ 460	$32^3 \times 64$	80
$a\mu^{val} = \{a\mu^{sea}, 0.0141, 0.0180, 0.0219\}$	0.0085	~ 490	$24^3 \times 48$	244

Table 1: Simulation details for the bare B_K parameter. The bare valence quark mass equal to 0.0141, 0.0180, 0.0219 have been chosen to allow for a smooth interpolation around the physical strange quark mass.

ensemble	$a\mu^{sea}$	am_{PCAC}^{sea}	aM^{sea}	θ^{sea}	$a\mu^{val}$	am_{PCAC}^{val}	#confs
1m	0.0085	-0.04125(13)	0.03308(10)	-1.3109(08)	set 1	-0.02116(2)	304
1p	0.0085	+0.04249(13)	0.03286(09)	+1.3091(08)	set 1	+0.01947(19)	304
7m	0.0085	-0.03530(13)	0.02851(10)	-1.2681(10)	set 1	-0.0216(2)	400
7p	0.0085	+0.03608(11)	0.02854(08)	+1.2683(09)	set 1	+0.01947(19)	400
8m	0.0020	-0.03627(11)	0.02804(08)	-1.4994(02)	set 1	-0.0216(2)	400
8p	0.0020	+0.03624(13)	0.02743(10)	+1.4978(03)	set 1	+0.01947(19)	400
3m	0.0180	-0.0160(2)	0.02191(09)	-0.6068(59)	set 2	-0.0160(2)	352
3p	0.0180	+0.0163(2)	0.02183(09)	+0.6015(57)	set 2	+0.0162(2)	352
2m	0.0085	-0.02091(16)	0.01815(11)	-1.0834(32)	set 1	-0.0213(2)	352
2p	0.0085	+0.0191(2)	0.01692(13)	+1.0445(45)	set 1	+0.01909(18)	352
4m	0.0085	-0.01459(13)	0.01404(08)	-0.9206(43)	set 2	-0.01459(13)	224
4p	0.0085	+0.0151(2)	0.01420(12)	+0.9289(64)	set 2	+0.0151(2)	224

Table 2: Details of the analysed ensembles for the RCs at $\beta = 1.95$; set 1 = {0.0085, 0.0150, 0.0203, 0.0252, 0.0298} and set 2 = {0.0060, 0.0085, 0.0120, 0.0150, 0.0180, 0.0203, 0.0252, 0.0298}. Here $M^{sea} = \sqrt{(Z_A m_{PCAC}^{sea})^2 + (\mu^{sea})^2}$, with a self-consistent Z_A -value.

The computation of the renormalization constants (RCs) for the relevant two- and four-fermion operators has been performed adopting the RI'-MOM scheme [9]. These RCs are computed by extrapolating to the chiral limit the RCs estimators measured at several quark mass values. We have performed dedicated runs with $N_f = 4$ degenerate sea quarks in order to be able to estimate the chiral limit of the RCs. In these $N_f = 4$ simulations working at maximal twist would imply a considerable fine tuning effort due to the difficulties in determining am_{PCAC} near $am_{PCAC} = 0$. Instead, working out of maximal twist the stability of the simulations increases and the $O(a)$ improvement of the RC estimators is achieved by averaging simulations with an equal value of the polar mass M^{sea} but opposite value of m_{PCAC}^{sea} and θ^{sea} , where $\tan \theta^{sea} = \frac{Z_A m_{PCAC}^{sea}}{\mu^{sea}}$ [10]. We label these ensembles as Ep/m where E=1,2... and p/m refers to the sign(θ^{sea}). In table 2 we report the parameters of the ensembles analysed to compute the RCs.

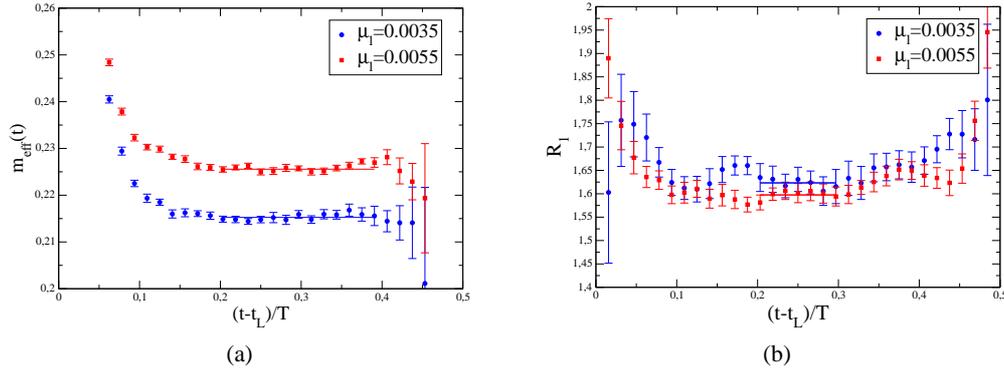


Figure 1: (a) Effective mass of the pseudoscalar mesons made out of two mass-degenerate quarks regularized with opposite Wilson parameters (see [12] for definitions) (b) Quality of the ratio plateaus at fixed value of $\mu_h = 0.0180$

