

## Multiplicity fluctuations of identified hadrons in central Pb+Pb collisions at the CERN SPS

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We report on the event-by-event multiplicity fluctuations of identified particles in central Pb+Pb collisions measured by the NA49 experiment at the CERN SPS. Employing a novel approach we unfolded the moments of the unknown multiplicity distributions of protons (p), kaons (K), pions ( $\pi$ ) and electrons. Using these moments we reconstructed an excitation function of the fluctuation measure  $v_{dyn}[A, B]$ , with A and B denoting different particle types. Specifically, we reconstructed  $v_{dyn}$  for the [p,  $\pi$ ], [p, K] and [K,  $\pi$ ] pairs. The energy dependence of  $v_{dyn}$  is in agreement with previously published NA49 results on the related measure  $\sigma_{dyn}$ . Moreover, for [K, p] and [K,  $\pi$ ] pairs, we discovered a dependence of the fluctuation measure  $v_{dyn}$  on the phase space coverage (acceptance). Interestingly for the [p,  $\pi$ ] case no significant acceptance dependence was observed. These observations provide a likely explanation of the reported differences between measurements of NA49 and those of STAR in central Au+Au collisions.

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## 1. Introduction

As the fundamental theory of strong interactions, Quantum Chromodynamics (QCD), is asymptotically free, the created matter at high temperature and/or high density may be dominated by the state of quasi-free quarks and gluons referred to as the Quark-Gluon Plasma (QGP). By colliding heavy ions at high energies one hopes to heat and/or compress the matter to energy densities at which the production of the QGP begins. Also lattice QCD calculations predict this new phase at high temperatures [1]. The motivation of investigating relativistic heavy ion collisions is the experimental study of hadronic matter under extreme conditions. A wealth of ideas in the past few decades have been proposed to explore the phase structure of strongly interacting matter. Among other possible probes event-by-event fluctuations of different observables may be sensitive to the transitions between hadronic and partonic phases. In particular, the location of the critical point may be signalled by a characteristic pattern in the energy and system size dependence of the measured fluctuation signals.

Pb+Pb reactions were investigated at the CERN SPS since 1994 by a variety of experiments. NA49 was the only experiment to participate in the entire energy scan, from projectile momenta of 158A down to 20A GeV/c. The explored energy range probes an important region in the phase diagram of strongly interacting matter. Indeed, the NA49 collaboration reported non-monotonic behavior in excitation functions of several hadronic observables [2, 3], which were in fact predicted within model calculations as a hint for the onset of deconfinement [4].

Motivated by these findings the NA49 Collaboration investigated the energy dependence of dynamical event-by-event fluctuations using the  $\sigma_{dyn}(A/B)$  measure. The collaboration reported an increasing trend of  $\sigma_{dyn}$  for both  $K/p$  and  $K/\pi$  ratios towards lower collision energies. Contrary to NA49 observations, recent results of the STAR experiment from the Beam Energy Scan (BES) at RHIC show practically no energy dependence of the event-by-event fluctuation measure  $v_{dyn}$  for  $[K, p]$  and  $[K, \pi]$  pairs. However, the comparisons between NA49 and corresponding STAR results were performed using the relation:

$$v_{dyn} = \text{sgn}(\sigma_{dyn})\sigma_{dyn}^2, \quad (1.1)$$

Note that the accuracy of this relation decreases inversely with multiplicity, i.e. at lower energies this relation is only approximate. In order not to rely on this approximation in this work we directly reconstruct the fluctuation measure  $v_{dyn}$  for the NA49 data.

Moreover, the published NA49 results are essentially performed at forward rapidities, whereas the STAR acceptance covers the mid-rapidity region without the low  $p_T$  range. This motivated our study of the acceptance dependence of event-by-event particle ratio fluctuations presented in this contribution. The paper is organized as follows: The information on analyzed data sets and selection criteria are introduced in section 2. In section 3 we discuss the elements of the analysis method, used in this work. Obtained results on  $v_{dyn}$  and their acceptance dependences are discussed in sections 4 and 5 respectively. Finally section 6 summarizes the paper.

