

$B_{(s)}$ meson decays in the framework of the covariant quark model

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The covariant quark model is introduced and some of its key features are briefly discussed. The model is then used for prediction of partial decay widths and branching fractions of B mesons in reactions $B_s \rightarrow J/\psi + \eta^{(\prime)}$ and $B \rightarrow K^{(*)} + 2\nu$ [1]. Our results stand: $\mathcal{B}(B_s \rightarrow J/\psi + \eta) = 4.67 \times 10^{-4}$, $\mathcal{B}(B_s \rightarrow J/\psi + \eta') = 4.04 \times 10^{-4}$, $\Gamma(B_s \rightarrow J/\psi + \eta')/\Gamma(B_s \rightarrow J/\psi + \eta) = 0.86$, $\mathcal{B}(B \rightarrow K\nu\bar{\nu}) = 0.63 \times 10^{-5}$ and $\mathcal{B}(B \rightarrow K^*\nu\bar{\nu}) = 7.9 \times 10^{-5}$. The most of them are in good agreement with experimental results [2, 3, 4, 5].

The European Physical Society Conference on High Energy Physics
18-24 July, 2013
Stockholm, Sweden

^{*}Speaker.

[†]The work was partly supported by Slovak Grant Agency for Sciences VEGA, grant No. 2/0009/10 (S. Dubnička, A. Z. Dubničková, A. Liptaj) and Joint research project of Institute of Physics, SAS and Bogoliubov Laboratory of Theoretical Physics, JINR, No. 01-3-1070 (S. Dubnička, A. Z. Dubničková, M. A. Ivanov and A. Liptaj).

1. Introduction

Physics of hadrons is nowadays an extensively studied field of particle physics. This is mainly due to new and ongoing colliding experiments, especially so-called heavy quark factories. The results are excellent: many new data, whose precision and amount is continuously increasing. Thanks to results from mass spectroscopy, lifetime measurements and branching fraction analysis we learn still more about hadrons and their constituents.

These results represent a challenge for theoretical physics. The low energy domain of quantum chromodynamics (QCD) remains a puzzling area for what concerns the first-principle calculations: perturbative approach loses its applicability and the enormous progress in lattice QCD is still not at the level to satisfactorily explain all measured data with an appropriate precision.

A model-dependent approach thus still remains the most adopted way of describing hadron dynamics. Here we present the covariant quark model (CQM), a model with many appealing features which has, till now, provided convincing results. It is based on a non-local Lagrangian density from which follows its full Lorentz invariance. In addition, standard techniques of quantum field theory (QFT) can be used for computation of physical observables. Further, besides mesons, the model is also able to incorporate higher-quark states (baryons, tetraquarks, ...) and has only one free parameter per hadron ¹.

2. Covariant quark model

The CQM introduces quark-hadron (in this case meson) interaction via the non-local Lagrangian (density)

$$\mathcal{L}_{\text{int}} = g_M \cdot M(x) \cdot J_M(x), \quad (2.1)$$

where $J_M(x)$ is the quark current

$$J_M(x) = \int dx_1 \int dx_2 F_M(x; x_1, x_2) \cdot \bar{q}_1^a(x_1) \Gamma_M q_2^a(x_2). \quad (2.2)$$

We assume the function $F_M(x; x_1, x_2)$ to have the form

$$F_M(x, x_1, x_2) = \delta^{(4)}(x - w_1 x_1 - w_2 x_2) \Phi_M [(x_1 - x_2)^2], \quad (2.3)$$

where the δ -function expresses the intuitive expectations about relative quark-hadron positions, i.e., for the choice we adopt

$$w_i = \frac{m_i}{m_1 + m_2}, \quad i = 1, 2 \quad (2.4)$$

the hadron is situated in the barycenter of the quark system. The interaction strength Φ_M is supposed to have a Gaussian form, which can be, in the momentum space, expressed as

$$\tilde{\Phi}_M(-p^2) = \exp\left(\frac{p^2}{\Lambda_M^2}\right) \quad (2.5)$$

with one free so-called size parameter Λ_M .

¹In the limit of large number of hadrons.