3. The K-meson bag parameter

The only four-fermion operator relevant for B_K in the Standard Model is ¹ (see [6, 11] for notation)

$$Q_1^+ = O_{VV+AA}^+ = [(\bar{s}'\gamma_\mu d)(\bar{s}'\gamma^\mu d') + (\bar{s}'\gamma_\mu\gamma_5 d)(\bar{s}'\gamma^\mu\gamma_5 d')] + [d \leftrightarrow d'] \quad (3.1)$$

It is multiplicatively renormalizable if the Wilson parameters appearing in the action of the OS fermions satisfy the relation $-r_s = r_d = r_{d'} = r_{s'}$ [6].

In order to estimate the B_K -parameter we compute a three point correlation function with an insertion of Q_1^+ free to move in lattice time t , and two "K-meson walls" with pseudoescalar quantum numbers and with a fixed time separation between them $|t_L - t_R| = T/2$. In this way, the lattice estimator for the bare B_K parameter can be obtained from the ratio $R_1(t)$ for times $t_L \ll t \ll t_R$

$$R_1(t) = \frac{C_{\bar{K}Q_1K}^{(3)}(t-t_L, t-t_R)}{C_{\bar{K}}^{(2)}(t-t_L)C_K^{(2)}(t-t_R)} \xrightarrow{t_L \ll t \ll t_R} \frac{8}{3} B_K \quad (3.2)$$

where the two- and three-point correlation functions are defined as in [12]. In figures 1a and 1b the quality of the plateaus of the effective mass and the ratio R_1 are shown for two values of the light quark mass and one typical value of $\mu_h = 0.0180$.

Chiral extrapolations in the light quark mass are performed using SU(2) Partially Quenched Chiral Perturbation Theory at NLO [13, 14]. We carry out three chiral fits, one for each simulated heavy mass, according to the fit ansatz

$$B_K(M_{ll}, M_{hh}) = B'_K(M_{hh}) \left[1 + b'(M_{hh}) \frac{(M_{ll})^2}{f_0^2} - \frac{(M_{ll})^2}{32\pi^2 f_0^2} \log \frac{(M_{ll})^2}{16\pi^2 f_0^2} \right] \quad (3.3)$$

as we illustrate in figure 2a. The decay constant f_0 has been obtained by analysing our data for the mass and decay constant of the ll pseudoscalar meson following the procedure of [4]. Our result is $f_0 = 120.99(09)$ MeV. In order to estimate the systematic errors affecting this extrapolation in the u/d -quark mass, we also tried a first order polynomial fit.

¹Note that it contains the same physical information as 1.1

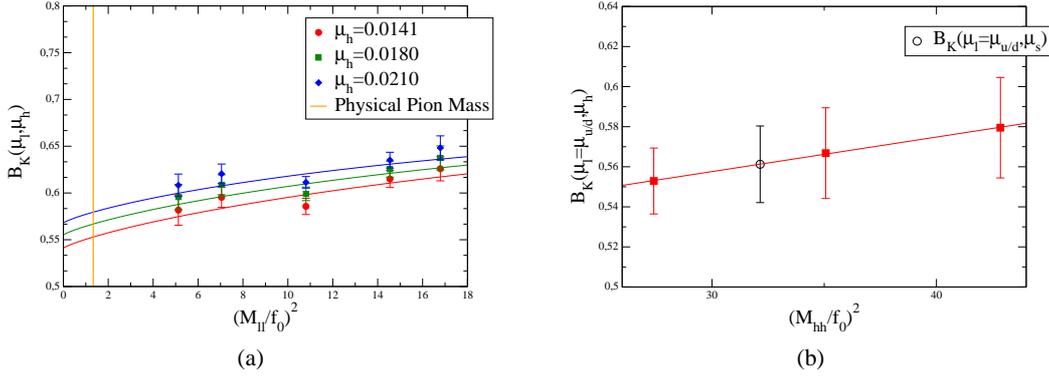


Figure 2: (a) Chiral fit according to the ansatz (3.3); (b) Interpolation in the vicinity of the physical strange quark mass.