## 2. Data sets and selection criteria

This paper presents the results for central Pb+Pb collisions at projectile momenta of 20A, 30A,

40A, 80A and 158A GeV/c recorded by the NA49 experiment (for detailed description of the NA49 apparatus cf. Ref. [5]). The particle identification in this analysis is achieved by simultaneous measurement of particle momenta and their specific energy loss in the gas volume of the Main Time Projection Chambers (MTPCs). The information from the Vertex Time Projection Chambers (VTPC1 and VTPC2) was only used for track reconstruction and selection.

The only event selection criterion used in this analysis is a centrality cut based on the energy loss of forward going projectile spectators ( $E_{Cal}$ ) measured in the forward veto calorimeter (VCAL). The datasets were recorded with 7% online VCAL cut for 20A - 80A GeV and 10% for 158A GeV Pb+Pb reactions respectively. Using an offline cut on  $E_{Cal}$  we selected event samples with 3.5 % centrality which in the Glauber Monte Carlo Model corresponds to about 367 wounded nucleons and an impact parameter range of  $b < 2.8$  fm. To ensure better particle separation only the tracks with large track length (better energy loss resolution) in the MTPCs were selected. For this purpose we distinguish between the number of potential and the number of reconstructed  $dE/dx$  points. The former one is estimated according to the position of the track in space together with the known TPC geometry, while the latter represents the number of points reconstructed by the cluster finder algorithm. The following track selection criteria referred to as the "loose cuts" are used for this analysis:

- Number of reconstructed points in the MTPCs:  $> 30$ .
- The ratio of the number of reconstructed points in all TPCs (VTPCs + MTPCs) to the number of potential points in all TPCs:  $> 0.5$ .

These selections reduce the acceptance to particles in the forward rapidity regions in the center-of-mass reference frame. In order to study the systematic uncertainties of final results due to the applied track cuts we employed another set of cuts ("tight cuts") in addition to the "loose cuts":

- The number of potential points in at least one of VTPC1 or VTPC2  $> 10$  and in the MTPCs  $> 30$ .
- The ratio of the number of reconstructed points to the number of potential points in the selected TPC(s):  $> 0.5$ .
- The track should be fitted to the primary vertex.
- The distance between the closest point on the extrapolated track to the main vertex position should be less than 4cm in  $x$  and less than 2cm in  $y$ .

### 3. Analysis Method

Practically all measures proposed for event-by-event fluctuations are defined as functions of moments of the unknown multiplicity distributions. In particular, the fluctuation measure  $v_{dyn}[A, B]$ , studied in this work, depends on the first and all second (pure and mixed) moments of the multiplicity distributions of particles  $A$  and  $B$ . The standard approach of finding the moments is to count the number of particles event-by-event. However, this approach is hampered by incomplete particle identification (overlapping  $dE/dx$  distribution functions), which is taken care of by either selecting

suitable phase space regions (where the distribution functions do not overlap) or by applying a fitting procedure event-by-event. The latter typically introduces artificial correlations which are corrected for by the event mixing technique. Here we employ a novel approach, called the *Identity Method* [13, 14]. The method follows a probabilistic approach which avoids the event-by-event fitting and, moreover, has a rigorous mathematical derivation. Furthermore, there is no need for corrections based on event mixing. The method is based on the fitted inclusive  $dE/dx$  distribution functions,  $\rho_j(x)$ , with  $j$  standing for proton, kaon, pion and electron. Each event has a set of measured  $dE/dx$  values,  $x_i$ , corresponding to each track in the event. For each track inside an event we estimate the probability of being a particle of type  $j$ :

$$w_j(x_i) \equiv \frac{\rho_j(x_i)}{\rho(x_i)}, \quad (3.1)$$

where  $\rho_j(x_i)$  are extracted from the lookup table storing the parameters of fitted  $dE/dx$  distribution functions in a given phase space bin and:

$$\rho(x_i) \equiv \sum_{j=p,K,\pi,e} \rho_j(x_i) \quad (3.2)$$

We further calculate an event variable  $W_j$  defined as:

$$W_j = \sum_{i=1}^n w_j(x_i) \quad (3.3)$$

where  $n$  is the number of particles in a given event.

As the introduced  $W$  quantities are calculated for each event, we obtain by straightforward averaging over the events all second moments of the  $W$  quantities. Finally, using the Identity Method we unfold the second moments of the multiplicity distributions from the moments of the  $W$  quantities. Note that the Identity Method was tested in numerous simulations as reported in Ref. [14]. We also provide a direct experimental verification of the method by studying the scaled variance  $\omega$ .

