

On the trace anomaly in 2+1 flavor QCD

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We report on recent progress by the HotQCD collaboration in studying the trace anomaly at non-zero temperature in 2+1 flavor QCD on lattices with the temporal extent $N_\tau = 4, 6, 8, 10$ and 12 using the highly improved staggered quark (HISQ) action as well as the asqtad action. We discuss the dependence of our lattice results on the scale setting procedure and compare them with hadron resonance gas (HRG) calculations as well as to resummed perturbative results.

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1. Introduction

At high temperatures strongly interacting matter undergoes an transition to a new state where quark and gluons are not subject to confinement; it shows up in a qualitative change in the equation of state (see [1] for a review). The most common way to calculate the QCD equation of state is the integral method. The pressure is calculated as an integral of the trace of the energy momentum tensor $\varepsilon - 3p$ or the trace anomaly

$$\frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^T dT' \frac{\varepsilon - 3p}{T'^5}. \quad (1.1)$$

The study of the trace anomaly is also interesting in its own right. At high temperature it is determined by the running of the QCD coupling constant α_s , and at leading order it is proportional $\alpha_s^2 T^4$. Thus it provides a sensitive test of the weakly coupled nature of the quark gluon plasma. For this reason it is also often called the interaction measure. Thermodynamic quantities are expected to have large cutoff effects at high temperatures when calculated using an unimproved fermion formulation, even if the lattice spacing is small in absolute units. These cutoff effects arise from the distortion of the quark dispersion relation on the lattice and can be cured by using an improved action [2, 3]. On the other hand, at low and intermediate temperatures the cutoff effects in thermodynamic quantities can be understood in terms of the cutoff effects of the hadron spectrum. In particular, the breaking of the so-called taste symmetry in the staggered fermion formulation could be the dominant source of cutoff effects when using staggered quarks. To control cutoff effects in pressure and other thermodynamic quantities, one first needs to understand the cutoff dependence of the trace anomaly. In this contribution we will study in detail the cutoff dependence of the trace anomaly and compare our lattice results with the hadron resonance gas model at low temperature and with resummed perturbation theory at high temperatures.

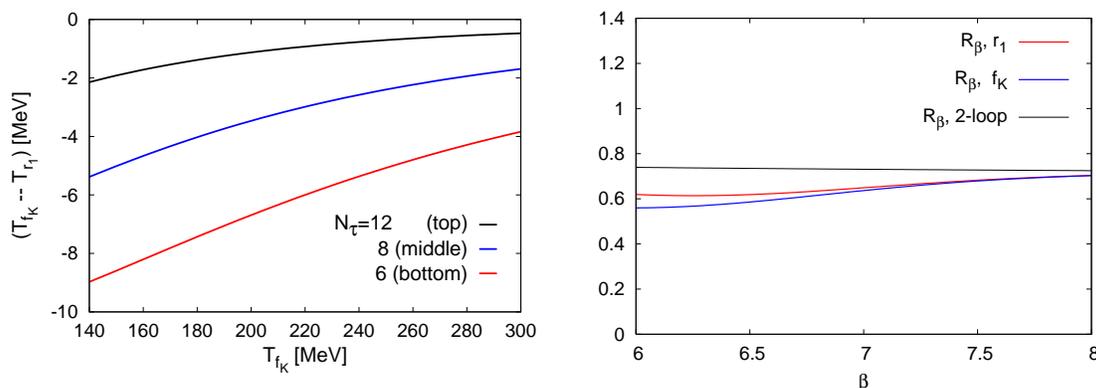


Figure 1: The difference in temperature (left) set by the r_1 and f_K scales, described in the text. The running of the gauge coupling (right).

2. Numerical results

We calculate the interaction measure in 2+1 flavor QCD using a tree-level improved action for gauge fields and the highly improved staggered quark (HISQ) action [4]. This combination

of gauge and quark action is called the HISQ/tree action. It can largely reduce cutoff effects in thermodynamic quantities both at low and high temperatures [5, 6].