Since we have obtained our lattice data for masses around the physical strange quark mass, we determine the bare B_K parameter through an interpolation in $(M_{hh})^2$ to the physical mass $M_{hh}^2 = 2M_K^2 - M_\pi^2$ as is shown in figure 2b.

Finally, the renormalized B_K parameter is given by the relation [12],

$$\hat{B}_K = \frac{Z_{11}^+}{Z_A Z_V} B_K \quad (3.4)$$

where Z_{11}^+ , Z_A and Z_V are the RCs of the operators Q_1^+ , the axial and the vector currenty. The relevant RCs both for the two- and four-fermion operators have been computed non-perturbatively in the RI'-MOM scheme following [9]. Our strategy to remove the inescapable $O(a)$ discretization effects consists in performing a θ^{sea} -average of the RCs estimators as described in [10].

Using the notation of [11], we compute the elements of the dynamical matrix D_{ij} for each value of $a^2 \tilde{p}^2 \equiv \sum_v \sin^2(ap_v)$, μ^{val} and Ep/m (and hence of M^{sea}). As described in [12], the valence chiral limit extrapolation is safely determined with the help of the following fit ansatz

$$D_{ij}(\tilde{p}^2, \text{Ep}/m; \mu^{val}) = D_{ij}(\tilde{p}^2; \text{Ep}/m) + A_{ij}(\tilde{p}^2; \text{Ep}/m) M^{val} + \frac{B_{ij}(\tilde{p}^2; \text{Ep}/m)}{(M_{PS}^{val})^2} \quad (3.5)$$

at fixed values of $a^2 \tilde{p}^2$ and Ep/m (and hence of M^{sea}). The last term takes into account the contribution of one Goldstone boson pole (GBP) (see Appendix A of [12] for details). In figure 3a we show the smooth dependence of D_{11} on $(aM_{PS}^{val})^2$ for a representative value of the momentum even before the GBP subtraction. Hence the one GBP contribution is strongly suppressed on the vertex D_{11} . Having performed the GBP subtraction and the valence chiral extrapolation, the $O(a)$ discretization effects are removed by averaging $D_{ij}(\tilde{p}^2; \text{Ep}/m; \mu^{val} = 0)$ over θ^{sea} . By combining the valence chiral limit estimator of D_{11} with the corresponding chiral limit of the quark field RC, Z_q , we obtain the intermediate quantities $Z_{11}^+(\tilde{p}^2, \mu^{val} = 0, \text{Ep}/m)$. Then, the sea chiral limit is taken at each fixed value of $(a\tilde{p})^2$ by fitting these intermediate estimators to a polynomial of first order in $(aM^{sea})^2$. We find that the dependence on the sea quark mass is very mild, as it can be seen from figure 3b. In this way, we get the chiral limit RC estimators, $Z_{11}^+(\tilde{p}^2)$.

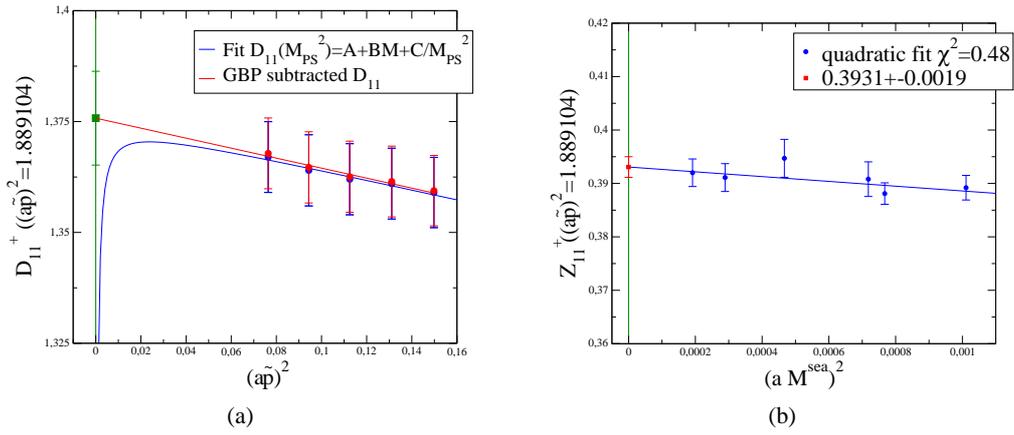


Figure 3: (a) Goldstone boson pole subtraction and valence chiral limit of D_{11} for the ensemble 2p, $\mu^{sea} = 0.0085$ and $(a\tilde{p})^2 = 1.889104$; (b) Sea chiral limit extrapolation of Z_{11}^+ for $(a\tilde{p})^2 = 1.889104$.