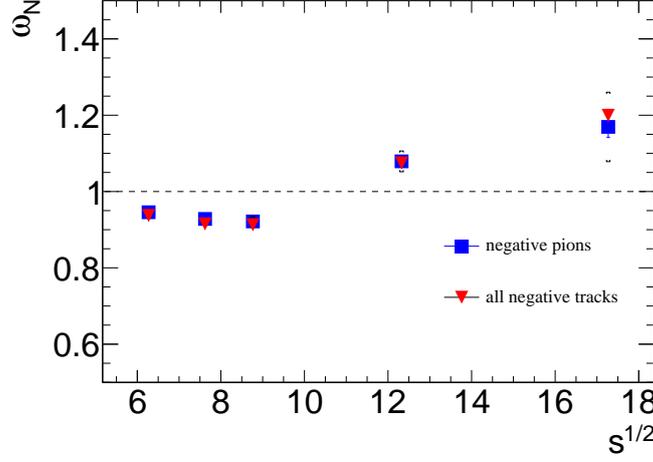
$$\omega = \frac{Var(N)}{\langle N \rangle} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} \quad (3.4)$$

For this purpose we perform two independent analyses: (i) by using the reconstructed moments for negatively charged pions (using the Identity Method) and (ii) by counting the negatively charged particles event-by-event (i.e without employing the Identity Method). The results are presented in Fig. 1 by blue squares for case (i) and by red triangles for case (ii). As the majority of negative particles are pions the remarkable agreement between the results of these two independent approaches is a direct experimental test of the Identity Method.

#### 4. Event by Event fluctuations in Pb+Pb collisions

The studied measure of dynamical particle ratio fluctuations  $v_{dyn}[A,B]$  is defined as [12]:

$$v_{dyn}[A,B] = \frac{\langle A(A-1) \rangle}{\langle A \rangle^2} + \frac{\langle B(B-1) \rangle}{\langle B \rangle^2} - 2 \frac{\langle AB \rangle}{\langle A \rangle \langle B \rangle}, \quad (4.1)$$



**Figure 1:** (Color Online) Energy dependence of scaled variance of negatively charged pions are plotted with blue boxes (Using the Identity Method). The red triangles are estimated by direct event-by-event counting of all negative particles (Without involving the Identity Method). The remarkable agreement between these results is an experimental verification of the Identity Method.

where A and B stand for multiplicities of different particle species and the brackets denote averaging over all events. Inserting the values of the reconstructed moments from the identity method in Eq.(4.1) we obtain  $v_{dyn}[p + \bar{p}, \pi^+ + \pi^-]$ ,  $v_{dyn}[K^+ + K^-, p + \bar{p}]$  and  $v_{dyn}[K^+ + K^-, \pi^+ + \pi^-]$ . Their energy dependence is presented by the blue squares in Fig. 2. Indicated statistical errors were determined using the sub sample approach. To estimate the systematic errors due to the applied track selection criteria, we calculate  $v_{dyn}$  separately for tracks selected by "loose" ( $v_{dyn}^{loose}$ ) and "tight" ( $v_{dyn}^{tight}$ ) cuts (see section 2). Final results are then presented as:

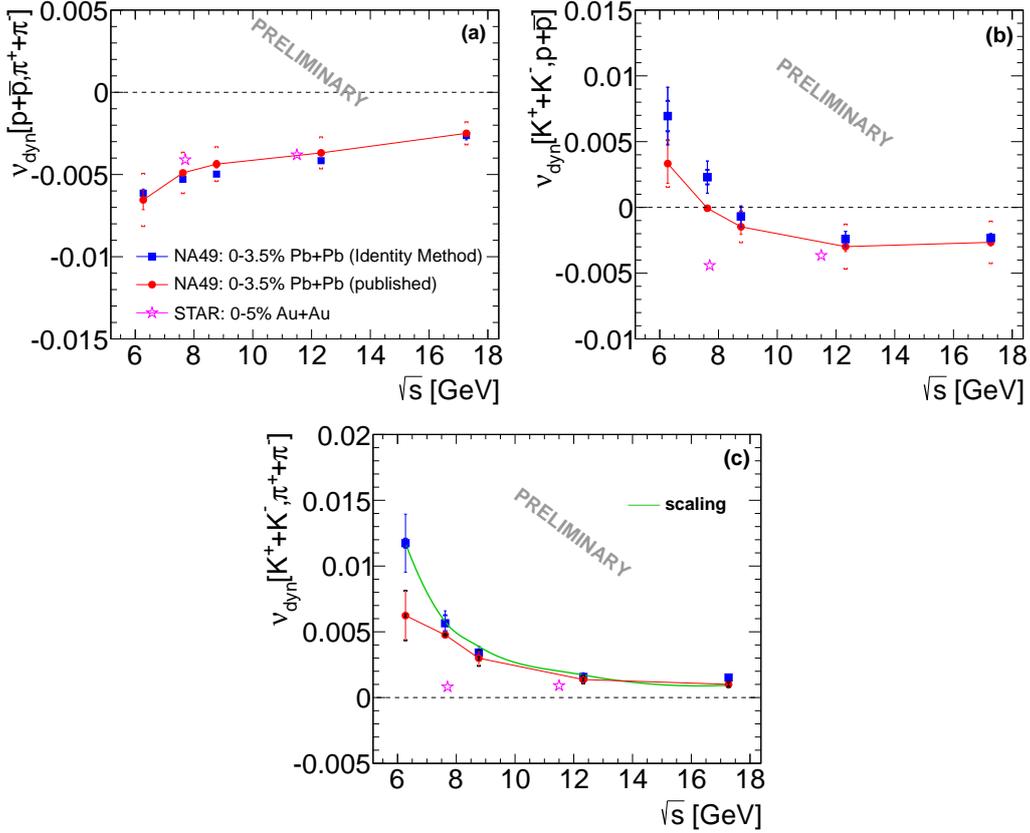
$$v_{dyn}[A, B] = \frac{\langle v_{dyn}^{loose} \rangle + \langle v_{dyn}^{tight} \rangle}{2} \pm \sigma_{stat} \pm \frac{1}{2} \left| \langle v_{dyn}^{loose} \rangle - \langle v_{dyn}^{tight} \rangle \right| \quad (4.2)$$

where  $\sigma_{stat}$  denotes the statistical error.

These results show trends similar to previously published NA49 findings indicated by red circles in Fig. 2. We conclude that the increasing trend of the excitation functions of  $v_{dyn}[K^+ + K^-, p + \bar{p}]$  and  $v_{dyn}[K^+ + K^-, \pi^+ + \pi^-]$  towards low energies are confirmed by our new independent analysis. We also present in Fig. 2 the STAR results for central Au+Au collisions (purple stars) from the RHIC Beam Energy Scan (BES) program [11]. In the next section we investigate the differences in  $v_{dyn}[K^+ + K^-, p + \bar{p}]$  and  $v_{dyn}[K^+ + K^-, \pi^+ + \pi^-]$  between the corresponding STAR and NA49 results.

## 5. Acceptance dependence of obtained results

A motivation for our work presented in this section is to shed light on differences between the results from STAR and NA49 on fluctuations of identified hadrons. The differences between NA49



**Figure 2:** (Color Online) Energy dependence of (a)  $v_{dyn}[p + \bar{p}, \pi^+ + \pi^-]$ , (b)  $v_{dyn}[K^+ + K^-, p + \bar{p}]$  and (c)  $v_{dyn}[K^+ + K^-, \pi^+ + \pi^-]$ . Results from the Identity Method for central Pb+Pb data of NA49 are shown by blue squares. Published NA49 results are indicated by red circles. Purple stars represent results of the STAR collaboration for central Au+Au collisions. In addition, for the case (c), we present the energy dependence of Eq.(5.1), which is consistent with the experimentally established trend.