The trace anomaly can be written in terms of the expectation values of gauge action and quark condensates as

$$\frac{\varepsilon - 3p}{T^4} = R_\beta [\langle S_{gauge} \rangle_0 - \langle S_{gauge} \rangle_T] - R_\beta R_m [2m_l (\langle \bar{l}l \rangle_0 - \langle \bar{l}l \rangle_T) + m_s (\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T)], \quad (2.1)$$

where the subscript “0” refers to $T = 0$ and “ T ” to finite temperature for observables evaluated at the same value of the cutoff. Furthermore, R_β and R_m are the lattice beta function and mass anomalous dimension that are defined below. The above expression is free of ultraviolet divergences. On a hypercubic lattice $N_s^3 \times N_\tau$ the physical temperature is set by the size of the temporal dimension and the lattice spacing as $T = 1/(N_\tau a)$. For $T = 0$ calculations we use $N_\tau \geq N_s$, and for $T > 0$ we keep $N_s/N_\tau = 4$ and at fixed N_τ vary the lattice spacing a by varying the gauge coupling $\beta = 10/g^2$. The continuum limit in this setup corresponds to $N_\tau \rightarrow \infty$. Therefore, we carried out this study on lattices with $N_\tau = 4, 6, 8, 10$ and 12 . The strange quark mass m_s is tuned to the physical value, while the two degenerate light quarks have masses $m_l = m_s/20$, slightly heavier than physical ($m_l \simeq m_s/27$). In the continuum limit these light quark masses correspond to the pion mass of about 160 MeV. To set the lattice spacing in physical units (fm) we use the r_1 scale defined through static quark potential

$$r^2 \frac{dV}{dr} \Big|_{r=r_1} = 1. \quad (2.2)$$

We use the value $r_1 = 0.3106$ fm [7]. Alternatively, the kaon decay constant, $f_K = 156.1$ MeV is used to set the scale. From Fig. 1 (left) one can see how the choice of reference observables affects the conversion of temperature from lattice units to MeV. Over the temperature range of interest on $N_\tau = 6$ lattices the difference is within 9 MeV, and on $N_\tau = 12$ within 2 MeV. Another effect of using different scales is related to the β -function and mass anomalous dimension that enter in Eq. (2.1)

$$R_\beta(\beta) = -a \frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m_s(\beta)} \frac{dm_s(\beta)}{d\beta}, \quad (2.3)$$

where $m_s(\beta)$ defines a line of constant physics (LCP), *i.e.*, the combination of the gauge coupling and strange quark mass such that the kaon mass (in MeV) stays approximately constant in the whole β range used in the simulation. R_β for r_1 and f_K scale is shown in Fig. 1 (right), together with the 2-loop perturbative result.

The present status of the interaction measure with the HISQ/tree action is shown in Fig. 2 together with the stout continuum estimate of Ref [8]. For temperature $T < 180$ MeV we see good agreement between our HISQ/tree results and the stout continuum estimate. At higher temperatures we see significant discrepancies between HISQ/tree results and the stout results, though the discrepancies seem to disappear at temperatures $T > 350$ MeV. The height of the peak decreases slightly when going from $N_\tau = 8$ to $N_\tau = 10$ and 12 . Thus one may expect that the discrepancies between HISQ/tree and stout actions in the peak region will be reduced once the continuum extrapolation for HISQ/tree is performed. It should be noted that in the previous calculations with p4 and asqtad actions [9, 10, 11] the trace anomaly was significantly smaller due to large cutoff effects.

