## Too many $X^{\prime} s, Y^{\prime} s$ and $Z^{\prime} s$ ?

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A number of resonances has been reported in the last few years in the context of heavy hadron spectroscopy. The nature of such states is in most cases still under controversy, suggesting that multiquark structures may appear. We analyze from a quark model perspective the possible contribution of meson-meson molecules to the meson spectra. Simple considerations based on coupled-channel effect restrict the number of channels that may accomodate multiquark structures.

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[^0]The discovery of charmonium $J / \Psi$ in 1974 together with a second narrow resonance at 3.7 $\mathrm{GeV}[1,2]$, or the so-called November revolution, changed our understanding of the structure of matter. They implied the existence of a new quark flavor, the charm quark, a heavy flavor that allows a nonrelativistic treatment in the study of the spectrum of the $c \bar{c}$ states with great precision. The existence of these two resonances implied the existence of further states belonging to the charmonium spectrum as well. We had found a wounderful example that showed the quark substructure of matter. During the next three decades, all charmonium-like states discovered nicely fitted into the simple spectroscopic predictions based on the one-gluon exchange postulated in 1975. The situation could be extrapolated with success to mesons made of a light and a heavy quark. However, again in 2003 two discoveries made this quiet period come to an end. The discovery by BABAR of the $D_{s_{0}}^{*}(2317)$ [3], an open-charm meson whose mass is in contradiction with the hitherto successful models of charm spectroscopy, was followed by the discovery by Belle of a charmonium-like state, the $X$ (3872) [4]. In spite of being firmly established, their puzzling properties indicate that they hardly fit into the standard $q \bar{q}$ description.

These were just the starting point of a series of new states discovered by the B factories. Most of these states still need confirmation, as the intriguing charged state $\mathrm{Z}(4430)$ seen by Belle but not by BABAR [5, 6]. Some of them can be understood following the simple $q \bar{q}$ scheme, as the $\mathrm{Z}(3930)$ recently identified as the $\chi_{c 2}(2 P)$ charmonium state. However, many others cannot be so easily accomodated, as the confirmed $\mathrm{X}(3872)$ or $\mathrm{Y}(4260)$, or the still dubious $\mathrm{Z}(4050)$ or $\mathrm{Z}(4250)$. At the same time, there are still free slots in the charmonium spectrum. Therefore nowadays we have a jungle of new particles whose nature has still to be clarified, in spite of the great effort of the physics community in the last decade. To that respect, one cannot forget that the wave function of a meson $(B=0)$ within the constituent quark model, where explicit gluon degrees of freedom are frozen in terms of a quark constituent mass, can be written as [7]:

$$
\begin{equation*}
|\Psi(B=0)\rangle=\alpha_{1}|q \bar{q}\rangle+\alpha_{2}|q q \bar{q} \bar{q}\rangle+\ldots \tag{1}
\end{equation*}
$$

where $\sum_{i}\left|\alpha_{i}\right|^{2}=1$. Ground state $\bar{q} q$ mesons have negative parity. Positive parity mesons can be reached either through a four-quark configuration in a $L=0$ state or by adding a unit of orbital angular momentum to the $\bar{q} q$ pair. In the constituent quark model, the mass of such a pair is around 600 MeV [8], whereas the mass shift produced by a unit of $L$ stands around $500-600 \mathrm{MeV}$ [9]. None of these two mechanisms is suppressed by the other and therefore the four quark piece is not negligible now. The challenging consequence of this so-called unquenching of the quark model [10], that seems to be unavoidable nowadays when pursuing a description of the excited hadronic spectrum, is the appearance of exotic states. They could be stable in nature, with a mass that is similar to the positive parity excitations, and they could not be described by the lowest order Fock space components of the naive quark model, $|q \bar{q}\rangle$.

