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Rho-gamma mixing and $e^+\ e^-$ vs. τ spectral functions

Robert Szafron*
Institute of Physics, University of Silesia, ul. Uniwersytecka 4, PL-40007 Katowice, Poland
E-mail: rszafron@us.edu.pl

Fred Jegerlehner
Humboldt-Universität zu Berlin, Institut für Physik, Newtonstrasse 15, D-12489 Berlin, Germany
Deutsches Elektronen-Synchrotron (DESY), Platanenallee 6, D-15738 Zeuthen, Germany
E-mail: fjeger@physik.hu-berlin.de

In this paper we present a simple model which enables us to calculate the effect of $\varrho^0-\gamma$ mixing for the pion form factor. We show that taking into account this effect together with other already known isospin breaking corrections solves the long standing e^+e^- vs. τ problem. After including the missing $\varrho^0-\gamma$ mixing effects in the isospin corrections of the τ data, τ-data based evaluations of the hadronic vacuum polarization contribution to the muon $g-2$ agree well with e^+e^--data based evaluations.

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1. Introduction

Isospin symmetry can be used to compare the data for the pion form factor measured in the process $e^+e^- \to \pi^+\pi^-$ with corresponding data obtained in τ decay ($\tau^- \to \pi^-\pi^0\nu_\tau$) [1–5]. In the limit of exact isospin symmetry we could express the cross section for $e^+e^- \to \pi^+\pi^-$ using the spectral function υ_- measured in τ decay in the form

$$\sigma^{I=1}_{e^+e^-\to\pi^+\pi^-} = \frac{4\pi\alpha^2}{s}\frac{\beta_0^3(s)}{\beta_-^3(s)}\upsilon_-, \tag{1.1}$$

where $\beta_0(s)$ and $\beta_-(s)$ are the pions velocities which depend on the phase spaces in both processes. Unfortunately, due to the differences in quarks masses, as well as the differences in electromagnetic interactions the isospin symmetry is broken and relation (1.1) is valid only approximately. The growing precision of the experiments in low energy hadron physics and its applications forces us to apply corrections for isospin breaking when both τ and e^+e^- data are to be compared. There are different types of corrections like: presence of omega meson in neutral channel, differences in masses of charged and neutral pions, different electromagnetic interaction in the final state [6–9], differences in masses and widths of a charged and neutral rho meson [10] and already mentioned different phase spaces.

All these isospin breaking corrections were not able to bring back into agreement the spectral functions measured in neutral and charged channel. This caused some problems for example in the determination of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment [11,12] (see also [13]). In this paper we will review the effect of $\varrho^0-\gamma$ mixing, discussed in detail in Ref. [14], which gives a significant contribution as an isospin breaking effect and enables us to understand the difference between e^+e^- and τ spectral functions.

2. The model

In order to understand the mixing effect between ϱ^0 and γ we need some input from a theoretical model. Within an effective field theory approach, we will use the simplest possible model which enables us to take into account $\varrho^0-\gamma$ interference which is scalar QED together with vector meson dominance. The Lagrangian of our model is given by

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\gamma\rho}, \tag{2.1}$$

$$\mathcal{L}_\pi = D_\mu\pi^+D^{+\mu}\pi^- - m_\pi^2\pi^+\pi^-; \quad D_\mu = \partial_\mu - i\,e\,A_\mu - i\,g_{\rho\pi\pi}\,\rho_\mu \tag{2.2}$$

$$\mathcal{L}_{\gamma\rho} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{M_\rho^2}{2}\rho_\mu\rho^\mu + \frac{e}{2\,g_\rho}\rho_{\mu\nu}F^{\mu\nu}. \tag{2.3}$$