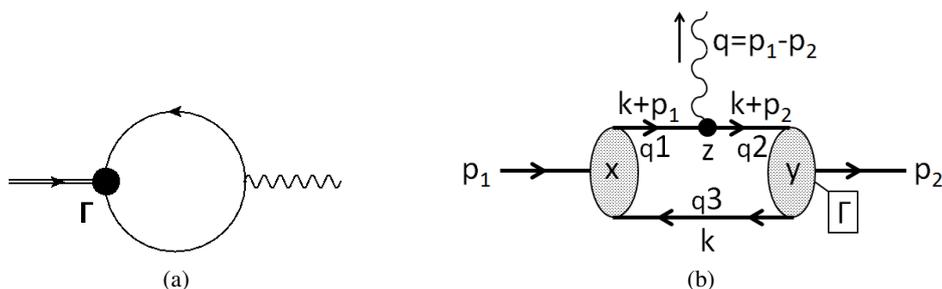


Figure 1: Diagrams representing a) a meson decay constant and b) a transition form factor.

In addition to size parameters (e.g. $\Lambda_\pi = 0.87$ GeV, $\Lambda_K = 0.87$ GeV, ...), the other free parameters of the model are the constituent quark masses (in GeV): $m_{u,d} = 0.235$, $m_s = 0.424$, $m_c = 2.16$, $m_b = 5.09$ and one additional cut-off parameter $\lambda_{cut-off} = 0.181$ related to the infrared confinement. So, to describe the dynamics of N hadrons, the model uses $N+5$ free parameters in total. The coupling constants g_M can be related to these parameter thanks to the so-called compositeness condition.

The compositeness condition concerns the question of a correct description of composite particles. This question was addressed already many years ago and reflects the fact, that the hadron and the quark fields appear in the Lagrangian (2.1) as elementary, although, in nature, the hadrons are made up of quarks. A. Salam [6] and S. Weinberg [7] argue, that the renormalization constant $Z_M^{1/2}$ can be interpreted as the matrix element between a physical state and the corresponding bare state. The condition $Z_M = 0$ then implies, that the physical state does not contain the bare state and is therefore properly described as a bound state. In the framework of the CQM this translates into the equation

$$Z_M = 1 - \frac{3g_M^2}{4\pi^2} \tilde{\Pi}'_M(m_M^2) = 0, \quad (2.6)$$

where $\tilde{\Pi}'_M$ is the derivative of the meson mass operator.

The observable quantities, when calculated within the CQM, are usually expressed via form factors and decay constants. These basic objects are calculated through evaluation of corresponding Feynman diagrams (see Fig. 1). The Schwinger representation of the quark propagator

$$\tilde{S}_q(k) = (m + \hat{k}) \int_0^\infty d\alpha e^{-\alpha(m^2 - k^2)} \quad (2.7)$$

is used, which leads to an expression of the Feynman graph, generally written as

$$\Pi = \int_0^\infty d^n \alpha \int [d^4 k]^\ell \Phi \exp\left[-\sum_{i=1}^n \alpha_i \left[m_i^2 - (K_i + P_i)^2\right]\right], \quad (2.8)$$

where Φ corresponds to numerator product of propagators and vertex functions, P_i refers to the linear combination of external momenta and K_i stands for the linear combination of loop momenta.

The evaluation of these graphs can be facilitated if done in a smart way. The operator identity

$$\int d^4 k P(k) e^{2kr} = \int d^4 k P\left(\frac{1}{2} \frac{\partial}{\partial r}\right) e^{2kr} = P\left(\frac{1}{2} \frac{\partial}{\partial r}\right) \int d^4 k e^{2kr} \quad (2.9)$$

allows for an elegant integration over loop momenta. The loop momentum k is removed from the polynomial P (which comes from the trace evaluation) so that the polynomial can be taken outside the integration. The resulting expression is further simplified using the identity

$$\int_0^\infty d^n \alpha P\left(\frac{1}{2} \frac{\partial}{\partial \mathbf{r}}\right) e^{-\frac{r^2}{a}} = \int_0^\infty d^n \alpha e^{-\frac{r^2}{a}} P\left(\frac{1}{2} \frac{\partial}{\partial \mathbf{r}} - \frac{\mathbf{r}}{a}\right) \mathbb{1}; \quad \mathbf{r} = \mathbf{r}(\alpha_i), \quad a = a(\Lambda_H, \alpha_i), \quad (2.10)$$

so that the derivative operator does not act on an exponential function but on an identity.