Next we study the effect of the scale setting on the trace anomaly. In Fig. 2 (right) the results for $N_\tau = 6$ and 10 are shown using r_1 and f_K to set the scale. Different scale settings affect the

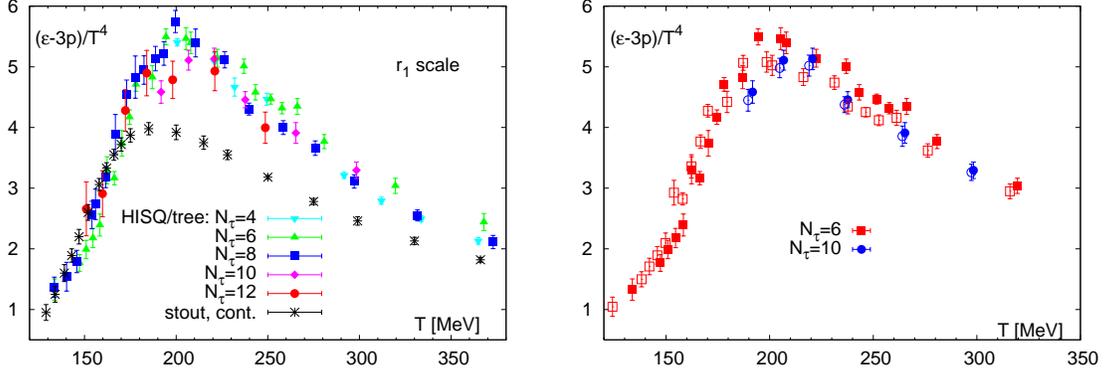


Figure 2: Comparison of the HISQ/tree interaction measure on $N_\tau = 4, 6, 8, 10$ and 12 lattices with the stout continuum estimate [8] (left), $N_\tau = 6$ and 10 HISQ/tree data (right). Filled (open) symbols in the right panel correspond to the r_1 (f_K) scale, see text.

$N_\tau = 6$ results, in particular in the peak region the trace anomaly is smaller if f_K is used to set the scale. However, for $N_\tau = 10$ there is almost no difference between the two scale setting procedures.

We also calculated the trace anomaly using the asqtad action. The expression for the trace anomaly in terms of expectation values of local operators is given in Ref. [10]. In Fig. 3 we compare our results obtained with HISQ/tree action for $N_\tau = 8$ and $N_\tau = 12$ using the scale set with both r_1 (filled symbols) and f_K (open symbols). If r_1 is used to set the scale, there is a significant discrepancy between the HISQ/tree and the asqtad result for $T < 200$ MeV for $N_\tau = 8$. As discussed above this is due to large cutoff effects at low temperatures when the asqtad action is used. The discrepancy is significantly smaller for $N_\tau = 12$, as expected. If we use f_K to set the scale, the discrepancies between HISQ/tree results and asqtad results are greatly reduced. Note, however, that the peak height in the $N_\tau = 8$ asqtad data is reduced when f_K is used to set the scale. Fig. 3 also shows that the difference between the two scale setting procedures for the asqtad action is reduced in the case of $N_\tau = 12$ data. This is again expected. There is, however still a significant reduction in the peak height if f_K scale is used.

The low-temperature behavior of the interaction measure is shown in Fig. 4 (left) and compared to the hadron resonance gas (HRG) calculation shown as the black line. At low temperatures, cutoff effects arising from the taste symmetry breaking of the staggered fermion formulation could be significant. To take into account possible cutoff effects in the comparison with HRG we replaced the contribution of each pion and kaon with a contribution averaged over the sixteen different tastes of pseudo-scalar mesons. The pseudo-scalar meson masses for each taste as function of the lattice spacing have been previously estimated by the HotQCD collaboration [6]. In Fig. 4 we show the HRG results with the modified pseudo-scalar meson sector as colored lines for each value of N_τ . We use the same color coding for the lattice results and the modified HRG result. Despite the fact that even for HISQ/tree action the effects of taste breaking in the pseudo-scalar sector are significant, the resulting cutoff effects are surprisingly small. They are of the same order or smaller than the statistical errors for all N_τ values, including $N_\tau = 6$. Fig. 4 shows that the lattice result starts to disagree with the HRG model at $T \sim 150 - 160$ MeV.