In this approach we treat the ϱ^0 as a massive gauge boson [15]. We include all contributions which are relevant in order to describe the process $e^+e^- \to \pi^+\pi^-$. This is an important extension of the well known Gounaris-Sakurai model in which some of the contributions were neglected [16]. We have calculated the pions loops (see figure 1) contributing to the self energy functions. In our model there are contributions to the γ and ϱ^0 propagators and also to the mixed propagator $\varrho^0-\gamma$. All these self energy functions are given by the same formula up to a multiplicative constant

$$\Pi_{\gamma\gamma} = \frac{e^2}{48\pi^2}f(q^2),\ \Pi_{\gamma\rho} = \frac{e g_{\rho\pi\pi}}{48\pi^2}f(q^2),\quad \Pi_{\rho\rho} = \frac{g^2_{\rho\pi\pi}}{48\pi^2}f(q^2), \tag{2.4}$$

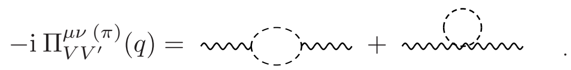

Figure 1: Pion loops contributing to the self energy functions ($V, V' = \gamma, \rho$).

with function $f(q^2)$ expressed in terms of one loop scalar integrals [17]

$$f(q^2) = \left(B_0(m_\pi, m_\pi; q^2)(q^2 - 4m_\pi^2) - 4A_0(m_\pi) - 4m_\pi^2 + \frac{2}{3}q^2 \right). \tag{2.5}$$

As usual we need to perform renormalization. Our problem is very similar to the well known $\gamma - Z$ mixing in the Standard Model [18]. We will not discuss the details of renormalization here, the only important thing is we diagonalize the $\gamma - \rho$ mass matrix which requires a rotation of the fields and this leads to the appearance of a direct coupling between ϱ^0 and the electrons - $g_{\rho ee} = 0.018149$[1] [14]. The renormalized self-energy functions are given by:

$$\Pi^{\text{ren}}_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(q^2) - q^2\,\Pi'_{\gamma\gamma}(0) \doteq q^2\,\Pi'^{\text{ren}}_{\gamma\gamma}(q^2), \tag{2.6}$$

$$\Pi^{\text{ren}}_{\gamma\rho}(q^2) = \Pi_{\gamma\rho}(q^2) - \frac{q^2}{M_\rho^2}\,\text{Re}\,\Pi_{\gamma\rho}(M_\rho^2), \tag{2.7}$$

$$\Pi^{\text{ren}}_{\rho\rho}(q^2) = \Pi_{\rho\rho}(q^2) - \text{Re}\,\Pi_{\rho\rho}(M_\rho^2) - (q^2 - M_\rho^2)\,\text{Re}\,\frac{d\Pi_{\rho\rho}}{ds}(M_\rho^2) \tag{2.8}$$

and we have used the fact that $\Pi_{\gamma\gamma}(0) = \Pi_{\gamma\varrho}(0) = \Pi_{\varrho\varrho}(0) = 0$ and $\Pi'_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(q^2)/q^2$.

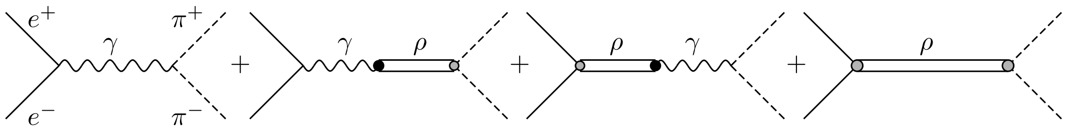

Figure 2: The different types of contribution to the pion form factor.

Before we proceed to the calculations of any observables we need to define the pion form factor. In our model it is given by a sum of four terms represented diagrammatically in figure 2

$$F_\pi(s) = \frac{e^2 D_{\gamma\gamma} + e(g_{\rho\pi\pi} - g_{\rho ee}) D_{\gamma\rho} - g_{\rho ee} g_{\rho\pi\pi} D_{\rho\rho}}{e^2 D_{\gamma\gamma}}. \tag{2.9}$$

The factor in the denominator comes from the normalization of the pion form factor in scalar QED, in which $F_\pi(s) = 1$, i.e., the vacuum polarization is absorbed into a running fine structure constant

$$\alpha \to \alpha(s) = \frac{\alpha}{1 + \Pi'^{\text{ren}}_{\gamma\gamma}(s)}. \tag{2.10}$$

The important point about (2.9) is that it takes into account in a proper way the interference effects. What we measure in e^+e^--annihilation is a coherent sum of the $I = 0$ and $I = 1$ parts, i.e., $|F^{(e)}_\pi(s)[I = 1] + F^{(e)}_\pi(s)[I = 0]|^2$, while in τ decay only the isovector part is contributing $|F^{(\tau)}_\pi(s)[I = 1]|^2$. Usual approximation is to neglect the interference term, but our model enables us to calculate the interference effects and the results presented in the next section shows that they are important.