The last important point to be discussed when describing the general features of the CQM is the infrared confinement, which needs to be implemented to prevent decays of heavy baryons to constituent quarks. It is achieved by introducing a cut-off in the integration over Schwinger parameters. The integration is improper in several dimensions. To overcome this difficulty we introduce a δ -function form of the identity

$$\mathbb{1} = \int_0^\infty dt \delta\left(t - \sum_{i=1}^n \alpha_i\right). \quad (2.11)$$

The initial integral can be then transformed into an integral over simplex convoluted with only one dimensional improper integral

$$\int_0^\infty d^n \alpha F(\alpha_1, \dots, \alpha_n) = \int_0^\infty dt t^{n-1} \int_0^1 d^n \alpha \delta\left(1 - \sum_{i=1}^n \alpha_i\right) F(t\alpha_1, \dots, t\alpha_n). \quad (2.12)$$

The cut-off is then naturally implemented as

$$\int_0^\infty dt t^{n-1} \dots \rightarrow \int_0^{1/\lambda^2} dt t^{n-1} \dots. \quad (2.13)$$

This procedure makes the integral a smooth function, where thresholds in the quark loop diagrams and corresponding branch points are removed. The value of the cut-off parameter is considered to be universal and was settled by fitting the model to experimental data.

3. Decay $B_s \rightarrow J/\psi + \eta^{(\prime)}$

The quark content of η and η' mesons can be written as

$$\eta : \frac{1}{\sqrt{2}} \sin \delta (u\bar{u} + d\bar{d}) - \cos \delta (s\bar{s}), \quad (3.1)$$

$$\eta' : \frac{1}{\sqrt{2}} \cos \delta (u\bar{u} + d\bar{d}) + \sin \delta (s\bar{s}), \quad (3.2)$$

where $\delta = \theta_1 + \theta_P = \arctan \frac{1}{\sqrt{2}} + \theta_P$ and $\theta_P = -13.34^\circ$ [8]. The corresponding interaction Lagrangian takes this mixing into account

$$\begin{aligned} \mathcal{L}_\eta(x) = & g_\eta \eta(x) \iint dx_1 dx_2 \delta\left(x - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) \times \phi_\eta \left[(x_1 - x_2)^2\right] \\ & \times \left\{ \frac{1}{\sqrt{2}} \cos(\delta) [\bar{u}(x_1) i\gamma^5 u(x_2) + \bar{d}(x_1) i\gamma^5 d(x_2)] - \sin(\delta) [\bar{s}(x_1) i\gamma^5 s(x_2)] \right\}. \end{aligned} \quad (3.3)$$

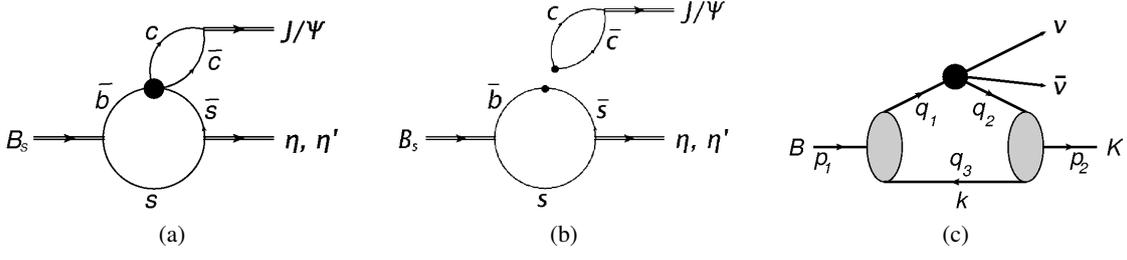


Figure 2: a) Diagram of the $B_s \rightarrow J/\psi + \eta^{(\prime)}$ decay and b) its factorization. c) Diagram of the decay $B \rightarrow K^{(*)} + 2\nu$.

B_s meson does not contain light constituent quarks, the dominant contribution to the decay thus comes from the s -quark channel (Fig. 2a).

The $\bar{b} \rightarrow \bar{s}$ transition is treated in the framework of an effective theory with four-fermion interaction and Wilson coefficients

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_i C_i Q_i, \quad (3.4)$$

where Q_i is an appropriate four-fermion operator ² and C_i are Wilson coefficients.