Let us get more insight into the four-quark systems by approaching them from two different techniques. We solved in first place the Schrödinger equation through the Hyperspherical Harmonic $(\mathrm{HH})$ method [11]. This is performed through an expansion of the trial wave function in terms of HH functions, generalizing thus the well-known Spherical Harmonic formalism. The main difficulty one has to fight with is the construction of base states with the proper symmetry, and this turns out to be different for $Q Q \bar{n} \bar{n}$ than for $Q \bar{Q} n \bar{n}$. The Pauli principle needs to be imposed in case some of the quarks in the system are identical. A constituent quark cluster model (CQC)


Figure 1: (a) $(I) J^{P}=(0) 1^{+} c c \bar{n} \bar{n}$ Fredholm determinant. The dashed line stands for a single channel calculation with the lightest two charmed mesons, the solid line includes the coupling to the relevant excited channels. (b) Same as (a) for bottomonium.
has been employed for the interaction. We performed a screening of all the $J^{P}$ channels for the hidden-charm $c \bar{c} n \bar{n}$ and the open-charm $c c \bar{n} \bar{n}$ systems. Whereas no non-exotic ( $c \bar{c} n \bar{n}$ ) deeply four-quark bound state can be found, there appears one single compact state in the open-charm sector with $(I) J^{P}=(0) 1^{+}$quantum numbers. However, when dealing with molecular-like objects that lie near threshold, the convergence of the HH expansion is slow since a large number of terms are required to determine the wave function and therefore it is computationally very expensive to analyze those regions within this formalism. For that purpose we face the same problem from a different perspective: the meson-meson interaction. In this framework we start from a physical system made of two mesons, $M_{1}$ and $M_{2}$ with quantum numbers $(I) J^{P}$ in a relative $S$ state. They interact through a potential $V$ that contains a tensor force. Then, in general, there is a coupling to the $M_{1} M_{2} D$-wave and to any other two-meson system that can couple to the same quantum numbers (I) $J^{P}$. Thus, if we denote $D_{1} \equiv M_{1} M_{2}, D_{2} \equiv M_{1}^{\prime} M_{2}^{\prime}$ and $D_{3} \equiv M_{1}^{\prime \prime} M_{2}^{\prime \prime}$, the Lippmann-Schwinger equation for the $M_{1} M_{2}$ scattering becomes

$$
\begin{align*}
t_{\alpha \beta ; j i}^{\ell_{\alpha} s_{\alpha}, \ell_{\beta} s_{\beta}}\left(p_{\alpha}, p_{\beta} ; E\right) & =V_{\alpha \beta ; j i}^{\ell_{\alpha} s_{\alpha}, \ell_{\beta} s_{\beta}}\left(p_{\alpha}, p_{\beta}\right)+\sum_{\substack{\gamma=D_{k} \\
(k=1,2,3)}} \sum_{\ell_{\gamma}=0,2} \int_{0}^{\infty} p_{\gamma}^{2} d p_{\gamma} V_{\alpha \gamma ; j i}^{\ell_{\alpha} s_{\alpha}, \ell_{\gamma} s_{\gamma}}\left(p_{\alpha}, p_{\gamma}\right) \\
& \times G_{\gamma}\left(E ; p_{\gamma}\right) t_{\gamma \beta ; j i}^{\ell_{\gamma} s_{\gamma} \ell_{\beta} s_{\beta}}\left(p_{\gamma}, p_{\beta} ; E\right), \alpha, \beta=D_{1}, D_{2}, D_{3}
\end{align*}
$$

where $t$ is the two-body scattering amplitude, $j, i$, and $E$ are the angular momentum, isospin and energy of the system, $\ell_{\alpha} s_{\alpha}, \ell_{\gamma} s_{\gamma}$, and $\ell_{\beta} s_{\beta}$ are the initial, intermediate, and final orbital angular momentum and spin, respectively, and $p_{\gamma}$ is the relative momentum of the two-body system $\gamma$. We solve the Lippmann-Schwinger equation searching for attractive channels that may lodge a mesonmeson molecule. The basic ingredient to solve the scattering problem are the interacting potentials. They are taken from the CQC model, the very same that we used for the Hyperspherical Harmonic


Figure 2: (a) $(I) J^{P C}=(0) 1^{++} c n \bar{c} \bar{n}$ Fredholm determinant. The dashed line stands for a calculation considering only charmed mesons, the solid line includes also the $J / \Psi \omega$ two-meson system. (b) Same as (a) for bottomonium.
formalism. To obtain them from the basic $\bar{q} q$ interaction we made use of a Born-Oppenheimer approximation.