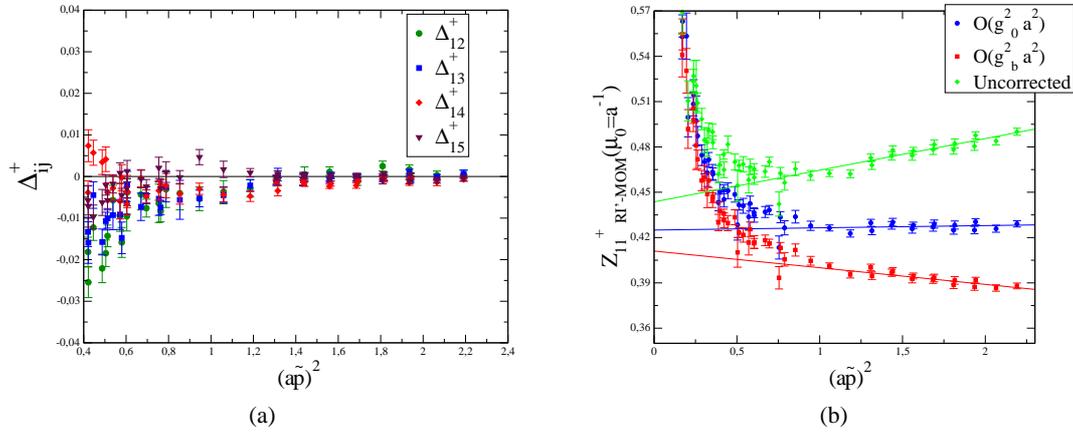


Figure 4: (a) Mixing coefficients in the valence and sea chiral limit; (b) RI'-MOM computation of the multiplicative renormalization factor Z_{11}^+ .

An important issue that arises in our analysis of the RCs is that wrong chirality mixings Δ_{ij} can affect the renormalization pattern of Q_1^+ at order a^2 or higher [6]. However, as it is clearly seen from figure 4a, these mixing coefficients are negligible within errors in our data.

Improved chiral limit RC estimators are obtained by analytically subtracting $O(a^2 g^2)$ discretization errors directly from the vertex D_{ij} , using the one-loop results in lattice perturbation theory calculated in [15]. The effect of this perturbative correction is illustrated in figure 4b where we compare the RI'-MOM values for Z_{11}^+ at the reference scale $\mu_0 = a^{-1}$ in three cases: uncorrected, corrected with a bare lattice coupling $g_0 = 6/\beta^2$ and with a boosted one $g_b^2 \equiv g_0^2/\langle P \rangle$, where the average plaquette $\langle P \rangle$ is computed non perturbatively.

After bringing $Z_{11}^+(\tilde{p}^2)$ to a common reference scale $\tilde{p}^2 = \mu_0^2 = a^{-2}$ by employing the known NLO running formula [16], we applied two different \tilde{p}^2 methods to remove the remaining $O(a^2 \tilde{p}^2)$ discretization errors [17]. In the M1-method we perform a linear fit in $(a\tilde{p})^2$ of $Z_{11}^+(\mu_0^2)$ in the interval $(a\tilde{p})^2 = [1.5 : 2.0]$ (see figure 4b). Alternatively, the M2-method consists in simply taking the weighted average of the RC in the momentum window $(a\tilde{p})^2 = [1.8 : 2.0]$.

Combining four- and two-fermion RCs computed as in [10], we obtain our final result for the RGI B_K at $\beta = 1.95$, corresponding to $a = 0.077$ fm, by employing the estimates of the RCs from the procedure M1 and eq.(3.3) for the extrapolation of our B_K data to the u/d-quark physical point

$$B_K^{RGI}(a = 0.077) = 0.747(05)(17)[18]$$

where the first error is statistical, estimated from a bootstrap analysis, and the second results from summing in quadrature several systematic uncertainties (0.014 from the spread in using eq.(3.3) or a polynomial fit in the extrapolation to the physical point and 0.009 associated to the renormalization). The error quoted in brackets is the total error obtained by a sum in quadrature of the systematic and statistical ones.

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