Finally, let us compare our results for the trace anomaly with earlier calculations using the p4 action [9, 10] as well as with resummed perturbation theory [12]. In Fig. 4 (right) we compare

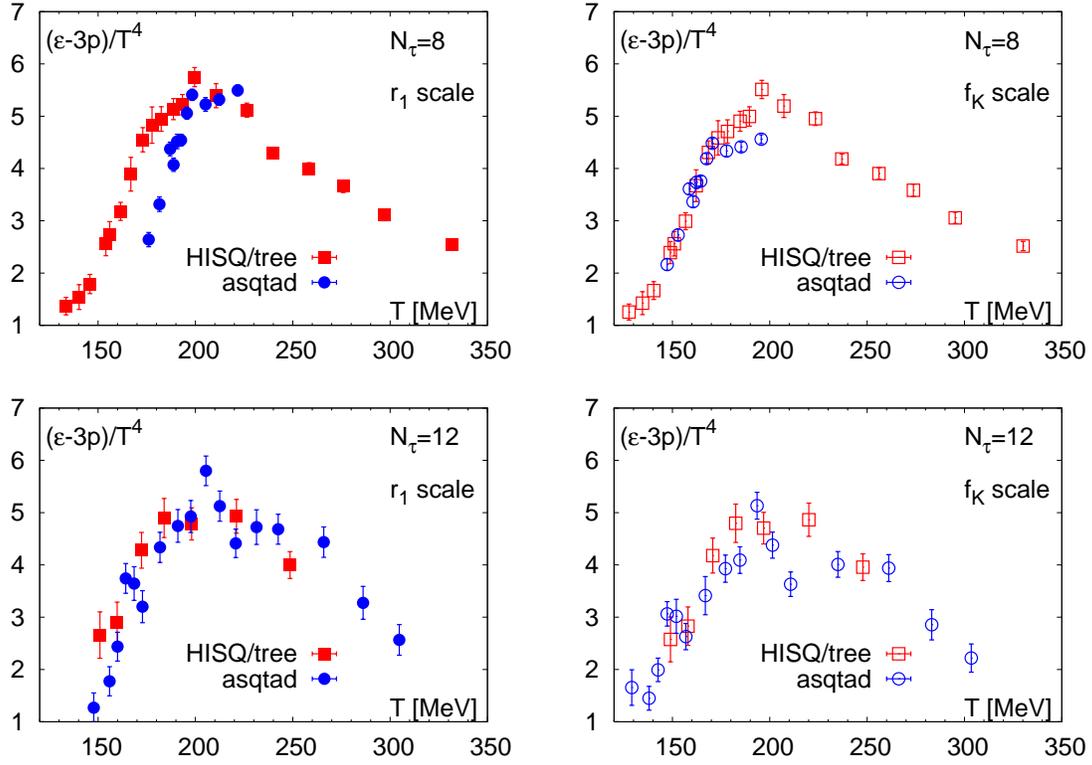


Figure 3: The interaction measure for the HISQ/tree action on $N_\tau = 8$ (top) and $N_\tau = 12$ (bottom) ensembles with r_1 (left) and f_K (right) scale.

our results for the HISQ/tree action with previous results obtained with p4 action and the stout continuum estimate. For $T > 400$ MeV different lattice data agree well with each other. The only exception is the $N_\tau = 4$ p4 data, which show the expected cutoff effects at high temperatures. The $N_\tau = 4$ HISQ/tree data should show similar cutoff dependence, but interestingly enough this is not observed. It is important to stress that at high temperatures the cutoff effects for p4 action should be small for $N_\tau \geq 6$. Thus the agreement between p4 results and HISQ/tree results is expected. We see quite good agreement with the resummed perturbative results [12].

3. Conclusion

We studied the trace anomaly in QCD using HISQ/tree and asqtad actions and lattices with temporal extent $N_\tau = 4, 6, 8, 10$ and 12 . Using different observables to set the scale, r_1 and f_K , allows for a crude estimate of the magnitude of cutoff effects, which seem to be small at the finest $N_\tau = 12$ lattices both for HISQ/tree and asqtad actions. Clearly to get reliable continuum results for the trace anomaly and thus for the equation of state the calculations on $N_\tau = 10$ and 12 lattices need to be significantly extended. We also compared our lattice results with HRG at low temperatures and resummed perturbative results at high temperatures. The trace anomaly is well described by HRG for $T < 150$ MeV. We also find a good agreement between lattice and resummed perturbative results at high temperatures.