Let us first consider the effect of allowing or not the coupling of channels by studying the case of the $(I) J^{P}=(0) 1^{+}$quantum numbers. We will start by the open flavor sector. Dashed line in Fig. 1(a) shows the single-channel $D D^{*}$ interaction, which is attractive -as indicated by a Fredholm determinant smaller than 1 - but not enough to form a bound state. A meaningful calculation has to include a complete physical basis, becoming necessary to include all possible vectors in the Hilbert space. This requires to include the vector-vector $\left(D^{*} D^{*}\right)$ channel in the calculation. This coupling enhances the attraction, moving the system close to a bound state at threshold, that is achieved when the Fredholm determinant becomes zero. For explicit flavor, stability is favored by increasing the mass of the heavy quark, as can be checked in Fig. 1(b), where both the single- and the coupled-channel cases gain attraction.

The situation is different when considering the hidden-charm sector, as it is plotted in Fig. 2. To account for all basis states, we allow for the coupling to charmonium-light two-meson systems. As we can see in the left panel and in the same line followed by open-charmed systems, the $D \bar{D}^{*}$ interaction is attractive but it only becomes bound when coupling to the $J / \Psi \omega$ channel. However, when moving to the bottom sector the effect is opposite. The single-channel $B \bar{B}^{*}$ interaction is much more attractive than the corresponding $D \bar{D}^{*}$ as the interaction in both is nearly the same but the larger mass of the $b$ quark implies a smaller kinetic energy. Now, the coupling to the corresponding charmonium-light two meson system $\Upsilon \omega$, instead of favoring the binding, cancels the possibility to form a bound state.

The difference between open- and hidden-charm sectors can be understood by looking at the physical thresholds. A four-quark system will be stable if its mass lies below all possible twomeson thresholds. Whereas for the $Q \bar{Q} n \bar{n}$ there are two different physical decay channels, i.e., $(Q \bar{Q})(n \bar{n})$ and $(Q \bar{n})(\bar{Q} n)$, for the exotic $Q Q \bar{n} \bar{n}$ there is only one possible final state, $(Q \bar{n})(Q \bar{n})$. This has important consequences if both systems (two- and four-quark states) are described within


Figure 3: Experimental masses of the different two-meson systems made of a heavy and a light quark and their corresponding antiquarks, $Q n \bar{Q} \bar{n}$ with $Q=s, c$, or $b$, for several sets of quantum numbers, $J^{P C}$. We have set as our origin of energies the $K \bar{K}, D \bar{D}$ and $B \bar{B}$ masses for the hidden strange, charm and bottom sectors, respectively.
the same two-body Hamiltonian; the $c \bar{c} n \bar{n}$ will hardly present bound states, because the system will reorder itself to become the lightest two-meson state, either $(c \bar{c})(n \bar{n})$ or $(c \bar{n})(\bar{c} n)$. In other words, if the attraction is provided by the interaction between particles $i$ and $j$, it does also contribute to the asymptotic two-meson state. This does not happen for the $c c \bar{n} \bar{n}$ if the interaction between, for example, the two quarks is strongly attractive. In this case there is no asymptotic two-meson state including such attraction, and therefore the system will bind.