[1]The value of the coupling $g_{\rho ee}$ can be obtained from the formula $g_{\rho ee} = \sqrt{12\pi\Gamma_{\rho ee}/M_\rho}$. This coupling $g_{\rho ee}$ is an important difference between our approach and the Gounaris-Sakurai model, which dose not contain any direct dependence on $\Gamma_{\rho ee}$.

3. Applications

In order to analyze the effects of the $\varrho^0 - \gamma$ mixing it is best to introduce the ratio

$$r_{\rho\gamma}(s) \equiv \frac{|F_\pi(s)|^2}{|F_\pi(s)|^2_{D_{\gamma\rho}=0}}. \tag{3.1}$$

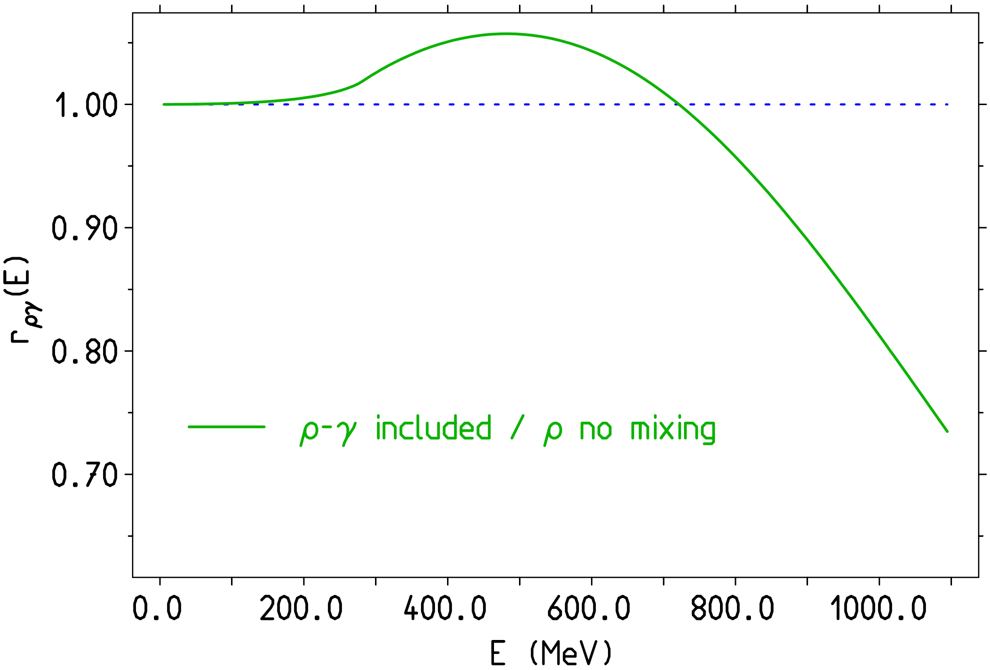

Figure 3: $r_{\rho\gamma}(s)$ correction as function of energy. A moderate positive interference below the ϱ^0 pole and a substantial negative interference above the ϱ^0 pole appears.

In figure 3 we plotted the correction $r_{\rho\gamma}(s)$ as a function of energy. As can be seen the $\varrho^0 - \gamma$ mixing gives a positive interference below the ϱ^0-pole and a negative interference above it. The correction must vanish at the ϱ^0 and at the γ poles due to the renormalization conditions.

