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Thermal monopoles in lattice QCD

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The properties of the thermal Abelian color-magnetic monopoles in the maximally Abelian gauge are studied for the first time in the deconfinement phase of the lattice $SU(3)$ gluodynamics and lattice QCD. The simulated annealing algorithm combined with multiple gauge copies is applied for fixing the maximally Abelian gauge to avoid effects of Gribov copies. We compute the density, correlators, interaction parameters of the thermal Abelian monopoles.

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1. Introduction

The nonperturbative properties of the nonabelian gauge theories such as confinement, deconfining transition, chiral symmetry breaking, etc. are closely related to the Abelian monopoles defined in the maximally Abelian gauge (MAG) [1, 2]. A number of arguments supports the statement that the Abelian monopoles found in the MAG are important physical fluctuations surviving the cutoff removal: scaling of the monopole density at $T = 0$ according to dimension 3 for infrared ($percolating$) cluster [3]; Abelian and monopole dominance for a number of infrared physics observables (string tension [3, 4, 5], chiral condensate [6], hadron spectrum [7]). It has been recently argued that the MAG is a proper Abelian gauge to find gauge invariant monopoles since t'Hooft-Polyakov monopoles can be identified in this gauge by the Abelian flux, but this is not possible in other Abelian gauges [8]. Listed above properties of Abelian monopoles survive the continuum limit and removal of the Gribov copy effects. Most of the results were obtained for $SU(2)$ gluodynamics and then confirmed for $SU(3)$ theory and QCD [9, 10]. It is worth noticing that removal of Gribov copy effects changes numerical values of monopole characteristics quite substantially [28].

In recent papers [11, 12] it has been suggested that color-magnetic monopoles contribution can explain the strong interactions in the quark-gluon matter which were found in heavy ion collisions experiments [13]. These proposals inspired studies of the properties and possible roles of the monopoles in the quark-gluon phase [14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

In Ref. [12] it has been shown that thermal monopoles in Minkowski space are associated with Euclidean monopole trajectories wrapped around the temperature direction of the Euclidean volume. So the density of the monopoles in the Minkowski space is given by the average of the absolute value of the monopole wrapping number. First numerical investigations of the wrapping monopole trajectories were performed in $SU(2)$ Yang-Mills theory at high temperatures in Refs. [24] and [25]. A more systematic study of the thermal monopoles was performed in Ref. [16]. It was found in [16] that the density of monopoles is independent of the lattice spacing, as it should be for a physical quantity. The density–density spatial correlation functions were also computed in [16]. It was shown that there is a repulsive (attractive) interaction for a monopole–monopole (monopole–antimonopole) pairs, which at large distances might be described by a screened Coulomb potential with a screening length of the order of 0.1 fm. In Ref. [17] it was proposed to associate the respective coupling constant with a magnetic coupling α_m. In the paper [19] trajectories which wrap more than one time around the time direction were investigated. It was shown that these trajectories contribute significantly to a total monopole density at T slightly above T_c. It was also demonstrated that Bose condensation of thermal monopoles, indicated by vanishing of the monopole chemical potential, happens at temperature very close to T_c. However, the relaxation algorithm applied in [16] to fix the MAG is a source of the systematic errors due to effects of Gribov copies. It is known since long ago that these effects are strong in the MAG and results for gauge noninvariant observables can be substantially corrupted by inadequate gauge fixing [28]. For the density of magnetic currents at zero temperature it might be as high as 20%.

For nonzero temperature the effects of Gribov copies were not investigated until recently. In a recent paper [21] this gap was partially closed. It was shown that indeed gauge fixing with SA algorithm and 10 gauge copies per configuration gives rise to the density of the thermal monopoles 20 to 30% lower (depending on the temperature) than values found in [16].

The quantitatively precise determination of such parameters as monopole density, monopole coupling and others is necessary, in particular, to verify the conjecture [11] that the magnetic monopoles are weakly interacting (in comparison with electrically charged fluctuations) just above transition but become strongly interacting at high temperatures.

The check of universality for the thermal monopole properties was made in [22]. It was found that the universality holds for thermal monopoles which do not form short range (ultraviolet) dipoles.

So far all lattice studies of the thermal monopoles were restricted to $SU(2)$ gluodynamics. In this paper we present preliminary results of our study of thermal monopoles in $SU(3)$ gluodynamics and in QCD. To avoid systematic effects due to Gribov copiesWe use the gauge fixing procedure as in Ref. [10] with 10 gauge copies.