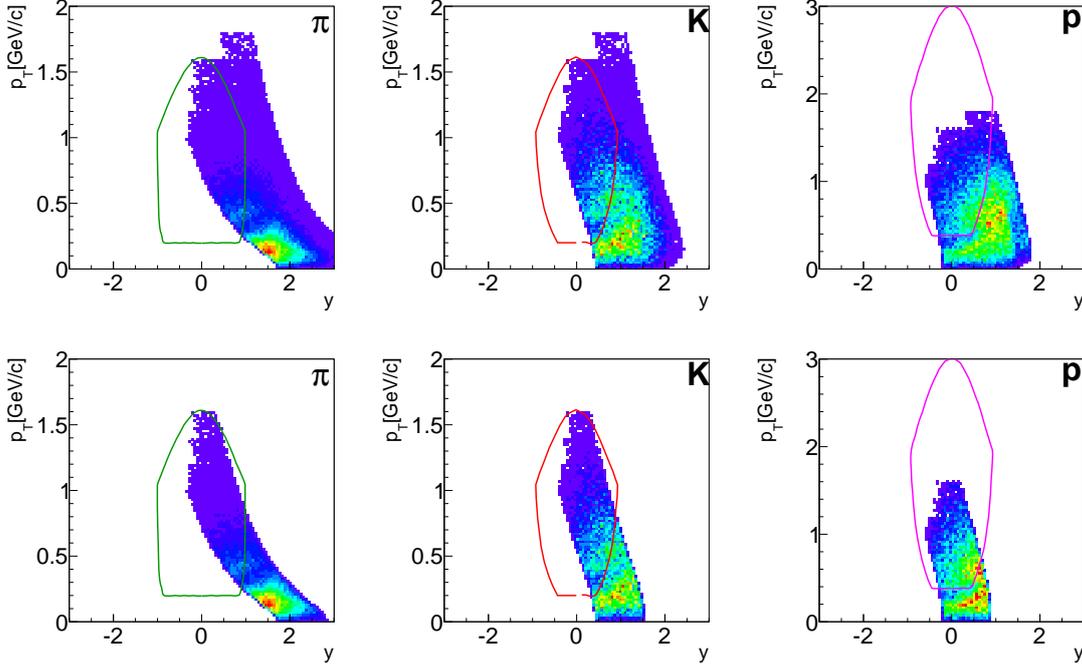
and the STAR results are partially related to the definition of  $v_{dyn}$ . Indeed, it was found in Ref. [15] that  $v_{dyn}$  exhibits an intrinsic dependence on the multiplicities of accepted particles, which leads to a trivial energy dependence of  $v_{dyn}$ :

$$v_{dyn}(x) = v_{dyn}(\sqrt{s}) \frac{\left[ \frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle} \right]_x}{\left[ \frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle} \right]_{\sqrt{s}}}, \quad (5.1)$$

where  $\sqrt{s}$  is a reference energy at which the value of measured  $v_{dyn}$  can be chosen.

The energy dependence of Eq.( 5.1), with reference energy of  $\sqrt{s} \approx 6.3$  GeV (corresponding to 20A GeV laboratory momentum), is illustrated in Fig. 2(c). It is consistent with the experimental observation. This remarkable consistency underlines the importance of phase space coverage (acceptance) of the experimental apparatus.

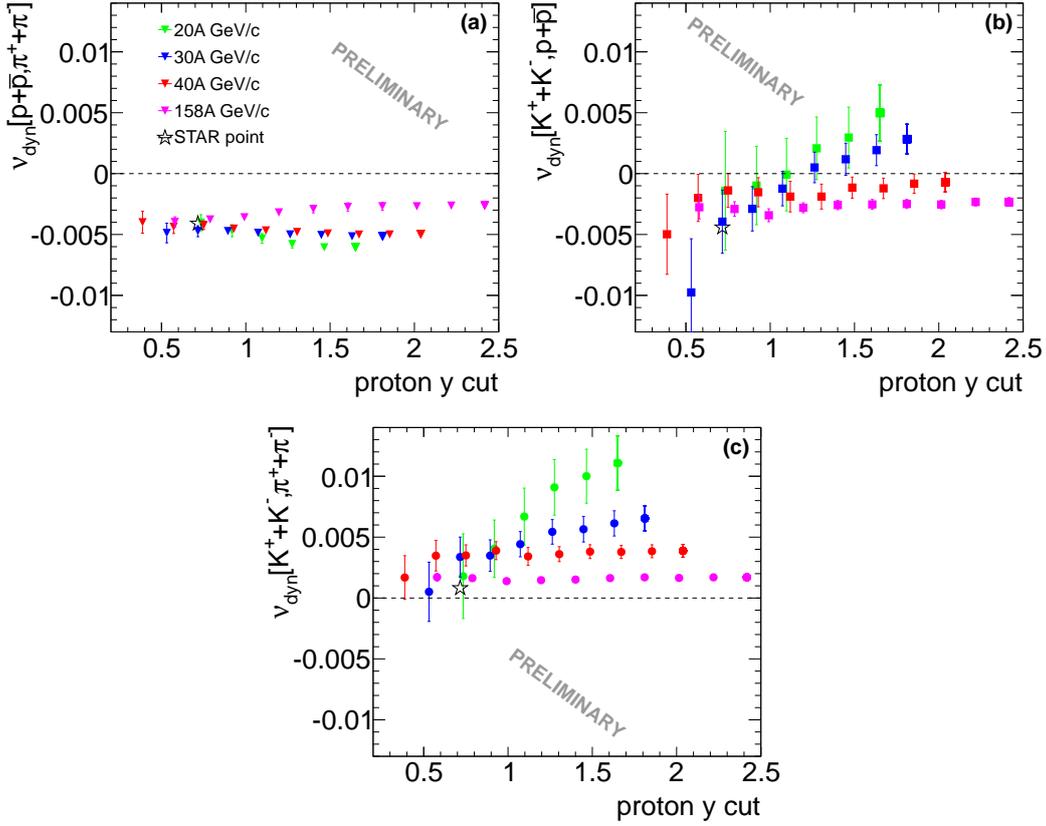
As we mentioned in the section 1 published NA49 and STAR measurements were performed in significantly different phase space regions. As an example we illustrate in Fig. 3 the phase space coverage of pions, kaons and protons at 30A GeV projectile momenta in the acceptance of