The consideration of physical thresholds may shed some light on the abundance of $X Y Z$ states recently discovered [13]. In Fig. 3 we have plotted the experimental mass [9] of the different twomeson systems made of a heavy and a light quark and their corresponding antiquarks for several sets of quantum numbers, $J^{P C}$, in three different flavor sectors: $Q=s$, hidden strange; $Q=c$, hidden charm; and $Q=b$, hidden bottom. In every flavor sector we represent the mass difference with respect to the mass of $K \bar{K}, D \bar{D}$ and $B \bar{B}$, respectively. In a constituent quark model picture, the four-quark state $Q n \bar{Q} \bar{n}$ could either split into $Q \bar{n}-n \bar{Q}$ or $Q \bar{Q}-n \bar{n}$. One observes how the general trend for all quantum numbers is that the mass of the $Q \bar{Q}-n \bar{n}$ system is larger than the mass of the $Q \bar{n}-n \bar{Q}$ state for $Q=s$, but it is smaller for $Q=c$ or $b$. It is remarkable the case of $J^{P C}=1^{++}$ for $Q=c$, where the $Q \bar{Q}-n \bar{n}$ and the $Q \bar{n}-n \bar{Q}$ states are almost degenerate. The reverse of the ordering of the masses of the $Q \bar{Q}-n \bar{n}$ and $Q \bar{n}-n \bar{Q}$ systems when increasing the mass of the
heavy quark for all $J^{P C}$ quantum numbers can be simply understood within the constituent quark model with a Cornell-like potential [12]. The binding of a coulombic system is proportional to the reduced mass of the interacting particles. Thus, in a heavy-light light-heavy four-quark system the $q \bar{q}$ binding does not change much when increasing the mass of the heavy flavor, due to the reduced mass of both subsystems being close to the mass of the light particle. However, in the heavy-heavy light-light system the binding increases with the mass of the heavy particle while that of the lightlight meson remains constant, becoming this threshold lighter than the heavy-light light-heavy two-meson structure, as seen in Fig. 3.

An immediate consequence, related to the possible existence of four-quark structures arises: they are more and more difficult to be found when moving to heavier flavors, the reason being the decrease of the mass of the heavy-heavy light-light threshold. The four-quark system would split up into a charmonium-light two-meson system. In such a case, the existence of attractive mesonantimeson channels would manifest as a bump in the scattering cross section but a bound state would never be possible. Therefore, for heavy flavors it is very rare that four-quark structures may exist. A remarkable exception is found in the charm sector for $(I) J^{P}=(0) 1^{++}$, where the $D \bar{D}^{*}$ and $J / \Psi$ channels are almost degenerate. Such a degeneration, together with an attractive interaction, gives rise to the $X(3872)$ particle [14]. Although the $B \bar{B}^{*}$ interaction is more attractive than $D \bar{D}^{*}$, move from the charm to the bottom sector makes it harder to find meson-antimeson molecules. The reason is that the coupling to the second physical threshold, which is lighter, prevents the formation of a bound state. As a consequence, based on the constituent quark model ideas, one shold not expect a twin of the $\mathrm{X}(3872)$ in the bottom sector like those pointed out in hadronic models based on the traditional meson theory of the nuclear forces or resorting to heavy quark symmetry arguments [15, 16].

The proximity between the charmonium-light and the meson-antimeson molecules is an important ingredient for the increase of the attraction that leads to a bound state. Channels with isospin 1 are coupled to a charmonium-light meson-meson pair that involves a pion, what means a very light threshold, canceling therefore the possibility of forming a bound state. The immediate consequence is the absence of charged partners of the $\mathrm{X}(3872)$.

In summary, the huge amount of data collected in the last decade has favored the discovery of many resonances in the context of heavy flavor spectroscopy. Called X's, Y's and Z's, they exhibit puzzling properties and do not seem to fit in the standard spectra made of $q \bar{q}$ pairs, maybe suggesting the existence of multiquark structures such as meson-meson molecules. We have studied the contribution of four-quark configurations from a quark-model perspective both from the hyperspherical harmonic and the Lippmann-Schwinger formalisms. Simple coupled-channel considerations allow to discard the contribution of meson-meson molecules to the meson spectrum in most of the channels, an exception being the $\mathrm{X}(3872)$. As a consequence, the existence of partners of the $\mathrm{X}(3872)$ either charged or in the bottom sector can be ruled out.

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