The Evolution of Transverse Momentum Distribution Functions at NNLL

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The definition and evolution of Transverse Momentum Dependent distributions have recently received much attention, both theoretically and experimentally. In this work we review some aspects of the evolution of these quantities and the methods to implement it.

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1. Introduction

In high-energy physics, transverse-momentum-dependent parton distribution functions (TMD-PDFs), with or without spin dependence, have proved to be an essential quantities for unraveling the internal structure of protons [1], as well as being an ingredients representing hadronic physics in a wide class of factorized physical observables. For example spin-dependent transverse momentum asymmetries provide a test for our understanding of the internal spin, angular momentum and 3-dimensional structure of the hadrons. A set of experiment will provide important pieces of information: HERMES, COMPASS, JLab, Belle, BNL, TeVatron, LHC and possibly EIC in the future, just to cite some of them. A relevant issue for the extraction of TMDs is that each experiment work at different energy, so that in order to compare their results it is fundamental to understand the evolution of TMDs. On the theoretical side, Sivers [2] and Collins [3] asymmetries have been intensely studied, and have attracted much attention recently [4, 5, 6, 7, 8, 9, 10] in contexts with fundamental theoretical concepts. Basically, some of the observed spin-asymmetries are linked to the presence of gauge links in non-collinear non-local correlators needed to maintain gauge invariance.

A complete formalism for the treatment of TMDs is still being developed [11, 12, 13, 14]. In this paper we concentrate on some properties of the evolution of the TMDs which have been evidenced in [15].

Definition of Quark-TMDPDFs: The definition of TMDPDFs is strictly related to the proof of factorization in physical processes. For the moment such proofs exist only for the simplest hadronic processes like Drell-Yan (DY), Semi Inclusive Deep Inelastic Scattering (SIDIS) and $e^+e^- \to 2j$. In order to fix the ideas we refer here to DY processes. In impact parameter space such factorization theorem for the hadronic tensor can be schematically written as:

$$\tilde{M} = H\,\tilde{F}_{f/P}\,\tilde{F}_{\bar{f}/\overline{P}}, \tag{1.1}$$

where H is the hard coefficient encoding the physics at the probing scale Q which is a polynomial of only $\log(Q^2/\mu^2)$. The functions $\tilde{F}_{f/P}$ and $\tilde{F}_{\bar{f}/\overline{P}}$ are the TMDs in impact parameter space.

The TMDPDFs of a polarized hadron collinear with the $+z$ direction with momentum P and spin $\vec{S}$ are defined extending the work done in [11, 13], as

$$\tilde{F}_{n,\alpha\beta} = \tilde{\Phi}^{(0)}_{n,\alpha\beta}(\eta)\,\sqrt{\tilde{S}(\eta,\eta)}\,, \tag{1.2}$$

where η is a generic rapidity regulator that separates collinear from soft modes. $\Phi_{n,\alpha\beta}$ stands for a purely collinear matrix element, i.e., a matrix element which has no overlap with the soft region [16], and it is given by the bilocal correlator

$$\Phi^{(0)}_{n,\alpha\beta} = \langle P\vec{S}|\,\left[\bar{\xi}_{n\alpha} W_n^T\right](0^+,y^-,\vec{y}_\perp)\,\left[W_n^{T\dagger}\xi_{n\beta}\right](0)\,|P\vec{S}\rangle\,. \tag{1.3}$$

The soft function S which encodes soft-gluon emission is given by

$$S = \langle 0|\,\mathrm{Tr}\,\left[S_n^{T\dagger}S_{\bar{n}}^T\right](0^+,0^-,\vec{y}_\perp)\,\left[S_{\bar{n}}^{T\dagger}S_n^T\right](0)\,|0\rangle\,. \tag{1.4}$$

To obtain the eight leading-twist quark-TMDs [17, 18], represented generically by $\tilde{F}$ below, one can simply take the trace of $\tilde{F}_{\alpha\beta}$ with the Dirac structures $\frac{\bar{n}\!\!\!/}{2}$, $\frac{\bar{n}\!\!\!/\gamma_5}{2}$ and $\frac{i\sigma^{j+}\gamma_5}{2}$ for unpolarized, longitudinally polarized and transversely polarized quarks, respectively, inside a polarized hadron.

The superscript T indicates transverse gauge-links $T_{n(\bar{n})}$, necessary to render the matrix elements gauge-invariant [19, 20]. The definitions of collinear ($W_{n(\bar{n})}$), soft ($S_{n(\bar{n})}$) and transverse ($T_{n(\bar{n})}$) Wilson lines for DY and DIS kinematics can be found in [11].