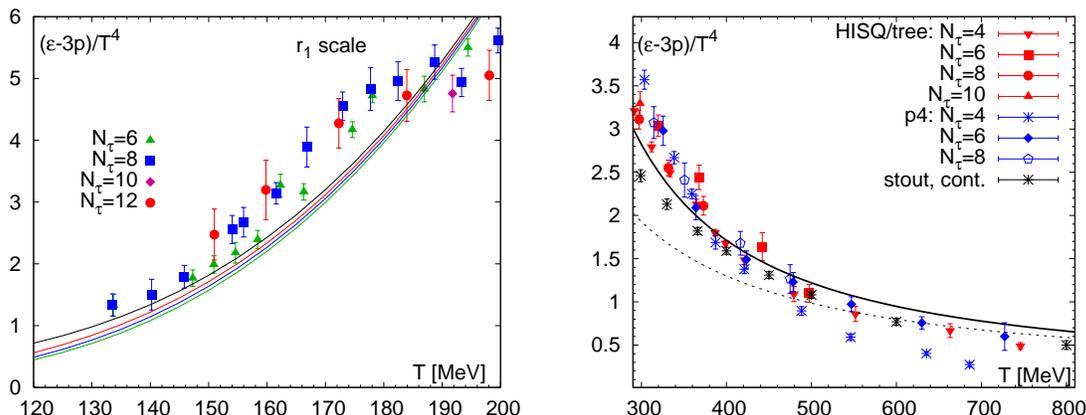


Figure 4: The interaction measure at low (left) and high (right) temperature. The curves on the left panel represent the HRG calculation, see text. On the right the dashed curve shows the resummed perturbative result with 2-loop running and the solid curve the resummed perturbative result with 1-loop running.

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References

- [1] P. Petreczky, *J. Phys. G* **39**, 093002 (2012) [arXiv:1203.5320 [hep-lat]].
- [2] U. M. Heller, F. Karsch and B. Sturm, *Phys. Rev. D* **60**, 114502 (1999) [hep-lat/9901010].
- [3] P. Hegde, F. Karsch, E. Laermann and S. Shcheredin, *Eur. Phys. J. C* **55**, 423 (2008) [arXiv:0801.4883 [hep-lat]].
- [4] E. Follana et al. (HPQCD/UKQCD Collaboration), *Phys. Rev. D* **75**, 054502 (2007), [arXiv:hep-lat/0610092].
- [5] A. Bazavov et al. (HotQCD Collaboration), *J. Phys. Conf. Ser.* **230**, 012014 (2010) [arXiv:1005.1131 [hep-lat]].
- [6] A. Bazavov et al. (HotQCD Collaboration), *Phys. Rev. D* **85**, 054503 (2012) [arXiv:1111.1710 [hep-lat]].
- [7] A. Bazavov et al. (MILC Collaboration), *PoS(Lattice 2010)074* (2010) [arXiv:1012.0868 [hep-lat]].
- [8] S. Borsanyi et al. (BW Collaboration), *JHEP* **1011**:077 (2010) [arXiv:1007.2580 [hep-lat]].
- [9] M. Cheng et al., *Phys. Rev. D* **77**, 014511 (2008) [arXiv:0710.0354 [hep-lat]].

- [10] A. Bazavov *et al.* (HotQCD Collaboration), Phys. Rev. D **80**, 014504 (2009) [arXiv:0903.4379 [hep-lat]].
- [11] M. Cheng *et al.*, Phys. Rev. D **81**, 054504 (2010) [arXiv:0911.2215 [hep-lat]].
- [12] J. O. Andersen, L. E. Leganger, M. Strickland and N. Su, Phys. Rev. D **84**, 087703 (2011) [arXiv:1106.0514 [hep-ph]].