**Figure 3:** (Color Online) The phase space coverage for pions, kaons and protons in the acceptance of the NA49 experiment for Pb+Pb collisions at 30A GeV/c. Upper and lower panels illustrate an example of two different selections of phase-space coverage. The solid lines represent the acceptance of the STAR apparatus at corresponding beam energy.

NA49. In the same figure the acceptance of the STAR apparatus at corresponding center-of-mass energy is presented by colored lines. We, therefore, studied the acceptance dependence of  $v_{dyn}$  by performing the analysis in different acceptance regions reducing them from forward rapidity to mid-rapidity. Technically different acceptance regions were selected by applying several upper momentum cuts for each reconstructed track and calculating the corresponding maximum rapidity of protons at  $p_T=0$ . Thereafter we will call this quantity a proton rapidity cut. In Fig. 4 we present the dependence of  $v_{dyn}$  for different combinations of particles at different energies. At 20A and 30A GeV/c  $v_{dyn}[K^+ + K^-, p + \bar{p}]$  and  $v_{dyn}[K^+ + K^-, \pi^+ + \pi^-]$  show a strong acceptance dependence, and eventually hit the STAR point in a particular acceptance bin. Interestingly the acceptance dependence weakens above 30A GeV/c where no difference was observed with STAR. It is also remarkable that  $v_{dyn}[p + \bar{p}, \pi^+ + \pi^-]$  shows little acceptance dependence. This detailed study of fluctuation results in different acceptance bins appears to explain the difference between the STAR BES and NA49 measurements.

Finally we would like to come back to the discussion of  $v_{dyn}$  from a different point of view. To reveal the underlying physics from the study of event-by-event fluctuations, the obtained fluctuation signals from heavy-ion (A+A) collisions should be compared systematically to a reference from nucleon-nucleon (N+N) collisions at corresponding energies per nucleon. It is however important to properly take into account trivial differences between A+A and N+N collisions e.g., in



**Figure 4:** (Color Online) Acceptance dependence of (a)  $v_{dyn}[p+\bar{p},\pi^+\pi^-]$ , (b)  $v_{dyn}[K^++K^-,p+\bar{p}]$  and (c)  $v_{dyn}[K^++K^-,\pi^+\pi^-]$  in central Pb+Pb collisions of NA49 (triangles, squares, dots). Stars show measurements of the STAR collaboration. Results are plotted versus the maximum of the proton rapidity at  $p_T=0$ .

the size of the colliding systems. In thermodynamics the quantities which are proportional to the volume of the system are called extensive quantities. For example, the mean number of particles in a relativistic gas within the Grand Canonical Ensemble (GCE) is an extensive quantity. The ratio of two extensive quantities does not depend on the system volume and is referred to as an intensive quantity. The studied fluctuation measure  $v_{dyn}$  is even more complicated as it is inversely proportional to the number of wounded nucleons, in a wounded nucleon model. An additional complication in the experimental study of fluctuations in A+A collisions are unavoidable volume fluctuations from event to event. These additional sources of fluctuations may well mask the fluctuation of interest. For example, the scaled variance defined in Eq. 3.4 is sensitive to the volume fluctuations. On the other hand, within the GCE, the value of  $v_{dyn}$  does not depend on volume fluctuations. However it does depend on the volume itself as mentioned above. It is rather suggestive to normalize  $v_{dyn}$  in order to remove the volume dependence but still keep it independent of volume fluctuations (within the GCE):

$$v_{dyn}[A,B]^{Scaled} = \frac{v_{dyn}[A,B]}{\frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle}} \quad (5.2)$$

Note that Eq.( 5.2) is a reformulated variant of Eq.( 5.1).