The anomalous dimension of each TMDs for the unpolarized case was given in [11]up to 3-loop order based on a factorization theorem for q_T-dependent observables in a Drell-Yan process. For the polarized case it can deduced at the same order considering the following facts. The hard coefficient H is built, perturbatively, by considering virtual Feynman diagrams only, i.e., no real gluon emission has to be considered. Moreover, the quantity H has to be free from infrared physics, no matter how the latter is regularized. This should be the case whether one works on or off-the-light-cone. The virtual contributions of the collinear functions Φ are also spin independent, and the soft function is spin independent. As a conclusion the scaling behavior of H is spin independent. Since the factorization theorem given above holds, at leading twist, also for spin-dependent observables, one can apply the same arguments, based on renormalization group invariance, as for the unpolarized case, to get a relation between the anomalous dimensions of $\tilde{F}$ and H, i.e., $\gamma_F = -\frac{1}{2}\gamma_H$, where $\gamma_H = 2\Gamma_{\rm cusp}\ln(Q^2/\mu^2) + 2\gamma_V$ is known at 3-loops level [21, 22, 23]. This crucial result can be automatically extended to all TMDs defined in Eq. (1.2), since the anomalous dimension is independent of spin structure.

2. Evolution Kernel

Starting from Eq. (1.2) the evolution of a generic quark-TMDPDF from an initial scale Q_i to a final scale Q_f is given by

$$\tilde{F}(x,b;Q_f) = \tilde{F}(x,b;Q_i)\,\tilde{R}(b;Q_i,Q_f), \tag{2.1}$$

where the evolution kernel $\tilde{R}$ is

$$\tilde{R}(b;Q_i,Q_f) = \exp\left\{\int_{Q_i}^{Q_f}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_F\left(\alpha_s(\bar{\mu}),\ln\frac{Q_f^2}{\bar{\mu}^2}\right)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-D(b;Q_i)}. \tag{2.2}$$

The D term can be obtained by noticing that the renormalized $\tilde{F}$ has to be well-defined when its partonic version is calculated pertubatively. This means that all divergences, other than genuine long-distance ones, have to cancel. This fundamental statement – that rapidity divergences cancel when the collinear and soft matrix elements are combined according to Eq. (1.2)– allows one to extract all the Q^2-dependence from the TMDs and exponentiate it with the function D, [11] thereby summing large logarithms of $\ln(Q^2/q_T^2)$. Applying the renormalization group invariance to $\tilde{M}$ we get $dD/d\ln\mu = \Gamma_{\rm cusp}$ by which we extract, for all the quark spin-dependent TMDs the function D at NNLO from the known cusp anomalous dimension at three-loops [23].

The function D in Eq. (2.2) contains large logarithms of the form $L_\perp = \ln\frac{Q_i^2 b^2}{4e^{-2\gamma_E}}$ that have to be resummed. The large logs appear in the Fourier transform of Eq. (2.2) and spoil the perturbative expansion of D when $\alpha_s L_\perp \gtrsim 1$. It is possible however to increase the convergence of D resumming all such logs in the following way. Matching the expansion

$$D(b,Q_i) = \sum_{n=1}^{\infty} d_n(L_\perp)a^n \tag{2.3}$$

where $a = \left(\frac{\alpha_s(Q_i)}{4\pi}\right)$ to the following ones: $\Gamma_{\rm cusp}(\alpha_s) = \sum_{n=1}^{\infty}\Gamma_{n-1}a^n$ and $\beta(\alpha_s) = -2\alpha_s\sum_{n=1}^{\infty}\beta_{n-1}a^n$ one gets the recursive differential equation

$$d_n'(L_\perp) = \frac{1}{2}\Gamma_{n-1} + \sum_{m=1}^{n-1} m\beta_{n-1-m}d_m(L_\perp)\,, \tag{2.4}$$

where $d_n' \equiv dd_n/dL_\perp$. It is possible to solve this equation recursively separating terms of order $\alpha_s^m(\alpha_s L_\perp)^n$, with $0 \le m \le n$. This can be seen writing explicitly the firsts solutions of Eq. (2.4)