If we want to apply our model to the calculation of the same observable based on both e^+e^- and τ data, then, if $\varrho^0 - \gamma$ mixing is not included in $F_0(s)$, we have the following relation between the spectral functions

$$v_0(s) = r_{\rho\gamma}(s) R_{\mathrm{IB}}(s) v_-(s), \tag{3.2}$$

where $R_{\mathrm{IB}}(s)$ represents other isospin breaking corrections [6–10]. We can now predict the measured branching fraction $B_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$ in terms of the e^+e^- spectral function as

$$B^{\mathrm{CVC}}_{\pi\pi^0} = \frac{2S_{\mathrm{EW}} B_e |V_{ud}|^2}{m_\tau^2} \int_{4m_\pi^2}^{m_\tau^2} ds\, R^{(0)}_{\pi^+\pi^-}(s) \left(1 - \frac{2}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \frac{1}{r_{\rho\gamma}(s) R_{\mathrm{IB}}(s)}. \tag{3.3}$$

The results are presented in table 1. Another thing we can calculate is the hadronic contribution to

Table 1: Calculated and predicted values of $B_{\pi\pi^0}$, for experimental values the first error is statistical and the second is systematic, while for predicted values the first error is experimental and the second is an uncertainty connected with the isospin breaking corrections, the details can be found in [14].

τ data	$B_{\pi\pi^0}[\%]$	e^+e^- data	$B^{\mathrm{CVC}}_{\pi\pi^0}[\%]$
ALEPH 97	$25.27 \pm 0.17 \pm 0.13$	CMD-2 06	$25.40 \pm 0.21 \pm 0.28$
ALEPH 05	$25.40 \pm 0.10 \pm 0.09$	SND 06	$25.09 \pm 0.30 \pm 0.28$
OPAL 99	$25.17 \pm 0.17 \pm 0.29$	KLOE 08	$24.82 \pm 0.29 \pm 0.28$
CLEO 00	$25.28 \pm 0.12 \pm 0.42$	KLOE 10	$24.65 \pm 0.29 \pm 0.28$
Belle 08	$25.40 \pm 0.01 \pm 0.39$	BaBar 09	$25.45 \pm 0.18 \pm 0.28$
combined	$25.34 \pm 0.06 \pm 0.08$	combined	$25.20 \pm 0.17 \pm 0.28$

the muon anomalous magnetic moment

$$a_\mu^{\mathrm{had,LO}}(\pi\pi) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\, R^{(0)}_{\pi\pi}(s)\frac{K(s)}{s}, \tag{3.4}$$

where $K(s)$ is a known kernel function (see e.g. [13]). Similarly as in the case of $B_{\pi\pi^0}$ our predictions based on e^+e^- and τ data are in agreement [14]. Our result for the lowest order hadronic vacuum polarization including τ and e^+e^- data is

$$a_\mu^{\mathrm{had,LO}} = 690.96(1.06)(4.63)\times 10^{-10}.$$

One remark is important here. We have analyzed the relationship between the neutral and the charged channel pion form factors by going to fields of the mass eigenstates. This produces an effective direct coupling of the ρ to the leptons, but also changes the coupling of the photon to the leptons in next to leading order. In this basis one would mess up completely standard calculations of $g-2$. This is not what we advocate, of course. Our calculation of a_μ and the use of the e^+e^- data is standard, but the correction required for the τ data becomes most transparent by adopting the mass eigenbasis. The value of an observable of course does not depend on the parametrization of the virtual fields (see e.g. [19] for an alternative approach based on the HLS model).

4. Summary and outlook

We have presented the simple model based on scalar QED and vector meson dominance which enabled us to calculate the effect of the $\varrho^0-\gamma$ mixing. We treated this effect as another isospin breaking correction, which needs to be applied before any comparison between e^+e^- and τ spectral function can be made. Using our correction we showed that e^+e^- and τ data lead to a consistent prediction for the hadronic contribution to the muon anomalous magnetic moment. Also the calculation of the branching fraction $B_{\pi\pi^0}$ showed that in average the e^+e^- and τ data are consistent. This results show that $\varrho^0-\gamma$ mixing effects are responsible for a large part of the discrepancy previously "observed".

Our model is very simple and needs to be extended in order to make more precise predictions for $F_\pi(s)$ which includes the ω, but even our simple model shows that effective field theory is a basic tool which needs to be taken serious when the results of different process in low energy hadronic physics are to be understood and compared with each others.

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