The fluctuation measure  $\Sigma^{AB}$  proposed in Ref. [16] depends neither on the system volume nor on its fluctuations (in the GCE or the wounded nucleon model formulation). Interestingly, using the definition of  $v_{dyn}$  from Eq.( 4.1) and the scaling property for  $v_{dyn}$  (see Eq. 5.2), one finds that  $\Sigma$  and the rescaled  $v_{dyn}$  are practically the same fluctuation measures:

$$v_{dyn}[A,B]^{Scaled} = \Sigma^{AB} - 1 \quad (5.3)$$

## 6. Summary

In summary we investigated several scenarios to understand differences between the NA49 and STAR results for excitation functions of  $v_{dyn}[p, K]$  and  $v_{dyn}[\pi, K]$ . First we replaced the event-by-event particle identification procedure previously used in NA49 by a different approach, the Identity Method, which reconstructs the moments of the multiplicity distributions of each particle type and thus allows to calculate the fluctuation measure  $v_{dyn}$ . The increasing trend of  $v_{dyn}[p, K]$  and  $v_{dyn}[\pi, K]$  towards lower energies reported in previous publications of NA49 in terms of  $\sigma_{dyn}$  is confirmed by this analysis. Second, the detailed study of  $v_{dyn}$  revealed a strong acceptance dependence at low energies for  $v_{dyn}[p, K]$  and  $v_{dyn}[\pi, K]$ . We conclude that the different energy dependence of  $v_{dyn}$  measured by NA49 and STAR ( BES program for central Au+Au collisions) is due to the different phase space coverages. As an outlook we mention that using the reconstructed second moments the energy and centrality dependence of the fluctuation measures proposed in Ref. [16] can be studied as well. These quantities are better suited for phase transition studies because within the grand canonical ensemble they depend neither on the volume no on its fluctuations.

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## References

- [1] Z. Fodor and S. D. Katz, JHEP 0404, 050 (2004)
- [2] C. Alt et al. (NA49 Collab.), Phys. Rev. C77, 024903 (2008)
- [3] A. Rustamov, Central Eur. J. Phys. 10, 1267-1270 (2012), arXiv:1201.4520v1 [nucl-ex] (2012)
- [4] M. Gazdzicki and M. Gorenstein, Acta. Phys. Pol. B30, 2705 (1999)

- [5] S. Afanasiev et al. (NA49 Collab.), Nucl. Instrum. Meth. A430, 210 (1999)
- [6] N. Antoniou et al. (NA61/SHINE Collab.), CERN-SPSC-2006-034, CERN-SPSC-P-330 (2006)
- [7] L. Landau, Journal of Physics (USSR), vol. 8, p. 201 (1944)
- [8] M. van Leeuwen, PhD thesis, NIKHEFF, Amsterdam (2003), CERN EDMS Id 816033
- [9] C. Alt et al. (NA49 Collab.), Phys. Rev. C79, 044910 (2009)
- [10] T. Anticic et al. (NA49 Collab.) Phys. Rev. C83, 061902(R) (2011)
- [11] T. J Tarnowsky (STAR Collab.) J. Phys. G: Nucl. Part. Phys. 38 124054 (2011)
- [12] C. Pruneau, S. Gavin, S. Voloshin, PRC 66, 044904 (2002)
- [13] M. Gazdzicki et al., Phys. Rev. C83, 054907 (2011); M. I. Gorenstein, Phys. Rev. C84, 024902 (2011)
- [14] A. Rustamov, M. I. Gorenstein, Phys. Rev. C86, 044906 (2012)
- [15] V. Koch and T. Schuster, Phys. Rev. C81, 034910 (2010)
- [16] M. I. Gorenstein and M. Gazdzicki, Phys.Rev. C84, 014904 (2011)