$$\begin{aligned}
d_1(L_T) &= \frac{\Gamma_0}{2\beta_0}(\beta_0 L_T) + d_1(0)\,,\\
d_2(L_T) &= \frac{\Gamma_0}{4\beta_0}(\beta_0 L_T)^2 + \left(\frac{\Gamma_1}{2\beta_0} + d_1(0)\right)(\beta_0 L_T) + d_2(0)\,,\\
d_3(L_T) &= \frac{\Gamma_0}{6\beta_0}(\beta_0 L_T)^3 + \frac{1}{2}\left(\frac{\Gamma_1}{\beta_0} + \frac{1}{2}\frac{\Gamma_0\beta_1}{\beta_0^2} + 2d_1(0)\right)(\beta_0 L_T)^2\\
&\quad + \frac{1}{2}\left(4d_2(0) + \frac{\beta_1}{\beta_0}2d_1(0) + \frac{\Gamma_2}{\beta_0}\right)(\beta_0 L_T) + d_3(0)\,.
\end{aligned} \tag{2.5}$$

The coefficients $d_n(0)$ can be fixed by matching with the perturbative calculation of the Drell-Yan cross-section [24, 15]. After some algebra, we can express the coefficients d_n in Eq. (2.3) in the following way,

$$\begin{aligned}
2d_n a^n &= \frac{\Gamma_0}{\beta_0}\frac{X^n}{n} + aX^{n-1}\left(\frac{\Gamma_0\beta_1}{\beta_0^2}\left(-1 + H_{n-1}^{(1)}\right)|_{n\ge3} + \frac{\Gamma_1}{\beta_0}|_{n\ge2}\right) + a^2X^{n-2}\\
&\times\left((n-1)2d_2(0)|_{n\ge2} + (n-1)\frac{\Gamma_2}{2\beta_0}|_{n\ge3} + \frac{\beta_1\Gamma_1}{\beta_0^2}s_n|_{n\ge4} + \frac{\beta_1^2\Gamma_0}{\beta_0^3}t_n|_{n\ge5} + \frac{\beta_2\Gamma_0}{2\beta_0^2}(n-3)|_{n\ge4}\right) + \ldots,
\end{aligned} \tag{2.6}$$

where $X = a\beta_0 L_\perp$, $H_n^{(r)}$ is the r-th order Harmonic Number function of n and $\psi(n)$ is the digamma function of n and

$$\begin{aligned}
s_n &= (n-1)H_{n-2}^{(1)} + \frac{1}{2}(5-3n)\,,\\
t_n &= \frac{1}{2}\left[(1-n)H_{n-1}^{(2)} + n + 1 + (n-1)(\psi(n) + \gamma_E - 2)(\psi(n) + \gamma_E)\right]\,.
\end{aligned} \tag{2.7}$$

Using Eq. (2.3) we can now perform the resummation of large logarithms in the D, obtaining,

$$\begin{aligned}
D^R &= -\frac{\Gamma_0}{2\beta_0}\ln(1-X) + \frac{1}{2}\left(\frac{a}{1-X}\right)\left[-\frac{\beta_1\Gamma_0}{\beta_0^2}(X + \ln(1-X)) + \frac{\Gamma_1}{\beta_0}X\right]\\
&+ \frac{1}{2}\left(\frac{a}{1-X}\right)^2\Bigg[2d_2(0) + \frac{\Gamma_2}{2\beta_0}(X(2-X)) + \frac{\beta_1\Gamma_1}{2\beta_0^2}(X(X-2) - 2\ln(1-X)) + \frac{\beta_2\Gamma_0}{2\beta_0^2}X^2\\
&+ \frac{\beta_1^2\Gamma_0}{\beta_0^3}\frac{+121X^6 - 188X^5 + 13X^4 + 30X^3 + 12X^2(1 - \mathrm{Li}_2(X)) + 12X(X+1)\ln(1-X)}{24X^2}\\
&+ \frac{\beta_1^2\Gamma_0}{2\beta_0^3}(1-X)^2\sum_{n=5}^{\infty}X^{n-2}(n-1)\left[H_{n-1}^{(1)}\right]^2\Bigg] + \ldots,
\end{aligned} \tag{2.8}$$

For the last term in this expression we have found an analytic form of the sum of the series by using the approximation $H^{(1)}_{n-1} = \ln(n) + \gamma_E + \frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} - \frac{1}{256n^6} + \ldots$ which is precise enough for our purposes.

This resummation in n of each term of Eq. (2.3) is valid for $|X| < 1$, however when analytically continued through Borel-summation its validity is extended to $X \to -\infty$, which corresponds to $b \to 0$. The maximum value of b where each term of this series can be used, corresponding to $X = 1$, is $b_X = 2e^{-\gamma_E}/Q_i \exp[2\pi/(\beta_0 \alpha_s(Q_i))]$.