2. Definitions and simulation details

MAG is determined by the gauge condition [1]

$$\sum_{c\neq 3,8}\left(\partial_\mu\delta_{ac}+\sum_{b=3,8}f_{abc}A^b_\mu(x)\right)A^c_\mu(x)=0\,,\quad a\neq 3,8 \tag{2.1}$$

Solutions of this equation are extrema (over g) of the functional $F_{\mathrm{MAG}}[A^g]$

$$F_{\mathrm{MAG}}[A]=\frac{1}{V}\int d^4x\sum_{a\neq 3,8}[A^a_\mu(x)]^2 \tag{2.2}$$

Abelian projection is defined as

$$A^a_\mu(x)T^a\to A^3_\mu(x)T^3+A^8_\mu(x)T^8 \tag{2.3}$$

On a lattice MAG gauge fixing functional and Abelian projection are of the form [2]

$$F(U)=\frac{1}{V}\sum_{x,\mu}\left(|U_\mu(x)^{11}|+|U_\mu(x)^{22}|+|U_\mu(x)^{33}|\right)\,,\ \ U_\mu(x)\to u_\mu(x)\in U(1)^2 \tag{2.4}$$

After Abelian projection one can define magnetic currents:

$$j^{(a)}_\mu\equiv\frac{1}{4\pi}\varepsilon_{\mu\nu\rho\sigma}\partial_\nu\overline{\Theta}^{(a)}_{\rho\sigma}=-\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\partial_\nu m^{(a)}_{\rho\sigma}\,,a=1,2,3 \tag{2.5}$$

were $\overline{\Theta}^{(a)}_{\rho\sigma}$ is lattice Abelian field strength. The magnetic currents satisfy the constraint

$$\sum_a j^{(a)}_\mu(x)=0\,, \tag{2.6}$$

on any link $\{x,\mu\}$ of the dual lattice. Furthermore magnetic currents form closed loops.

Thermal monopoles are related to clusters of magnetic currents wrapped in T dimension. Wrapping number for given cluster N^a_{wr} is equal to:

$$N^a_{wr}=\frac{1}{3L_t}\sum_{j^a_4(x)\in cluster}j^a_4(x) \tag{2.7}$$

β	κ	L_t	L_s	T/T_c	N_{meas}
$SU(3)$					
6.061	-	6	24	1.33	200
6.061	-	4	24	2.0	190
QCD					
5.25	0.13605	8	24	1.5	250
5.25	0.13605	6	24	2.0	535
5.25	0.13605	4	16	3.0	1200

Table 1: Simulation parameters.

Then respective density is

$$\rho = \frac{\langle\ \sum_{clusters,a} |N^a_{wr}|\ \rangle}{3L_s^3 a^3} \tag{2.8}$$

Parameters of our simulations for $SU(3)$ gluodynamics and QCD are presented in Table 1. In both theories we varied temperature by variation of L_t. $SU(3)$ lattice gluodynamics was simulated with Wilson action on lattices $24^3\cdot 6$ and $24^3\cdot 4$ at $\beta = 6.061$. This is critical value of β for $L_t = 8$. Thus respective temperatures are $T = 4/3T_c$ and $T = 2T_c$.

Configurations of $N_f = 2$ lattice QCD were produced on lattices $24^3\cdot 8$, $24^3\cdot 6$ and $16^3\cdot 4$ with non-perturbatively $O(a)$ improved Wilson fermionic action at parameters $\beta = 5.25$, $\kappa = 0.13605$. It had been found by DIK collaboration that these parameters determine the crossover transition point on lattices with $L_t = 12$. It was also found that at this crossover point $T_c \approx 200$ MeV, $m_\pi \approx 400$MeV. Temperatures for three QCD lattices are $T = 1.5T_c, T = 2T_c$ and $T = 3T_c$ (see Table 1).

3. Results

In Figure 1 we present our results for the density of thermal monopoles ρ/T^3 for QCD and $SU(3)$ gluodynamics. For comparison we also show density for $SU(2)$ gluodynamics [22]. The density is plotted as function of the ratio T/T_0, where $T_0 = 270$ MeV was chosen. One can see that the density in $SU(3)$ gluodynamics is somewhat smaller than that in $SU(2)$ gluodynamics, while density in QCD is substantially higher.

We also computed the correlation functions for charges of same sign ($g_{MM}(r)$) and for charges of opposite sign ($g_{AM}(r)$) :

$$g_{MM}(r) = \frac{\langle \rho^a_M(0)\rho^a_M(r)\rangle}{2\rho^b_M\rho^b_M} + \frac{\langle \rho^a_A(0)\rho^a_A(r)\rangle}{2\rho^b_A\rho^b_A} \tag{3.1}$$

$$g_{AM}(r) = \frac{\langle \rho^a_A(0)\rho^a_M(r)\rangle}{2\rho^b_A\rho^b_M} + \frac{\langle \rho^a_M(0)\rho^a_A(r)\rangle}{2\rho^b_A\rho^b_M} \tag{3.2}$$