Within the CQM the diagram of decay process actually factorizes into two parts: one proportional to the transition form factor and the other to the decay constant (Fig. 2b). The diagram factorization is a general feature of the model when describing this type of processes and allows for reusing previously calculated objects (form factors) for different reactions.

The calculation of decay widths is achieved by standard techniques of QFTs. Before the numerical results could be carried out, we had to fix free parameters (size parameters $\Lambda_{\eta}^{\bar{q}q}$, $\Lambda_{\eta}^{\bar{s}s}$, $\Lambda_{\eta'}^{\bar{q}q}$ and $\Lambda_{\eta'}^{\bar{s}s}$), because our aim was a prediction and not a parameter fitting. We used a set of processes $\eta \rightarrow \gamma\gamma$, $\eta' \rightarrow \gamma\gamma$, $\rho \rightarrow \eta\gamma$, $\phi \rightarrow \eta\gamma$, $\phi \rightarrow \eta'\gamma$, $B_d \rightarrow J/\psi \eta$, $B_d \rightarrow J/\psi \eta'$, $\omega \rightarrow \eta\gamma$ and $\eta' \rightarrow \omega\gamma$ for that purpose. Each of them corresponds to a decay that has been previously studied within the CQM and could be straightforwardly included into the fit.

The branching fractions of the $B_s \rightarrow J/\psi + \eta^{(\prime)}$ decay as predicted by the CQM are

$$\mathcal{B}_{\text{CQM}}(B_s \rightarrow J/\psi \eta) = 4.67 \times 10^{-4}, \quad (3.5)$$

$$\mathcal{B}_{\text{CQM}}(B_s \rightarrow J/\psi \eta') = 4.04 \times 10^{-4}. \quad (3.6)$$

The corresponding experimental values [2] stand

$$\mathcal{B}(B_s \rightarrow J/\psi \eta) = 5.10 \pm 1.12 \times 10^{-4}, \quad (3.7)$$

$$\mathcal{B}(B_s \rightarrow J/\psi \eta') = 3.71 \pm 0.95 \times 10^{-4}. \quad (3.8)$$

Interesting is the prediction of the relative branching fraction

$$R = \frac{\Gamma(J/\psi + \eta')}{\Gamma(J/\psi + \eta)} = \underbrace{\frac{|\mathbf{q}_{\eta'}|^3}{|\mathbf{q}_{\eta}|^3}}_{\approx 1.04} \tan^2 \delta \times \underbrace{\left(\frac{F_+^{B_s \eta'}}{F_+^{B_s \eta}} \right)^2}_{\approx 0.83} \approx 0.86, \quad (3.9)$$

²E. g. $Q_1 = (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A}$, $Q_2 = \dots$ with $(\bar{\psi}\psi)_{V-A} = \bar{\psi}\gamma^\mu (1 - \gamma^5) \psi$ and $(\bar{\psi}\psi)_{V+A} = \bar{\psi}\gamma^\mu (1 + \gamma^5) \psi$

where one observes the importance of the model-dependent form factor ratio. The experiment gives

$$R = \begin{cases} 0.73 \pm 0.14 \pm 0.02 & [2], \\ 0.90 \pm 0.09_{-0.02}^{+0.06} & [3]. \end{cases} \quad (3.10)$$

4. Decay $B \rightarrow K^{(*)} + 2\nu$

The evaluation of the $B \rightarrow K^{(*)} + 2\nu$ decay diagram (Fig. 2c) has many similarities with the previous case: an effective theory with four fermion interaction is used to describe $\bar{b} \rightarrow \bar{s}$ transition and a factorization of the diagram makes appear the appropriate form factors. These differ for K and K^* particles. The K meson is spinless particle and is therefore characterized by three form factors F_+ , F_- and F_T :

$$\langle P'_{[\bar{q}_3, q_2]}(p_2) | \bar{q}_2 O^\mu q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \rangle = F_+(q^2) P^\mu + F_-(q^2) q^\mu, \quad (4.1)$$

$$\langle P'_{[\bar{q}_3, q_2]}(p_2) | \bar{q}_2 (\sigma^{\mu\nu} q_\nu) q_1 | P'_{[\bar{q}_3, q_1]}(p_1) \rangle = \frac{i}{m_1 + m_2} (q^2 P^\mu - q \cdot P q^\mu) F_T(q^2). \quad (4.2)$$