It is interesting then to study the behavior of the kernel when the impact parameter approaches b_X, being this the most subtle region. Using Eq. (2.8) we get the asymptotic expression of D^R when $X \lesssim 1$, up to NNLL,

$$D^R|_{X\to 1^-} = -\frac{\Gamma_0}{2\beta_0}\ln(1-X)\left[1 + \left(\frac{a}{1-X}\right)\frac{\beta_1}{\beta_0} + \left(\frac{a}{1-X}\right)^2 \frac{\beta_1\Gamma_1}{\beta_0\Gamma_0} + \ldots\right]$$
$$\stackrel{n_f=5}{=} -\frac{\Gamma_0}{2\beta_0}\ln(1-X)\left[1 + \left(\frac{a}{1-X}\right)5.04 + \left(\frac{a}{1-X}\right)^2 34.84 + \ldots\right], \qquad (2.9)$$

where we have checked that the last term in Eq. (2.8) with the Harmonic Number function does not give any divergent contribution. Although we do not have a general proof, we are assuming that $\ln(1-X)$ can be factored out in Eq. (2.9) to all orders in perturbation theory, which is likely to be the case given that, as we have checked, it holds even up to second order.

Another point to be noticed is that what appears in the evolution kernel is actually the exponential of $-D^R$, which guarantees that when $b \to b_X^-$ ($X \to 1^-$), one has $\tilde{R} \to 0$, due to the sign of the exponent. For the leading order term in Eq. (2.8) we have

$$\lim_{b\to b_X^-} D_0^R = \lim_{b\to b_X^-}\left[-\frac{\Gamma_0}{2\beta_0}\ln(1-X)\right] \to +\infty, \qquad (2.10)$$

and this limit is not spoiled by higher order corrections, as it is obvious from Eq. (2.9).

From this discussion we deduce that the kernel with D^R appears as a series in powers of $a/(1-X)$ (once the $\ln(1-X)$ is factored out, as in Eq. (2.9)) and that its validity is determined by the range of convergence of this series. The coefficients of this series are fixed by a combination of coefficients of the QCD β-function and the cusp anomalous dimension $\Gamma_{\rm cusp}$, and we have shown the result up to order $[a/(1-X)]^2$. In order to account for this restricted range of validity of $\tilde{R}$ we write it as

$$\tilde{R}(b; Q_i, Q_f) = \exp\left\{\int_{Q_i}^{Q_f} \frac{d\bar{\mu}}{\bar{\mu}}\gamma_F\left(\alpha_s(\bar{\mu}), \ln\frac{Q_f^2}{\bar{\mu}^2}\right)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-D^R(b;Q_i)} \theta(b_c - b), \qquad (2.11)$$

and we study its behavior for various values of b_c. In our examples we have considered two possible values of b_c: b_{c1}, such that $a/(1-X) < 1$, and also b_{c2}, such that $a/(1-X) < 0.2$.

Notice that the singularity at $b = b_X$, for which $X = 1$ does not correspond to the Landau pole. In fact writing $\Lambda_{QCD} = Q_i \exp G(t_{Q_i})$ where $t_{Q_i} \equiv -2\pi/(\beta_0\alpha_s(Q_i))$ and

$$G(t) = t + \frac{\beta_1}{2\beta_0^2}\ln(-t) - \frac{\beta_1^2 - \beta_0\beta_2}{4\beta_0^4}\frac{1}{t} - \frac{\beta_1^3 - 2\beta_0\beta_1\beta_2 + \beta_0^2\beta_3}{8\beta_0^6}\frac{1}{2t^2} + \ldots, \qquad (2.12)$$

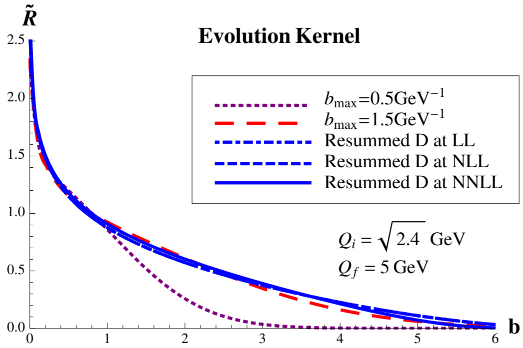

Figure 1: *Evolution kernel from $Q_i = \sqrt{2.4}$ GeV up to $Q_i = 5$ GeV within several schemes*

we have $b_X = \frac{2e^{-\gamma_E}}{\Lambda_{QCD}} \exp\left(-t_{Q_i} + G(t_{Q_i})\right) > \frac{2e^{-\gamma_E}}{\Lambda_{QCD}}$. Notice that the evolution kernel allows the resummation of logs between two energy scales Q_i and Q_f and that the impact parameter b is not related to these two scales. On the other side it turns out that the range of b where the TMDPDFs are different from zero is shorter that the interval where $\tilde{R}$ converges. In this way the evolution results basically independent from non-perturbative models.