In Figure 2 correlators $g_{MM}(r)$ for SU(3) gluodynamics and QCD for $T/T_c = 2$ are shown. The correlators are fitted to functions [16, 17]

$$g_{MM,AM}(r) = e^{-U(r)/T}\ , \tag{3.3}$$

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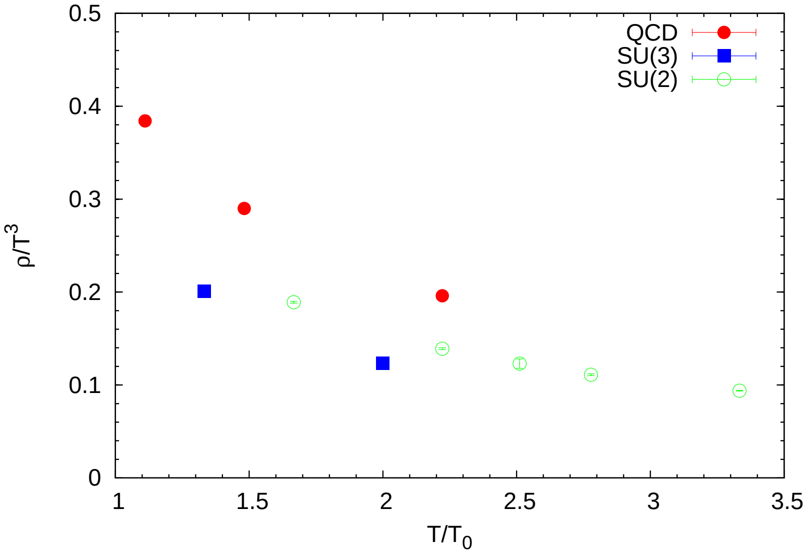

Figure 1: Thermal monopoles density vs T/T_0

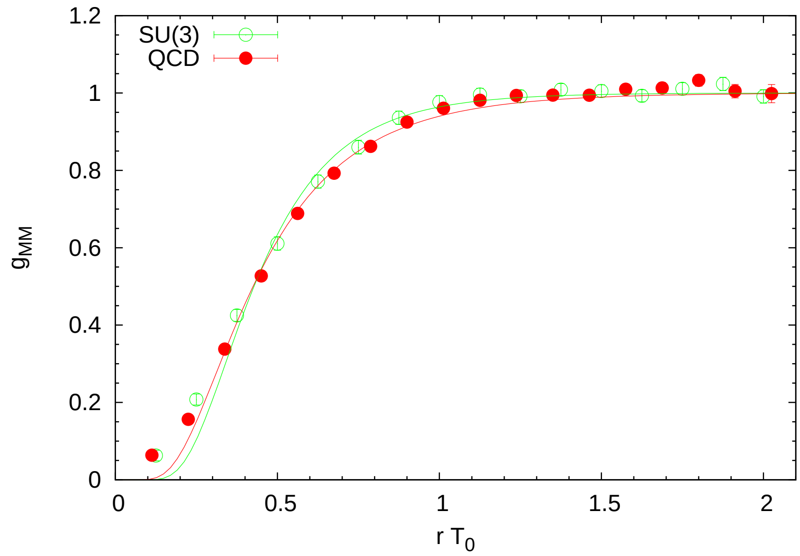

Figure 2: Thermal monopoles correlation functions for $T/T_c = 2$

where

$$U(r) = \frac{\alpha_m}{r} e^{-m_D r} \tag{3.4}$$

In Table 2 results for parameters α_m and m_D are shown. One can see that in QCD mass m_D and especially magnetic coupling α_m are substanially smaller than respective parameters in $SU(3)$ gluodynamics.

m_D/T_0	α_m	Γ	T/T_c
$SU(3)$			
4.44(41)	3.26(57)	3.1(6)	1.33
3.11(25)	2.43(36)	2.0(3)	2.0
QCD			
2.18(14)	1.24(9)	1.5(2)	1.5
2.65(12)	1.46(13)	1.6(2)	2.0
2.74(19)	1.89(25)	1.8(2)	3.0

Table 2: Monopole interaction parameters.

Coulomb plasma parameter

$$\Gamma = \alpha_m \left(\frac{4\pi\rho}{3T^3} \right)^{1/3} \tag{3.5}$$

can be now computed. Results are also shown in Table 2. Γ in QCD is smaller than in SU(3) gluodynamics, but it is still above 1. Thus our data indicate that thermal monopoles are in a liquid state in QCD.

4. Conclusions

We have found that density of thermal monopoles in QCD is higher than in gluodynamics. This is similar to zero temperature case. The magnetic coupling α_m is substantially lower in QCD and as a result plasma parameter Γ are also lower. Still obtained values of Γ corresponds to liquid state.

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