The vector particle K^* is in total described by seven form factors:

$$\begin{aligned} \langle V_{[\bar{q}_3, q_2]}(p_2, \varepsilon_2) | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3, q_1]}(p_1) \rangle = \\ = \frac{\varepsilon_\nu^\dagger}{m_1 + m_2} \left[-g^{\mu\nu} P \cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V(q^2) \right], \end{aligned} \quad (4.3)$$

$$\begin{aligned} \langle V_{[\bar{q}_3, q_2]}(p_2, \varepsilon_2) | \bar{q}_2 [\sigma^{\mu\nu} q_\nu (1 + \gamma^5)] q_1 | P_{[\bar{q}_3, q_1]}(p_1) \rangle = \\ = \varepsilon_\nu^\dagger \left[-\left(g^{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) P \cdot q a_0(q^2) + \left(P^\mu P^\nu - q^\mu P^\nu \frac{P \cdot q}{q^2} \right) a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta g(q^2) \right]. \end{aligned} \quad (4.4)$$

The (subset of these) form factors can be related to differential decay widths as a function of $s = q^2/m_P^2$ through formulas [9]

$$\frac{d\Gamma[P \rightarrow P' \nu \bar{\nu}]}{ds} = \frac{G_F^2 m_P^5 |\lambda_t|^2 \alpha_{em}^2 |D(x_t)|^2}{2^8 \pi^5 \sin^4 \theta_W} |F_+|^2 \Phi_{P'}^{\frac{3}{2}}, \quad (4.5)$$

$$\frac{d\Gamma[P \rightarrow V \nu \bar{\nu}]}{ds} = \frac{3G_F^2 m_P^5 |\lambda_t|^2 \alpha_{em}^2 |D(x_t)|^2}{2^8 \pi^5 \sin^4 \theta_W} \Phi_V^{\frac{1}{2}} \left[s\alpha_1(A_0, V) + \frac{\Phi_V}{3} \beta_1(A_0, A_+, V) \right]. \quad (4.6)$$

Before getting results, we had to determine the K^* meson size parameter. We did it using $\tau \rightarrow K^* + \nu_\tau$ decay, which can be easily described by the model. The differential decay width distributions were integrated numerically to arrive to our results:

$$\begin{aligned} \mathcal{B}_{\text{CQM}}(B \rightarrow K + 2\nu) &= 0.63 \times 10^{-5}, \\ \mathcal{B}_{\text{CQM}}(B \rightarrow K^* + 2\nu) &= 7.9 \times 10^{-5}, \end{aligned}$$

which are to be compared with limits established by Belle [4] and BaBar [5]

$$\begin{aligned} \mathcal{B}_{\text{Belle}}(B \rightarrow K + 2\nu) &< 5.5 \times 10^{-5}, \\ \mathcal{B}_{\text{Belle}}(B \rightarrow K^* + 2\nu) &< 4.0 \times 10^{-5}, \end{aligned}$$

and

$$\begin{aligned}\mathcal{B}_{\text{BaBar}}(B \rightarrow K + 2\nu) &< 3.2 \times 10^{-5}, \\ \mathcal{B}_{\text{BaBar}}(B \rightarrow K^* + 2\nu) &< 7.9 \times 10^{-5}.\end{aligned}$$

We observe a good agreement, except the Belle result for K^* . This experimental limit however does not have any uncertainty, and we prefer not to draw preliminary conclusions. A final result with uncertainty will provide a better apprehension of our number.

5. Summary and outlook

We have calculated widths of selected decays of B and $B_{(s)}$ mesons within the covariant quark model. The results are nicely describing the recent experimental measurements and favor the model as an appropriate framework for studying hadron dynamics. The model has several appealing features and its predictions are heavily over-constrained. More details about the CQM can be found in Refs. [10, 11].

In a near future we plan to continue this work and predict observable quantities for the decay $B \rightarrow K^{(*)} + ll$, which is very similar to what is described in this text. Furthermore, many interesting and rare reactions were recently measured or are about to be measured, such as $B_s \rightarrow J/\Psi f_0(980)$, $B_s^0 \rightarrow \pi^+ \pi^-$, $B^0 \rightarrow K^+ K^-$, $B_{(s)}^0 \rightarrow \mu^+ \mu^-$ or $B_s^0 \rightarrow J/\Psi K_S^0$. These processes are very well suited to be studied within the CQM and we will certainly provide our predictions for them.

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