Plotting $\tilde{R}$ in the range of b where it overlaps with $\tilde{R}$ in Fig. 1 we can see that the convergence of $\tilde{R}$ has no problem in going from NLL to NNLL. Moreover in the region $b > 6.07$, which is not plotted, $\tilde{R}$ is negligibly small. In Fig. 1 we plot also the same function $\tilde{R}$ according to the usual CSS method. Within this method, large $L_\perp$ logarithms in the evolution kernel are cancelled by choosing $\mu_b = 2e^{-\gamma_E}/b$ [26]. To avoid the Landau pole that appears when Fourier transforming back to momentum space, one writes $b^* = b/\sqrt{1+(b/b_{\max})^2}$ instead of b. $b_{\max}$ is an arbitrary cutoff that accounts for the separation between perturbative and non-perturbative regimes. We have implemented the BLNY model to compare our approach with CSS with the following parameters: $g_K(b) = \frac{g_2}{2} b^2$, with $g_2 = 0.68$ GeV^2 for $b_{\max} = 0.5$ GeV^{-1} [27] and $g_2 = 0.184$ GeV^2 for $b_{\max} = 1.5$ GeV^{-1} [25].

Results and Conclusions:

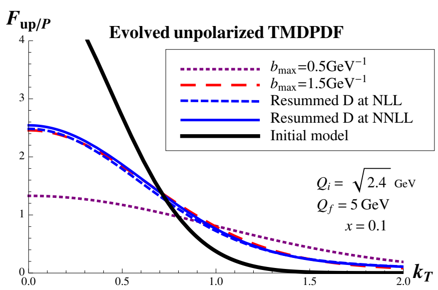

Figure 2: *Up quark unpolarized TMD evolved from $Q_i = \sqrt{2.4}$ GeV up to $Q_f = 5$ GeV with different approaches to the evolution kernel.*

In order to perform the resummation of large logarithms consistently up to N^{i}LL order (or N^{i-1}LO in RG-improved perturbation theory) one needs the input shown in table 1. In our approach one takes the resummed series in Eq. (2.8) up to the corresponding order i.

Order	Accuracy $\sim \alpha_s^n L^k$	γ_V	Γ_{cusp}	D^R
N^{i}LL	$n+1-i \le k \le 2n$ (α_s^{i-1})	α_s^i	α_s^{i+1}	$(\alpha_s/(1-X))^i$

Table 1: Approximation schemes for the resummed TMD, where $L = \ln(Q_f^2/Q_i^2)$ and α_s^i indicates the order of the perturbative expansion.

The unpolarized quark-TMDPDF at low energy is modeled as a Gaussian: $f_{up/P}(x;Q_i)\exp[-\sigma b_T^2]$ with $\sigma = 0.38/4\,\text{GeV}^2$ [29], and we have taken the MSTW data set for the integrated PDFs [30].

The model-dependence within the CSS resummation method, manifests itself as dependence on the parameter called $b_{\max}$ and the fit parameters of, for example, the BLNY model (g_2). However, within our method this model-dependence is almost absent. Varying b_c from to b_{c1}, b_{c2}, b_X we obtain lines which almost overlap one another and cannot be distinguished in the plots.

As can be seen in Fig. 2, our implementation of CSS is consistent with [4, 5]. From this figure it is clear that if one uses CSS rather than ours, the value of $b_{\max}$ has to be close to 1.5 GeV^{-1}, as was indeed found in [25] by a comparison with experimental data. Previous fits did not consider $b_{\max}$ as a free parameter, but rather set it to $0.5\,\text{GeV}^{-1}$ right from the start, fitting just the parameters of the non-perturbative model.

The definition of quark-TMDPDFs given in Eq. (1.2) and the new approach to determine the evolution kernel can be extended to gluon-TMDs [31] and quark/gluon TMD Fragmentation Functions. This new resummation technique can be applied as well to the evolution kernel of the complete hadronic tensor $\tilde{M}$ (built with two TMDs).

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